Name : ____ ______________________

ST GEORGE CHRISTIAN BROS.

Yr 11 EXTENSION 1 MATHEMATICS 2014
ASSESSMENT TASK #1

Date : Wednesday 19th March 2014.
Time Allowed : 50 minutes.

INSTRUCTIONS :

- Show all necessary working out or full marks may not be given.
- Keep all your answers in numerical order.
- Clearly label all your work.
- Write in blue or black ink. Complete diagrams in pencil.
- Please submit this question paper with your answer booklet(s).

Final Mark for This Task	/59	%

MARKING GRIDS – For Teacher Use Only

QUESTION NUMBER	1 – 3	4	5	6a	6b	7abd	7c	8	9ab	9c	10
Knowledge and Skills Marks	3		2	3		6		6	6		
Reasoning and Communication Marks		15			5		4			3	6

Total Knowledge and Skills Marks = 26	44 %
Total Reasoning and Communication Marks = 33	56 %

SECTION I – MULTIPLE CHOICE QUESTIONS (1 Mark each)

Choose the letter that corresponds to your choice of answer.
WRITE THE LETTER OF YOUR ANSWER IN YOUR ANSWER BOOKLET.
Do **NOT** write your answer in the question booklet.

QUESTION 1 (1 Mark)

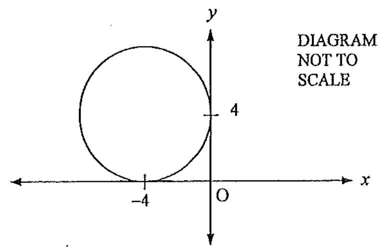


DIAGRAM NOT TO SCALE

The equation of the circle shown on the left is:

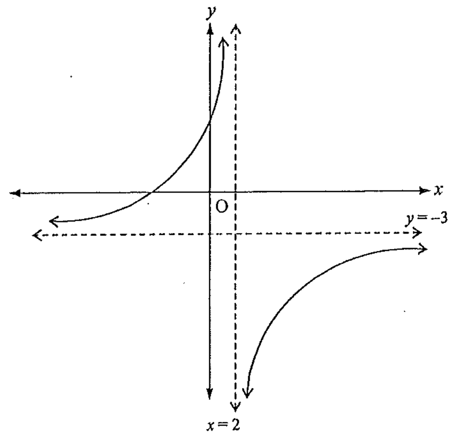
A) $(x+4)^2+(y-4)^2=16$

B) $(x+4)^2+(y+4)^2=16$

C) $(x-4)^2+(y-4)^2=16$

D) $(x-4)^2+(y-4)^2=16$

QUESTION 2 (1 Mark)



The equation of the hyperbola shown on the left could be:

A) $y+3=\frac{1}{x+2}$

B) $y+3=\frac{1}{x-2}$

C) $y-3=\frac{1}{x-2}$

D) $y-3=\frac{1}{x+2}$

QUESTION 3 (1 Mark)

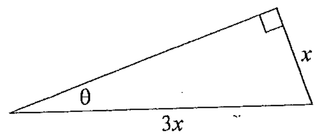


DIAGRAM NOT TO SCALE

The size of angle θ is:

A) 18° 26’ B) 18° 43’

C) 19° 28’ D) 19° 47’

Extended Answer Questions begin on the next page…

SECTION 2 – EXTENDED ANSWER QUESTIONS

Begin each question on a new page

QUESTION 4 (15 Marks)

For the following functions :

- i) sketch the graph showing all the important features including any asymptotes and/or intercepts if they exist.
- ii) find the domain.
- iii) find the range.

a) $y = 8^x - 2$ (5 Marks)

b) $y - 1 = -\sqrt{4-(x-3)^2}$ (5 Marks)

c) $y = |6x - x^2|$ (5 Marks)

QUESTION 5 (2 Marks)

Determine whether the following functions are odd, even or neither.

a) $y = x^5 - 10x^3 + 9x$

b) $y = \dfrac{x}{x^3 - 1}$

QUESTION 6 (8 Marks)

a) Find the point(s) of intersection of the graphs $y = x^2 - 4$ and $y = 2 - x$. (3 Marks)

b) Sketch the intersection of the regions $y \le x^2 - 4$ and $y > 2 - x$. Show working to justify your answer. (5 Marks)

QUESTION 7 (10 Marks)

a) Write the equation $x^2 - 40x + y^2 + 16y + 175 = 0$ in the general form for an equation of a circle i.e. $(x-a)^2 + (y-b)^2 = r^2$. (3 Marks)

b) State the centre and radius of this circle. (1 Mark)

c) Sketch this circle on a number plane. Ensure you indicate the intercept(s). (4 Marks)

d) State the domain and range of this circle. (2 Marks)

