NAME:_____

THE SCOTS COLLEGE

YEAR 11 2 UNIT MATHEMATICS
40% PRELIMINARY ASSESSMENT TASK
Monday 18th September 2006

INSTRUCTIONS:

* Time allowed: **2 hours.**
* **Approved calculators may be used.**
* All questions are to be answered on the paper provided.
* Start each question on a new page.
* **All** necessary working must be shown.
* Marks will not be awarded for careless or poorly arranged work.

Outcomes Being Assessed.

P3 Performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities

P4 Chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

P5 Understands the concept of a function and the relationship between a function and its graph

P6 Relates the derivative of a function to the slope of its graph.

P7 Determines the derivative of a function through routine application of the rules of differentiation.

P8 Understands and uses the language and notation of calculus.

QUESTION 1 **(8 Marks)** **Start a new page.**

a) Calculate $\frac{5.82}{\sqrt{3.71+2.9}}$, give your answer correct to 3 significant figures. (2marks)

b) Rationalise the denominator of $\frac{2}{5-\sqrt{3}}$. (2marks)

c) Express $0.5\dot{7}\dot{6}$ as a fraction. (2marks)

d) Simplify $\frac{4x}{5}-\frac{2x+1}{3}$ (2marks)

QUESTION 2 **(11 Marks)** **Start a new page.**

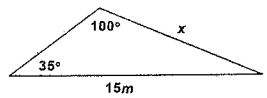
a) Simplify $\frac{x+4}{x^2-4}\times\frac{4x+8}{x^2+x-12}$ (2marks)

b) Graph the solution of $|3x+2|\leq 8$ on a number line. (2marks)

c) Simplify $3\tan^2 60^0+\cos 60^0 \cos ec 30^0$ (2marks)

d) Evaluate $\lim_{x\longrightarrow 3}\frac{x^2-8x+15}{x-3}$ (1mark)

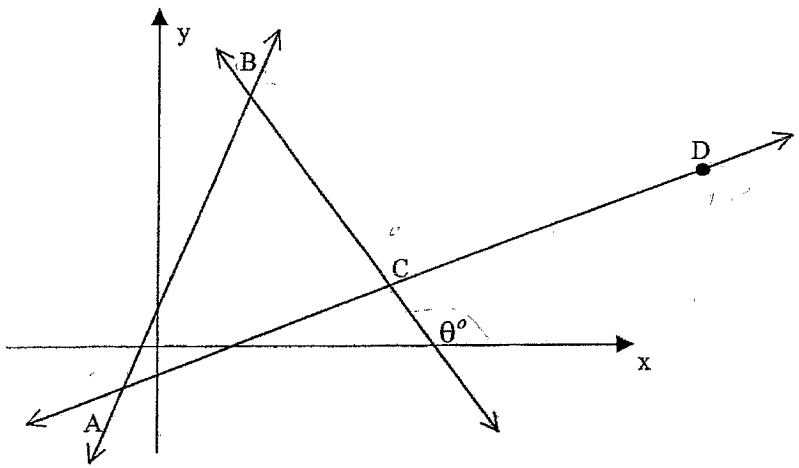
e) Find the length of x correct to 1 decimal place. (2marks)



f) Solve $2\cos\theta^0=-\sqrt{3}$ for $0^0\leq\theta\leq 360^0$ (2marks)

QUESTION 3 **(10 Marks)** **Start a new page.**

The diagram shows the points A(-2,-2), B(2,5), C(4,1) and D(x,4).



a) Calculate the length of AB leaving your answer in surd form. (1mark)

b) Show that the gradient of the line BC is -2. (1mark)

c) Find the angle θ that the line BC makes with the positive direction of the x-axis. Write your answer to the nearest minute. (1mark)

d) D is a point which lies on the line AC. If C is the midpoint of interval AD, show that the co-ordinates of point D are (10,4). (1mark)

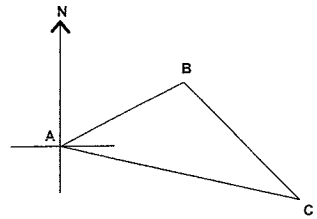
e) Show that the line BC is perpendicular to the line AD. (2marks)

f) Given that the gradient of the line AB is $\frac{7}{4}$.
Find the equation of line AB. (2marks)

g) Calculate the perpendicular distance of the point D from the line AB. Give your answer correct to 1 decimal place. (2marks)

QUESTION 4 **(12 Marks)** **Start a new page.**

a) A ship sails from port A on a bearing of 050^0 for $230km$ to port B. It then changes its bearing to 120^0 and continues sailing for $400km$ to port C.



i) Copy or trace the diagram into your booklet.
Show all the given information on your diagram.

