Name: _____

THE SCOTS COLLEGE

YEAR 11 MATHEMATICS
ASSESSMENT TASK 3
Thursday 26th July 2007

25% Assessment Task

TOTAL MARKS : 41

INSTRUCTIONS:

* Time allowed: **50 minutes.**
* Approved calculators may be used.
* Start each question on a new page.
* **All necessary working must be shown.**
* Marks will not be awarded for careless or badly arranged work.

Outcomes Being Assessed.

P3 • performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities

P4 • chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

P5 • understands the concept of a function and the relationship between a function and its graph

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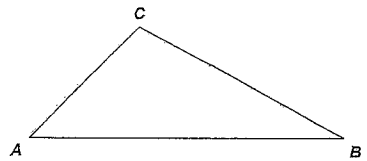
QUESTION 1 **(12 Marks)**

a) In triangle ABC, $AB = 8cm$, $BC = 7cm$ and $CA = 5cm$.



i) Show that $\angle CAB = 60^0$. [2]

ii) Hence find the **exact** area of triangle ABC. [2]

b) A ship leaves Port P and sails on a bearing of 040^0 for 320 nautical miles to reach point A. It then sails on a bearing of 150^0 for 510 nautical miles to reach point B.

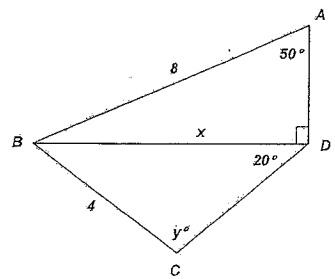
i) Draw a diagram to clearly represent all the given information.

ii) Find the distance between the point B and Port P. Give your answer in nautical miles correct to 1 decimal place.

iii) Use your answer from part ii) to find the bearing of B from P. Give your answer correct to the nearest degree.

[5]

c)



i) Use the given diagram to show that $x = 8\sin 50^0$. [1]

ii) Hence show that $\sin y = 2\sin 20^0 \sin 50^0$ and find the value of y correct to the nearest minute. [2]

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QUESTION 2 **(8 Marks)** **START A NEW PAGE**

a) Find the exact value of $\tan 30^0 + \sec 300^0$ [2]

b) If $\sin x = \frac{2}{3}$ and $90^0 < x < 180^0$, find the exact value of $\cot x$. [2]

c) Solve $2\cos x + 1 = 0$ for $0^0 \le x \le 360^0$ [2]

d) Prove that $\cot\theta + 2\sec\theta = \frac{1-\sin^2\theta + 2\sin\theta}{\sin\theta\cos\theta}$ [2]

QUESTION 3 **(14 Marks)** **START A NEW PAGE**

a) i) Sketch the curve $y = 2x^2 - 9x - 5$ by first finding where the curve crosses the co-ordinate axes.

ii) Find the equation of the axis of symmetry of the curve $y = 2x^2 - 9x - 5$.

iii) Hence, or otherwise, find the co-ordinate of the turning point. [4]

b) Find the values of k for which the expression $kx^2 + (k+3)x + 4$ is positive definite. Explain why there are no values of k for which the expression is negative definite. [3]

c) Find the values of A, B and C for which

$$2x^2 - 3x + 5 = A(x-1)(x-2) + B(x-1) + C$$ [2]

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d) If α and β are the roots of $2x^2 - 6x + 8 = 0$ find the value of

i) $\alpha + \beta$

ii) $\alpha\beta$

iii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ [3]

e) Solve $x^4 + 8x^2 - 9 = 0$ [2]

QUESTION 4 **(7 Marks)** **START A NEW PAGE**

a) Write the equation of the locus of a point P (x, y) that moves so that its distance from the point $(0,3)$ is equal to its distance from the line $y = -3$. [1]

b) i) Write the equation $8y = x^2 - 4x - 28$ in the form $(x-h)^2 = 4a(y-k)$.

ii) Hence, write down the co-ordinates of the focus of this parabola. [3]

c) Sketch the parabola $y^2 = -8(x-2)$. Clearly draw and label the focus, directrix and vertex on your diagram. [3]

