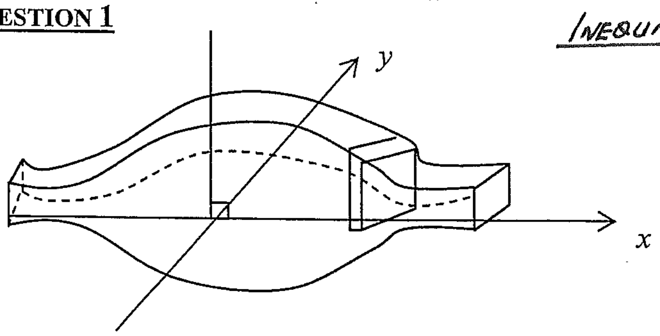
SCOTS COLLEGE - EXT 2 - PRACTICE PAPER (JUNE 09)

TOPICS: VOLUMES; COMPLEX NUMBERS; POLYNOMIALS

INEQUALITIES.

QUESTION 1



A solid is formed with a square cross section at right angles to the x axis as shown above. The equation of the base is $4y^2+(xy)^2=1$.

(i) Show that the volume of a thin slice is approximately equal to $\frac{4}{4+x^2}\delta x$, where δx is the thickness of the slice.

(ii) Hence calculate the volume of the solid bounded by $-a \le x \le a$.

(iii) What is the limiting value of the volume of the solid as a approaches infinity?

QUESTION 2

a. The region bounded by the curve $y=x^3$, the line $y=1$ and the y axis is rotated about the line $y=-1$. Using cylindrical shells, find the volume of the solid of revolution. [4 MARKS]

b. A solid has a base in the form of a circle with centre the origin and radius 6 units. If every section perpendicular to the x axis is an equilateral triangle, show that the volume of the solid is $288\sqrt{3}$ cubic units. [4 MARKS]

c. **(i)** Sketch the ellipses $x^2+25y^2=100$ and $25x^2+y^2=100$ on the same diagram, showing their intercepts with the coordinate axes. [2 MARKS]

(You do not need to show their foci or directrices).

(ii) A child's spinning top is made by revolving the total area enclosed by these ellipses around the x axis. [3 MARKS]

Show that its volume is given by:

$$V = 4\pi\left[\int_0^a x\sqrt{100-25x^2}\,dx + \tfrac{1}{5}\int_a^{10} x\sqrt{100-x^2}\,dx\right]$$

where a is the positive x coordinate of the points of intersection of the ellipses.

(iii) Hence or otherwise find the volume in terms of a. [2 MARKS]

Question 3

a. Given that $z=-1+\sqrt{3}i$ is a root of the equation $z^4-4z^2-16z-16=0$, find the other roots.
[4 marks]

b. Given that α, β and γ are the roots of the cubic equation $x^3-x^2+5x-3=0$, find:
[5 marks]

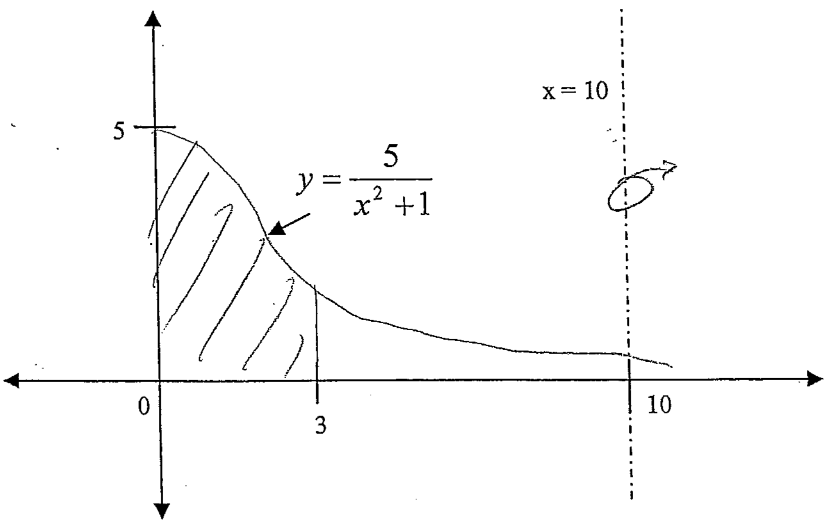
(i) the equation whose roots are $-\alpha, -\beta, -\gamma$.

(ii) the equation whose roots are $\alpha\beta$, $\alpha\gamma$, $\beta\gamma$.

c. For what values of m does the equation $x^3-12x^2+45x-m=0$ have three distinct solutions?
[6 marks]

QUESTION 4

(a)



A circular flange is formed by rotating the region bounded by the curve $y=\dfrac{5}{x^2+1}$, the x axis and the lines x = 0 and x = 3, about the line x = 10.
(All measurements are in cm)

(i) Use the **method of cylindrical shells** to show the volume generated V cm^3 of the flange is given by $V = \int_0^3 \frac{(100 - 100x)\pi}{x^2 + 1} dx$.

(ii) Hence find the volume of the flange correct to the nearest cm^3

(b) Use the **method of slices** to find the volume generated when the area bounded by $y = x^2 - 3x^4$ and the x-axis is rotated about the y-axis. Begin with a suitable sketch.

(i) Evaluate the volume.

(ii) Give two reasons why using the **"method of cylindrical shells"** in this question would have been easier.

(iii) Use the **method of cylindrical shells** to evaluate the volume

(c) Show the area of an isosceles right angled triangle with hypotenuse h is given by $\frac{h^2}{4}$

The base of a solid is the region enclosed by the curve $y = 9 - x^2$ and the x-axis.
Each cross-section perpendicular to the y-axis is an isosceles right angled triangle with hypotenuse lying in the base.
Use integration to find the volume of the solid.

Question 5

a. (i) Show that $2 + i$ is a zero of $P(x) = x^3 - 11x + 20$.

(ii) Hence or otherwise solve $x^3 - 11x + 20 = 0$.

b. The equation $x^3 + 2x - 1 = 0$ has roots α, β, γ **[6 marks]**

(i) Find the value of $\Sigma\alpha$ and the value of $\Sigma\alpha\beta$.

(ii) Find the value of $\alpha^3 + \beta^3 + \gamma^3$

(iii) Find the equation of a polynomial with roots $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

c. (i) Find the coordinates of the stationary points of $P(x) = x^3 - 3px^2 + 4q$, where p and q are positive real constants.

(ii) Show that the equation $P(x) = 0$ has three real distinct roots if $p^3 > q$.

Question 6

(a) $\sqrt{3} + i$ is one root of $x^4 + px^2 + q = 0$, where p and q are real.

(i) Find p and q **3**

(ii) Factor $x^4 + px^2 + q$ into quadratic factors with real coefficients. **2**

(b) The quadratic equation $x^2 - x + k = 0$ where k is a real number, has two distinct positive real roots α and β

(i) Show that $0 < k < \frac{1}{4}$ **2**

(ii) Show that $\alpha^2 + \beta^2 = 1 - 2k$ and deduce that $\alpha^2 + \beta^2 > \frac{1}{2}$ **2**

(iii) Show that $\frac{1}{\alpha^2} + \frac{1}{\beta^2} > 8$ **3**