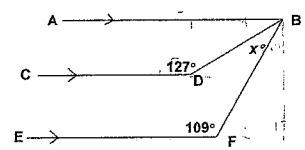
ii) Show that $\angle ABC = 110^0$. (1mark)

iii) Calculate how far, to the nearest km, the ship is from its starting point. (2marks)

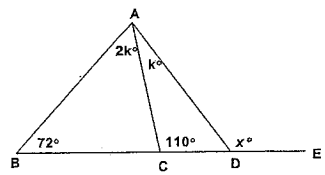
b) Show that $\frac{1+\cot^2\theta}{1+\tan^2\theta} = \cot^2\theta$ (3marks)

c) Find the value of x in each of the following diagrams. **Reasons are not required.**

i)

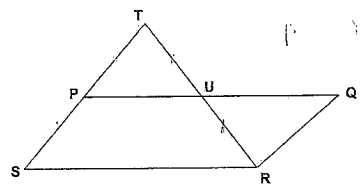


ii)



(3marks)

d) PQRS is a parallelogram and SP is extended to T so that SP=PT. Using congruent triangles or otherwise prove that U is the midpoint of PQ. (3marks)



QUESTION 5 **(13 Marks)** **Start a new page.**

a) On a number plane shade in the region given by the following two conditions
$x^2+y^2 \le 16$ and $x+y \ge 2$ (3marks)

b) For the function $y=3^{-x}+2$ state the domain and range. (2marks)

c) Prove that $y=\sqrt{9-x^2}$ is an even function. (2marks)

d) The function $f(x)$ is defined as

$$f(x)=\begin{cases}\dfrac{4}{x} & for\ -2 \le x \le 2\\ -5x & for\ \ 2 < x \le 5\end{cases}$$

Find $f(3)+f(2)$. (2marks)

e) The focus of a parabola is the point (-2, 1) and the directrix is the line $y=-3$.

i) Sketch the parabola and indicate the co-ordinates of the vertex. (2marks)

ii) Write down the focal length of the parabola. (1mark)

iii) Find the equation of the parabola. (1mark)

QUESTION 6 **(8 Marks)** **Start a new page.**

a) Solve $4^x-9(2)^x+8=0$ (2marks)

b) Find the values for k for which $y=x^2+(k-5)^2x+25$ is positive definite. (2marks)

c) If α and β are the roots of $2x^2-5x+3=0$ find:

i) $\alpha+\beta$

ii) $\alpha\beta$

iii) $(\alpha+2)(\beta+2)$ (4marks)

QUESTION 7 **(7 Marks)** **Start a new page.**

Differentiate with respect to x.

a) $y=5x^3-7x^2+12$ (1mark)

b) $y=4x^2(6x+2)^5$ (3marks)

c) $y=\dfrac{x^2-3}{4x+1}$ (3marks)

QUESTION 8 **(10 Marks)** **Start a new page.**

a) i) If $y=\sqrt{5-x^2}$ show that $\dfrac{dy}{dx}=\dfrac{-x}{\sqrt{5-x^2}}$. (2marks)

ii) Hence, find the equation of the normal to the curve $y=\sqrt{5-x^2}$ at the point (1, 2). (2marks)

b) Consider the curve given by $y=(x+2)(x-1)^2$

i) Find any turning points and determine their nature. (3marks)

ii) Sketch the curve $y=(x+2)(x-1)^2$ for the domain $-3 \le x \le 3$.
Mark clearly on the graph the x and y-intercepts, the co-ordinates of the turning points and the endpoints. (3marks)

END OF PAPER

[SCOTS COLLEGE] SOLUTIONS – YR 11 2-UNIT FINAL. (18/9/06)

Quest ①

a) $2 \cdot 26$

b) $\frac{2(5+\sqrt{3})}{(5-\sqrt{3})(5+\sqrt{3})} = \frac{10+2\sqrt{3}}{5-3} = 5+\sqrt{3}$

c) $x = 0 \cdot 57676767\dot{6}$.