END OF PAPER

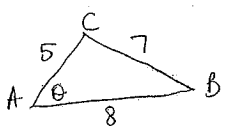
Year 11 Mathematics marking Q1 Q3-LF Q2-KG Q4-AP

Task #3 - 26th July 2007

Time Allowed: 50 minutes (Total: 43 marks)

Question 1 (12 marks)

a) i)



$\cos A = \frac{5^2+8^2-7^2}{2\times5\times8}$

$\cos A = 0.5$ (1)

$A = \cos^{-1} 0.5$

$A = 60°$ (1)

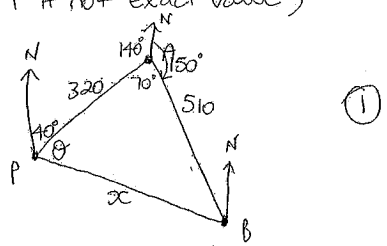
ii) Area $= \frac{1}{2}\times5\times8\times\sin 60$

$= 20\times\frac{\sqrt{3}}{2}$

$= 10\sqrt{3}\ cm^2$ (2)

(-1 if not exact value)

b) i)



(1)

ii) $x^2 = 320^2 + 510^2 - 2\times320\times510\cos 70°$

$x^2 = 250\,864.6252$ (1)

$x = 500.8638....$

$x = 500.9$ nautical miles (1)

iii) $\frac{\sin\theta}{510} = \frac{\sin 70}{500.9}$

$\sin\theta = \frac{\sin 70}{500.9}\times510$

$\sin\theta = 0.95676...$ (1)

∴ Bearing of B from P is

$= 40 + 73$

$= 113°$ (1)

c) i) $\sin 50 = \frac{x}{8}$

$\therefore x = 8\sin 50°$ (1)

ii) $\frac{\sin y}{x} = \frac{\sin 20}{4}$

$\sin y = \frac{\sin 20}{4}\times x$

from i)

$\sin y = \frac{\sin 20}{4}\times8\sin 50$ (1)

$\therefore \sin y = 2\sin 20\sin 50$

$\sin y = 0.5240...$

$y = 31.6013...$

$= 31°36'$

Question 2 (10 marks)

a) $\tan 30 + \sec 300$

$= \frac{1}{\sqrt{3}} + \frac{1}{\cos 300}$

$= \frac{1}{\sqrt{3}} + \frac{1}{\cos 60}$

$= \frac{1}{\sqrt{3}} + \frac{1}{\frac{1}{2}}$ (1)

$= \frac{1}{\sqrt{3}} + 2$

b) $\sin x = \frac{2}{3}$

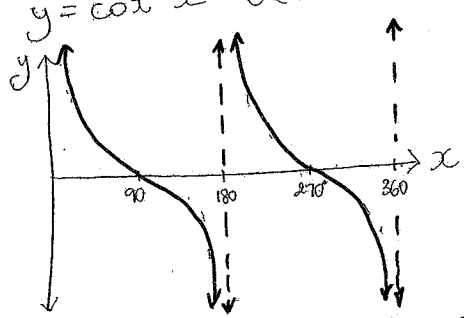
$3^2 - 2^2 = 5$

$\sin x$ $90° < x < 180°$

$\tan x < 0 \therefore \tan x = -\frac{2}{\sqrt{5}}$ (1)

$\therefore \cot x = -\frac{\sqrt{5}}{2}$ (1)

c) $y = \cot x \quad 0° \le x \le 360°$



d) $2\cos x + 1 = 0$ for $0° \le x \le 360°$

$\cos x = -\frac{1}{2}$

S | A
T | C

$\therefore x = (180-60), (180+60)$ (1)

$= 120°, 240°$ (1)

e) Prove

$\cot\theta + 2\sec\theta = \frac{1-\sin^2\theta+2\sin\theta}{\sin\theta\cos\theta}$

RHS $= \frac{\cos^2\theta + 2\sin\theta}{\sin\theta\cos\theta}$

$= \frac{\cos^2\theta}{\sin\theta\cos\theta} + \frac{2\sin\theta}{\sin\theta\cos\theta}$ (1)

$= \frac{\cos\theta}{\sin\theta} + \frac{2}{\cos\theta}$

$= \cot\theta + 2\sec\theta$ (1)