QUESTION 8 (6 Marks)

a) For what values of x is the function $f(x) = \dfrac{x^2 - 2x - 3}{x^2 - 9}$ discontinuous? (1 Mark)

b) Evaluate the following limits:

i) $\lim\limits_{x \to 3} \dfrac{x^2 - 2x - 3}{x^2 - 9}$ (3 Marks)

ii) $\lim\limits_{x \to \infty} \dfrac{x^2 - 2x - 3}{x^2 - 9}$ (2 Marks)

QUESTION 9 (9 Marks)

a) A beam of light from the top of a lighthouse has an angle of depression of 34°12' to a nearby reef. The reef is 110m from the base of the lighthouse.

i) Draw a diagram of the information required to calculate h, the height of the lighthouse. Indicate the angle of depression with the symbol "θ". (1 Mark)

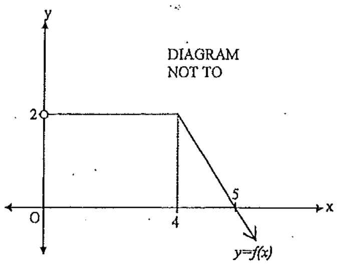
ii) Calculate h, the height of the lighthouse correct to 1 decimal place. (3 Marks)

b) Solve for x in the following equation: (2 Marks)

$\cos(9x - 24)^\circ = \sin 78^\circ$

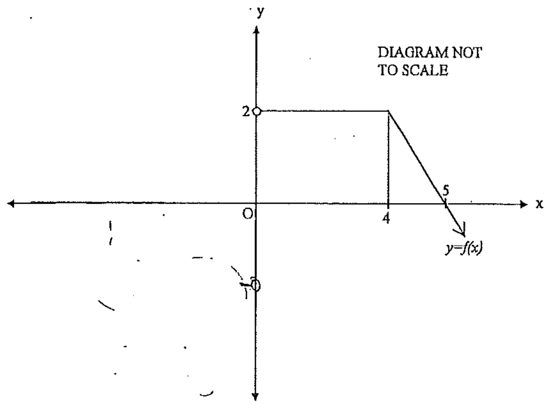
c) Sketch $y = \cos x$ on the domain $0^\circ \le x \le 360^\circ$ (3 Marks)

QUESTION 10 (6 Marks)

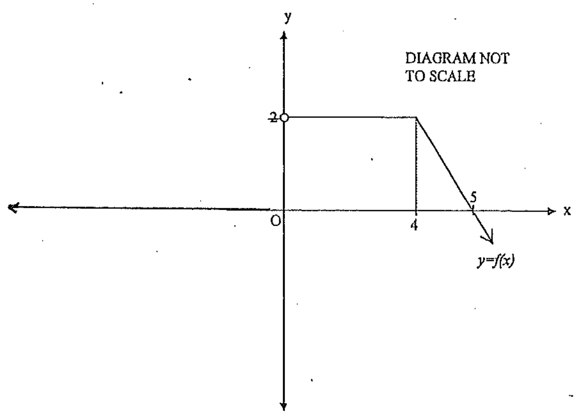


The diagram above shows part of a function. **On the diagrams below** NOT IN YOUR ANSWER BOOKLET complete the sketch of the function if it is : (SHOW ALL THE NECESSARY FEATURES)

a) An odd function (3 Marks)



b) An even function (3 Marks)



c) In your answer booklet write the rule for the **even** function i.e. $f(x) =$..... (5 Marks)

END OF ASSESSMENT TASK

Please check your work carefully to ensure that you have not made any careless mistakes.

YEAR 11 EXTENSION 1 MATHEMATICS (2014) ASSESSMENT TASK #3 SOLUTIONS

Q1 A

2 B

3 $\sin\theta = \frac{opp}{hyp} \doteq \frac{x}{3x} = \frac{1}{3}$ $\theta = \sin^{-1}(\frac{1}{3}) \doteq 19°28'$ C

Q4 a) i) x-intercepts ⇒ y=0

$0 = 8^x - 2$ (+2)

$2 = 8^x$

$8^x = 2$

$(2^3)^x = 2$

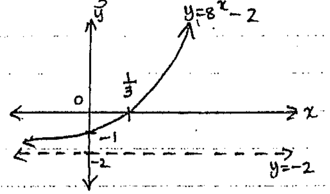
$2^{3x} = 2^1$

$3x = 1$ (÷3)

$x = \frac{1}{3}$

y-intercepts ⇒ x=0

$y = 8^0 - 2 = 1 - 2 = -1$



ii) Domain: all real x

iii) Range: $y > -2$

b) i) $y - 1 = -\sqrt{4-(x-3)^2}$

$(y-1)^2 = 4 - (x-3)^2$ (+ $(x-3)^2$)