$10x = 5 \cdot 767676767\dot{6}$ ← $990x = 571$

$100x = 57 \cdot 67676767\dot{6}$

$1000x = 576 \cdot 767676767\dot{6}$ ← $x = \frac{571}{990} = \frac{32}{55}$

d) $\frac{12x - 10x - 5}{15} = \frac{2x-5}{15}$.

Quest ②

a) $\frac{x+4}{(x+2)(x-2)} \times \frac{4(x+2)}{(x+4)(x-3)} = \frac{4}{(x-2)(x-3)}$

b) $-8 \le 3x+2 \le 8$

$-10 \le 3x \le 6 \rightarrow -3\frac{1}{3} \le x \le 2$

c) $= 3(\sqrt{3})^2 + (\frac{1}{2}) \times (\frac{2}{1}) = 10$

d) $= \lim_{x \to 3} \frac{(x-3)(x-5)}{(x-3)}$

$= \lim_{x \to 3} (x-5) = -2$.

e) $\frac{x}{\sin 35^\circ} = \frac{15}{\sin 100^\circ} \Rightarrow x \approx 8 \cdot 7$ m

f) $\cos\theta = -\frac{\sqrt{3}}{2}$ (2ND & 3RD quadrants)

& $\cos^{-1}(\frac{\sqrt{3}}{2}) = 30^\circ$

$\therefore \theta = 150^\circ$ or 210°

Question ③

a) $AB = \sqrt{4^2 + 7^2} = \sqrt{65}$

b) $m_{BC} = \frac{(5)-(1)}{(2)-(4)} = \frac{4}{-2} = -2$

c) $\tan\theta = -2 \quad \therefore \theta = 180^\circ - 63^\circ 26'$

$\therefore \theta = 116^\circ 34'$

d) Let $D = (x, y) \therefore C = (\frac{x-2}{2}, \frac{y-2}{2}) = (4, 1)$

$\therefore \frac{x-2}{2} = 4 \rightarrow x = 10$

& $\frac{y-2}{2} = 1 \rightarrow y = 4$.

e) $m_{AD} = \frac{(4)-(-2)}{(10)-(-2)} = \frac{6}{12} = \frac{1}{2}$

& Since $m_{BC} \times m_{AD} = -2 \times \frac{1}{2} = -1$

$\therefore AD \perp BC$

f) $y - 5 = \frac{7}{4}(x-2)$

$4y - 20 = 7x - 14$

$\therefore 7x - 4y + 6 = 0$

g) $p = \frac{|7(10) - 4(4) + 6|}{\sqrt{7^2+4^2}} = \frac{60}{\sqrt{65}} = \frac{12\sqrt{65}}{13}$

$\approx 7 \cdot 4$

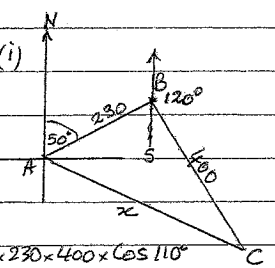
Question ④

ii) $\angle ABS = 50^\circ$

$\angle CBS = 60^\circ$

$\therefore \angle ABC = 110^\circ$

(i)



(iii) $x^2 = 230^2 + 400^2 - 2 \times 230 \times 400 \times \cos 110^\circ$

$= 212900 - 184000(-0 \cdot 34202)$

$\approx 275831 \cdot 7$

$\Rightarrow x \approx 525$ km.

b) LHS $= \frac{\operatorname{Cosec}^2\theta}{\sec^2\theta}$

$= \frac{1}{\sin^2\theta} \div \frac{1}{\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$

$=$ RHS

c) i) $x = 71^\circ - 53^\circ = 18^\circ$

ii) $2k + 72 = 110 \rightarrow k = 19^\circ$

$x = 110 + 19 \rightarrow x = 129^\circ$

Quest ④ con't

d) PQRS is a parallelogram $\therefore$

$\therefore SP = QR$ and $ST \parallel RQ$.

