THE SCOTS COLLEGE

Assessment Task

Year 12

Ext-1 Mathematics

HSC Task 1: Tuesday 21st February, Week 4, Term 1 2012

Weighting of Task:10%

Time allowed: 45 minutes

Directions:

- All 12 questions must be attempted on your own paper.
- Questions 1 to 5 are Multiple Choice A, B, C, or D only.
- Write your name on each page.
- Start a new page for questions 6 to 12.
- Show all necessary working in questions 6 to 12.
- Approved calculators may be used.

Topics Assessed

Integration of Trigonometric functions.(Q,9)	**/5**
Solution of trigonometric equations.(Q,3,6,12)	**/8**
Polynomials and Numerical Estimation of Roots.(Q,1,7,10)	**/10**
Inverse Functions and Inverse Trigonometric Functions.(Q,2,4,5,8,11)	**/17**

Total Marks: **/40**

Questions 1 to 5 ONLY are multiple choice (1 mark each).
For questions 1 to 5, write A, B, C or D on your answer sheet.

1. When $x^2+4x+5b$ is divided by x the remainder is -10.

   The value of b is

(A) 2 (B) -12 (C) -2 (D) 0

2. The domain of $y=-2\sin^{-1}(1-4x)$ is

(A) $\frac{-\pi}{2}\le x\le\frac{\pi}{2}$ (B) $\frac{-1}{2}\le x\le\frac{1}{2}$ (C) $0\le x\le\frac{1}{2}$ (D) $\frac{-\pi}{8}\le x\le\frac{\pi}{8}$

3. $2\cos(t-\frac{\pi}{3})$ can be rewritten as

(A) $\cos t-\sqrt{3}\sin t$ (B) $2\cos t-2\cos\frac{\pi}{3}$

(C) $\cos t+\sqrt{3}\sin t$ (D) $\sin t-\sqrt{3}\cos t$

4. $\cos^{-1}(-x)=$

(A) $\pi-\cos^{-1}x$ (B) $\pi+\cos^{-1}x$ (C) $\sin^{-1}x$ (D) $-\sin^{-1}x$

5. $\sin(2\tan^{-1}\frac{2}{3})=$

(A) $\frac{\sqrt{13}}{12}$ (B) $\frac{12}{\sqrt{13}}$ (C) $\frac{6}{\sqrt{13}}$ (D) $\frac{12}{13}$

End of Multiple Choice Section

For Questions 6 to 12 start each question on a new page and show all working.

6. $5\text{Cos}\theta + 12\text{Sin}\theta \equiv \text{RCos}(\theta - \alpha)$ where R>0 and α is acute.

(a) Find the value of R. (2)

(b) Find the size of α to 3 decimal places. (1)

(c) Hence solve to 3 decimal places

$5\text{Cos}\theta + 12\text{Sin}\theta = 13$ for $0 \le \theta \le 4\pi$. (2)

7.

(a) Show that $\sin x = x - 1$ has a root near $x = 2$ (2)

(b) Use Newton's Method once to find a better approximation to this root. (Answer to 2 decimal places) (2)

8. The function $f(x) = 3x - x^3$ has a minimum turning point at (-1,-2) and a maximum turning point at (1,2).

(a) Sketch $y = f(x)$ showing given turning points and intercepts on both axes. (2)

(b) Find the largest domain containing the origin for which $f(x)$ has an inverse function $y = f^{-1}(x)$. (1)

(c) Find the domain and range of $y = f^{-1}(x)$. (2)

(d) Sketch $y = f^{-1}(x)$ clearly showing the end points. (3)

9.

(a) Find $\int \sin^2 3x\,dx$ (2)

(b) Find in terms of π the volume of the solid formed when $y = \cos x$ is rotated about the x-axis from $x = 0$ to $x = \frac{\pi}{2}$. (3)

10.

(a) If α, β, γ are the roots of $x^3 + 4x - 9 = 0$ find $\alpha(\beta + 1) + \beta(\gamma + 1) + \gamma(\alpha + 1)$ (2)

(b) Given that $Q(x) = 4x^3 + kx + 6$ has a root at $x = -3$.

(i) Find k. (1)

(ii) Write $Q(x)$ in the form $(x+3)(ax^2 + bx + c)$ (2)

11. Given the function $f(x) = 2\sin^{-1}(\frac{x}{3})$

(a) Find $f(0)$ (1)

(b) State the domain and range of $y = f(x)$ (2)

(c) Draw the graph of $y = f(x)$ showing the end points. (3)

12. Given $\sqrt{2}\cos\theta = 1$

(a) Write the general solution for this equation in terms of π. (1)

(b) Solve for n = -1 and n = 2. Answer in terms of π. (1)

END OF EXAMINATION

NAME: _____

TEACHER: SOLUTIONS

PART A – MULTIPLE CHOICE ANSWER SHEET (5 MARKS)

Mark the correct answer by filling in the circle. To make a correction, neatly place a cross over the circle and then fill in the correct circle.