$= LHS$

Question 5 (14 Marks)

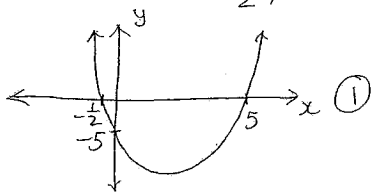
a) i) $y = 2x^2 - 9x - 5$.

y-int is $y = -5$.

x-int is $2x^2 - 9x - 5 = 0$

$(2x+1)(x-5) = 0$

$x = -\frac{1}{2}, x = 5$ (1)



(1)

ii) $x = -\frac{b}{2a} = \frac{9}{4}$

$\therefore x = 2.25$

iii) $y = 2(\frac{9}{4})^2 - 9(\frac{9}{4}) - 5$

$= -15$

$\therefore$ turning point is $(2.25, -15\frac{1}{8})$

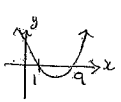
b) If positive definite

$k > 0$ and $\Delta < 0$

$(k+3)^2 - 4k \times 4 < 0$

$k^2 + 6k + 9 - 16k < 0$

$k^2 - 10k + 9 < 0$



$(k-9)(k-1) < 0$ (1)

$1 < k < 9$ (1)

There are no values for negative definite because if negative definite $k < 0$ and this cannot happen if $\Delta < 0$ since $1 < k < 9$. (1)

c) $2x^2 - 3x + 5 \equiv A(x-1)(x-2) + B(x-1) + C$

$\equiv A(x^2 - x - 2x + 2) + B(x-1) + C$

$\equiv Ax^2 - 3Ax + 2A + Bx - B + C$

(1) $\equiv Ax^2 + (-3A + B)x + 2A - B + C$

$\therefore A = 2 \quad -3A + B = -3 \quad 2A - B + C = 5$

$-6 + B = -3 \quad 4 - 3 + C = 5$

(1) $B = 3 \quad C = 4$

d) $2x^2 - 6x + 8 = 0$

i) $\alpha + \beta = -\frac{b}{a} = \frac{6}{2} = 3$ (½)

ii) $\alpha\beta = \frac{c}{a} = \frac{8}{2} = 4$ (½)

iii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$= \frac{\beta^2 + \alpha^2}{\alpha^2\beta^2}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$= \frac{3^2 - 2(4)}{4^2}$

$= \frac{9-8}{16}$

$= \frac{1}{16}$.

e) $x^4 + 8x^2 - 9 = 0$

let $m = x^2$

$m^2 + 8m - 9 = 0$

$(m+9)(m-1) = 0$

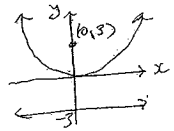
$m = -9 \quad m = 1$

$\therefore x^2 = -9 \quad x^2 = 1$

$x = \pm\sqrt{-9} \quad x = \pm\sqrt{1}$

no solutions. $x = \pm 1$

Question 7 (7 Marks)

a) 

$x^2 = 4ay$

$x^2 = 12y$ (1)

b) i) $8y = x^2 - 4x - 28$

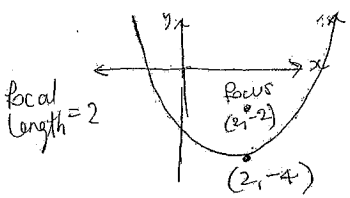
$x^2 - 4x = 8y + 28$

$x^2 - 4x + 4 = 8y + 28 + 4$

$(x-2)^2 = 8y + 32$

$(x-2)^2 = 8(y+4)$ (2)

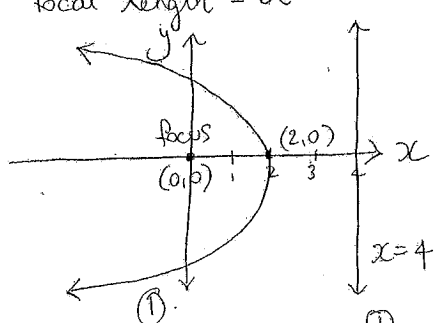
ii) focus = $(2, -2)$ (1)



c) $y^2 = -8(x-2)$

vertex = $(2,0)$ (1)

focal length = 2



(1) (1)