$\therefore$ Since $TP = SP$ (given)

Then $TP = QR$. ——— (*)

Also $\angle PTU = \angle QRU$ (Alternate $\angle$'s $ST \parallel RQ$)

& $\angle PUT = \angle QUR$ (Vert. opp. $\angle$'s)

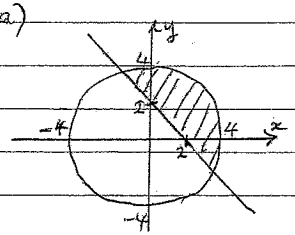
$\therefore \Delta PTU \equiv \Delta QUR$ (AAS)

$\therefore$ all corresponding sides are equal

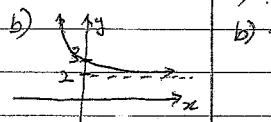
$\Rightarrow PU = UQ \quad \therefore U$ is midpoint of PQ

Question ⑤

a)



b)



$D = \{x : x \text{ is all real nos}\}$

$R = \{y : y \ge 2\}$

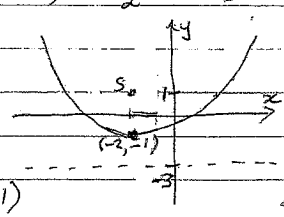
c) $f(x) = \sqrt{9-x^2}$

$f(-x) = \sqrt{9-(-x)^2} = \sqrt{9-x^2} = f(x)$

d) $f(3) + f(2) = -5(3) + \frac{4}{2} = -13$

e) i) $V = (-2, -1)$

ii) $a = 2$



iii) $(x+2)^2 = 4a(y+1)$

$\therefore (x+2)^2 = 8(y+1)$

Question ⑥

a) $2^{2x} - 9(2^x) + 8 = 0$ let $u = 2^x$

$u^2 - 9u + 8 = 0$

$(u-1)(u-8) = 0$

$\therefore u = 1$ or $8 \quad \therefore 2^x = 1 \rightarrow x = 0$

or $2^x = 8 \rightarrow x = 3$

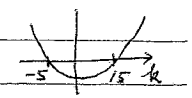
b) Require $\Delta < 0$ and $a > 0$ $(a = 1)$

$\therefore \Delta = (k-5)^2 - 4 \times 25 \times 1 < 0$

need $k^2 - 10k + 25 - 100 < 0$

$k^2 - 10k - 75 < 0$

$(k-15)(k+5) < 0$



$\therefore -5 < k < 15$.

c) i) $\alpha + \beta = \frac{5}{2}$ (ii) $\alpha\beta = \frac{3}{2}$

iii) $\alpha^2\beta + 2\alpha + 2\beta + 4 = (\frac{3}{2}) + 2(\frac{5}{2}) + 4 = 10\frac{1}{2}$

Question ⑦

a) $y' = 15x^2 - 14x$

b) $y' = 8x(6x+2)^5 + 120x^2(6x+2)^4$

$= 8x(6x+2)^4\{6x+2+15x\}$

$= 8x(6x+2)^4(21x+2)$

c) $y' = \frac{(4x+1) \cdot 2x - (x^2-3) \times 4}{(4x+1)^2} = \frac{4x^2+2x+12}{(4x+1)^2}$

Question ⑧

a) i) $y = (5-x^2)^{\frac{1}{2}}$

$y' = \frac{1}{2}(5-x^2)^{-\frac{1}{2}} \times (-2x) = \frac{-x}{\sqrt{5-x^2}}$

ii) at $x = 1 \quad y' = -\frac{1}{2} \leftarrow (m)_t = 2$.

$\therefore y - 2 = 2(x-1) \rightarrow 2x - y = 0$

b) $y' = 1.(x-1)^2 + (x+2) \cdot 2(x-1)$

$= 3(x-1)(x+1) = 0$ when $\begin{cases} x = 1 \\ y = 0 \end{cases}$ $\begin{cases} x = -1 \\ y = 4 \end{cases}$

$y'' = 6x$

$\therefore y''(1) > 0$ MIN T.P. $y''(-1) < 0$ MAX T.P.