EXAMPLE:

	A	B	C	D
	○	⊗	●	○

	A	B	C	D
Question 1	○	○	●	○
Question 2	○	○	●	○
Question 3	○	○	●	○
Question 4	●	○	○	○
Question 5	○	○	○	●

✓ 5/5

Question 6 ..... Name: _____

Teacher: _____

5/5

$5\cos\theta + 12\sin\theta = R\cos(\theta - \alpha)$

a) $R = \sqrt{a^2+b^2}$

$R = \sqrt{5^2+12^2}$

$R = 13$ //

b) $\tan\alpha = \frac{b}{a}$

$\tan\alpha = \frac{12}{5}$

$\alpha = 1.17\ldots$ radians

$\alpha = 1.176$ (3dp)

c) $13\cos(\theta - 1.176) = 13$

$\cos(\theta - 1.176) = 1$

let $\theta - 1.176 = u$ $\cos x = 1$ $x = 0, 2\pi, 4\pi$.

$\therefore \theta - 1.176 = 0$

$\theta = 1.176$

$\theta - 1.176 = 2\pi$

$\theta = 2\pi + 1.176$

$\theta = 1.76 + 4\pi$

$\theta = 4\pi + 1.76$

but $\theta \neq 4\pi + 1.76$ since domain is $0 \leq \theta \leq 4\pi$

$\therefore \theta = 1.176, 2\pi + 1.176$

Question 7

Name:

Teacher:

a) $\sin x = x - 1$ / root near $x = 2$, $\Rightarrow f(x) = \sin x - x + 1 = 0$

when $x = 2$.

Try $f(1) = \sin 1 - 1 + 1 > 0$

$f(3) = \sin 3 - 3 + 1 < 0$

$\therefore$ a root exist near $x = 2$

$\sin 2 = 2 - 1$ ✗

LHS: $\sin 2 = 0.909\ldots$

RHS: $2 - 1 = 1$

LHS ≠ RHS ∴ there must be a root near $x = 2$.

b) $x = x - \frac{f(x_n)}{f'(x_n)}$

$f(x) = \sin x - x + 1$

$f'(x) = \cos x - 1$

let $x = 2$

$x = 2 - \left(\frac{-0.09\ldots}{-1.416\ldots}\right)$ //

$x = 2 - (0.06\ldots)$

$x = 1.94$ (2dp).

Question 8

Name:

Teacher:

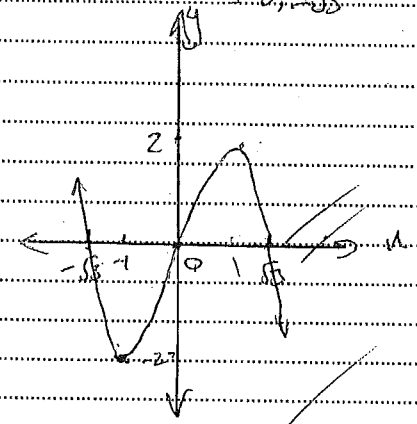
a. ∅ is final intercept.

when $x = 0$, $f(x) = 0$

when $f(x) = 0$, $0 = 3x - x^3$

$0 = x(3 - x^2)$ $\quad x^2 = 3$

$x = 0, \pm\sqrt{3}$ $\quad x = 0$



b. $-1 \le x \le 1$

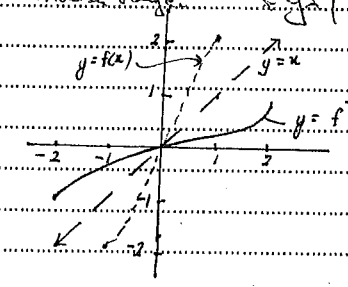
c. Original domain: $-1 \le x \le 1$

original range: $-2 \le y \le 2$

∴ inverse domain: $-2 \le x \le 2$ //

inverse range: $-1 \le y \le 1$

d.