$(x-3)^2 + (y-1)^2 = 4$

NB: This the whole circle. The graph required is the lower semicircle

Centre = (3, 1) radius $= \sqrt{4} = 2$

x-intercepts ⇒ y=0

$(x-3)^2 + (0-1)^2 = 4$

$(x-3)^2 + 1 = 4$ (−1)

$(x-3)^2 = 3$

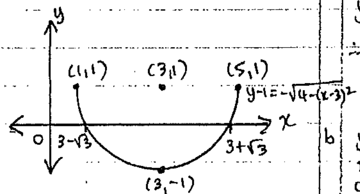
$x - 3 = \pm\sqrt{3}$ (+3)

$x = 3 \pm \sqrt{3}$

y-intercepts ⇒ x=0

$y - 1 = \sqrt{4-(0-3)^2} = \sqrt{4-9} = \sqrt{-5}$

∴ no y-intercepts



ii) Domain: $1 \le x \le 5$

iii) Range: $-1 \le y \le 1$

c) i) $y = |6x - x^2| = |x(6-x)|$

x-intercepts ⇒ y=0

$0 = x(6-x)$

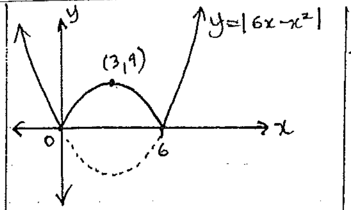
∴ $x = 0,\ x = 6$

y-intercepts ⇒ x=0

$y = |6x - x^2| = |6\times0 - 0^2| = 0$

when x = 3

$y = |6\times3 - 3^2|$



ii) Domain: all real x

iii) Range: $y \ge 0$

Q5 a) $f(x) = x^5 - 10x^3 + 9x$

$f(-x) = (-x)^5 - 10(-x)^3 + 9(-x)$

$= -x^5 - 10x - x^3 - 9x$

$= -(x^5 - 10x^3 + 9x)$

$= -f(x)$ ∴ odd

b) $f(x) = \frac{x}{x^3-1}$

$f(-x) = \frac{-x}{(-x)^3 - 1}$

$= \frac{-x}{-x^3-1}$ (÷−1, ÷−1)

$= \frac{x}{x^3+1}$

$\ne f(x)$

$\ne -f(x)$ ∴ neither

Q6 a) $y = x^2 - 4$ ①

$y = 2 - x$ ②

① = ②

$x^2 - 4 = 2 - x$ (+x −2, −2 +x)

$x^2 + x - 6 = 0$ a+b=1, ab=−6, 3, −2

$x^2 + 3x - 2x - 6 = 0$

$x(x+3) - 2(x+3) = 0$

$(x+3)(x-2) = 0$

$x + 3 = 0$ or $x - 2 = 0$

$x = -3$ $x = 2$

subs into ②

when x=−3: $y = 2 - -3 = 5$

when x=2: $y = 2 - 2 = 0$

∴ points of intersection are: (−3, 5) and (2, 0)

b) $y = x^2 - 4$

x-intercepts ⇒ y=0

$0 = x^2 - 4$ (+4)

$4 = x^2$

$x^2 = 4$

$x = \pm 2$

y-intercepts ⇒ x=0

$y = 0^2 - 4 = -4$

$y = 2 - x$

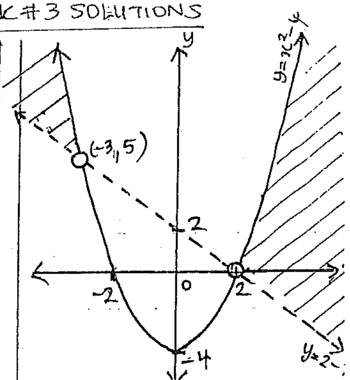
x-intercepts ⇒ y=0

$0 = 2 - x$ (+x)

$x = 2$

y-intercepts ⇒ x=0

$y = 2 - 0 = 2$



Test $y \le x^2 - 4$ $y > 2 - x$

(0,0) $0 \nleq 0^2 - 4$ ✗

(0,3) $3 \nleq 0^2 - 4$ ✗

(0,−5) $-5 \le 0^2 - 4$ ✓ $-5 > 2 - 0$

(3,0) $0 \le 3^2 - 4$ ✓ $0 > 2 - 3$

(−4,7) $7 \le (-4)^2 - 4$ ✓ $7 > 2 - -4$

Q7 a) $x^2 - 40x + y^2 + 16y + 175 = 0$ (−175, −175)

$x^2 - 40x + (-20)^2 + y^2 + 16y + 8^2 = -175 + (-20)^2 + 8^2$

$(x-20)^2 + (y+8)^2 = 289$

$(x-20)^2 + (y+8)^2 = 17^2$

b) centre = (20, −8)

radius = 17

c) x-intercepts ⇒ y=0

$(x-20)^2 + (0+8)^2 = 289$

$(x-20)^2 + 64 = 289$ (−64, −64)

$(x-20)^2 = 225$

$x - 20 = \pm\sqrt{225} = \pm 15$ (+20, +20)

$x = 5, 35$

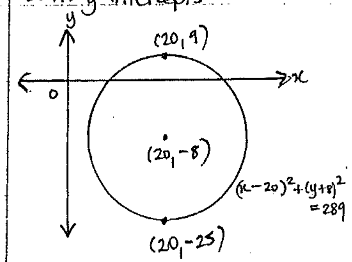
y-intercepts ⇒ x=0

$(0-20)^2 + (y+8)^2 = 289$

$400 + (y+8)^2 = 289$ (−400, −400)

$(y+8)^2 = -111$

∴ no y-intercepts



d) Domain: $3 \le x \le 37$

Range: $-25 \le y \le 9$

Q8 a) $x^2 - 9 = 0$ (+9, +9)

$x^2 = 9$

$x = \pm\sqrt{9}$

$x = \pm 3$

b) i) $x^2 - 2x - 3$ a+b=−2, ab=−3, −3, 1

$= x^2 + x - 3x - 3$

$= x(x+1) - 3(x+1)$

$= (x+1)(x-3)$

$x^2 - 9 = (x+3)(x-3)$

$\lim_{x\to3} \frac{x^2-2x-3}{x^2-9}$

$= \lim_{x\to3} \frac{(x+1)(x-3)}{(x+3)(x-3)}$

$= \lim_{x\to3} \frac{x+1}{x+3}$

$= \frac{3+1}{3+3}$

$= \frac{4}{6}$

$= \frac{2}{3}$

ii) $\lim_{x\to\infty} \frac{x^2-2x-3}{x^2-9}$ (÷x^2, ÷x^2)

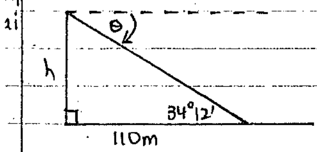
$= \lim_{x\to\infty} \frac{\frac{x^2}{x^2} - \frac{2}{x} - \frac{3}{x^2}}{\frac{x^2}{x^2} - \frac{9}{x^2}}$

$= \lim_{x\to\infty} \frac{1 - \frac{2}{x} - \frac{3}{x^2}}{1 - \frac{9}{x^2}}$

$= \frac{1 - 0 - 0}{1 - 0}$

$= 1$

Q9 a) i)



ii) $\tan\theta = \frac{opp}{adj}$

$\tan 34°12' = \frac{h}{110}$

$h = 110 \tan 34°12'$

$\doteq 74.7559...$

$\doteq 74.8$ m (1 dp)

b) $\cos(9x - 24) = \sin 78°$

$\cos(90 - \theta) = \sin\theta$

∴ $\theta = 78°$

$9x - 24 = 90 - \theta$

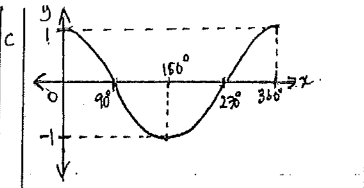
$= 90 - 78$

$= 12$ (+24, +24)

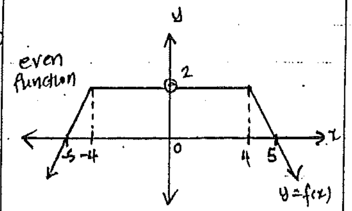
$9x = 36$ (÷9, ÷9)

$x = 4$

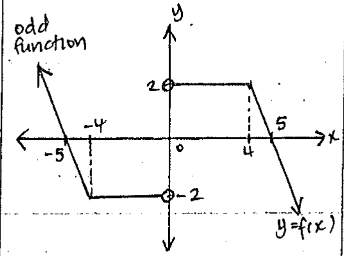
c)



Q10 b) even function



a) odd function



c) even function

RHS passes through (4, 2) & (5, 0)

$m = \frac{rise}{run} = \frac{-2}{1} = -2$

$y = mx + b$

$0 = -2\times5 + b$ (+10, +10)

$10 = b$

∴ equation is $y = -2x + 10$

LHS passes through (−5, 0)

$m = \frac{rise}{run} = \frac{2}{1} = 2$

$y = mx + b$

$0 = 2\times-5 + b$ (+10, +10)

$10 = b$

∴ $b = 10$

∴ equation is $y = 2x + 10$

middle section equation is $y = 2$ except at $x = 0$.

$$\therefore f(x) = \begin{cases} y = 2x + 10, & x < -4 \\ y = 2, & -4 \le x < 0,\ 0 < x \le 4 \\ y = -2x + 10, & x > 4 \end{cases}$$