Question 9 — 3/5

Name:

Teacher:

a $\int \sin^2 3x \, dx$

$\int \frac{1}{2}(1-\cos 6x)\, dx$ ✓

$\int \left(\frac{1}{2} - \frac{1}{2}\cos 6x\right) dx$ ✓

$\frac{1}{2}x - \frac{1}{12}\sin 6x + c$ ✓ (2)

(b) $V_x = \pi \int_0^{\frac{\pi}{2}} y^2 \, dx$ where $y = \cos x$

$= \pi \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ (But $\cos 2x = 2\cos^2 x - 1$)

$= \pi \int_0^{\frac{\pi}{2}} \frac{\cos 2x + 1}{2} \, dx$

$= \frac{\pi}{2}\left[\frac{\sin 2x}{2} + x\right]_0^{\pi/2}$

$= \frac{\pi}{2}\left[(0 + \frac{\pi}{2}) - (0+0)\right]$

$= \frac{\pi^2}{4}$ cu. units.

Question 10 — 4/5

Name:

Teacher:

a $\alpha\beta\gamma = -\frac{(-6)}{1}$

$\alpha + \beta + \gamma = 0$

expand: $\alpha\beta + \beta\gamma + \gamma\alpha = 4$

$\alpha\beta\gamma = 9$

$\alpha\beta + \alpha\gamma + \beta\gamma + \gamma\alpha$

$(\alpha\beta + \beta\gamma + \gamma\alpha) + (\alpha + \beta + \gamma) = 4 + 0 = 4$

$\alpha\beta + \beta\gamma + \gamma\alpha = -\frac{5}{a} = 0$ (i)

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = -9$ (ii)

$f = -9 + (-9)$

$f = -18$ ✗ (1)

b i remainder theorem

$Q(-3) = 9(-3)^3 + (-3)(k) + 6$

$Q(-3) = 0 = -108 - 3k + 6$

$102 = -3k$

$k = -34$ ✓

ii $4x^2 - 12x + 3$

$x+3 \overline{)\, 4x^3 + 0x^2 - 33x + 6}$

$4x^3 + 12x^2$

$-12x^2 - 33x$

$-12x^2 - 36x$

$3x + 6$

$3x + 6$

$Q(x) = (x+3)(4x^2 - 12x + 3)$ ✓ (3)

6

Question 11 Name: [illegible]

Teacher:

$f(x) = 2\sin^{-1}\left(\frac{x}{3}\right)$

a) $f(0) = 2\sin^{-1}\left(\frac{0}{3}\right)$

$f(0) = 0$ ✓

b) original inverse

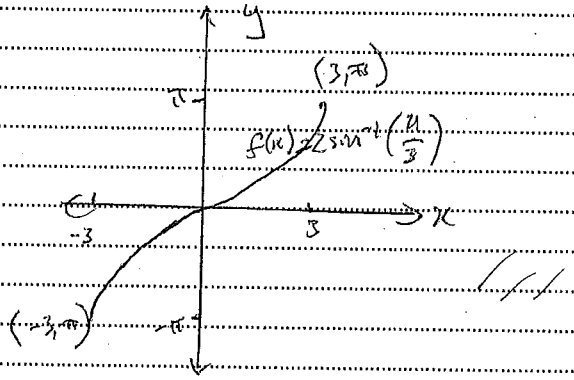
Domain: $-1 \le x \le 1$

Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

New: $-1 \le \frac{x}{3} \le 1 \Rightarrow -3 \le x \le 3$ ✓

New: $-\frac{\pi}{2} \le \frac{y}{2} \le \frac{\pi}{2} \Rightarrow -\pi \le y \le \pi$ ✓

c.



✓✓

(1)

Question 12 Name: [illegible]

Teacher: Lee

a) Given $\sqrt{2}\cos\theta = 1$

$\cos\theta = \frac{1}{\sqrt{2}}$ $\theta = \frac{\pi}{4}$

$\theta = \frac{7\pi}{4}$ ✗

general solution: $2\pi n \pm \frac{\pi}{4}$, $n \in \mathbb{Z}$

b) $2\pi n \pm \frac{\pi}{6} = 2x$ solve for x

when $n = -1$

$-2\pi \pm \frac{\pi}{6} = 2x$

$-2\pi + \frac{\pi}{6} = 2x$ $x = -2\pi - \frac{\pi}{6}$

$x = -\frac{11}{6}\pi$ $x = -2\frac{1}{6}\pi$

when $n = 2$

$4\pi \pm \frac{\pi}{6} = 2x$

$x = 4\pi + \frac{\pi}{6}$ $x = 4\pi - \frac{\pi}{6}$

$x = 4\frac{1}{6}\pi$ $x = 3\frac{5}{6}\pi$

for $n = -1$ $x = -\frac{11\pi}{6}, -\frac{13\pi}{6}$

for $n = 2$ $x = \frac{23\pi}{6}, \frac{25\pi}{6}$

✓ f4