Name : ________________

Year 11
Semester 1 Examination
2014

Extension 1 Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time - $1\frac{1}{2}$ hours.
- Write using a blue or black pen.
- Complete diagrams in pencil
- Approved calculators may be used.
- Answer the Multiple Choice Questions on the Answer Sheet provided
- Answer questions in Section II in the Answer Booklets provided
- Begin each question of Section II in a new booklet.
- All necessary working should be shown for questions in Section II.
- Marks will be deducted for careless or untidy work.
- This exam question paper must be submitted with your answer booklets at the end of this examination.

Total marks (42)

- Attempt all questions.

SECTION I (6 Marks)

- Questions 1 – 6 consist of 6 Multiple Choice Questions worth 1 Mark each
- Allow about 15 minutes for this section

SECTION II (36 Marks)

- Questions 7 – 9 inclusive are all worth 12 Marks each
- Allow about 1 hour 15 minutes for this section

For Teacher use only:

Section I	Section II	Q7	Q8	Q9	Total	
/6		/12	/12	/12	/42	%

SECTION I

Total marks (6)
Attempt Questions 1– 6
Allow about 15 minutes for this section

FILL IN THE LETTER OF YOUR ANSWER ON THE ANSWER SHEET

QUESTION 1

Which of the following is equal to $\frac{1}{2\sqrt{5}-\sqrt{3}}$?

A) $\frac{2\sqrt{5}-\sqrt{3}}{7}$

B) $\frac{2\sqrt{5}+\sqrt{3}}{7}$

C) $\frac{2\sqrt{5}-\sqrt{3}}{17}$

D) $\frac{2\sqrt{5}+\sqrt{3}}{17}$

QUESTION 2

Which expression is a correct factorisation of a^3-27?

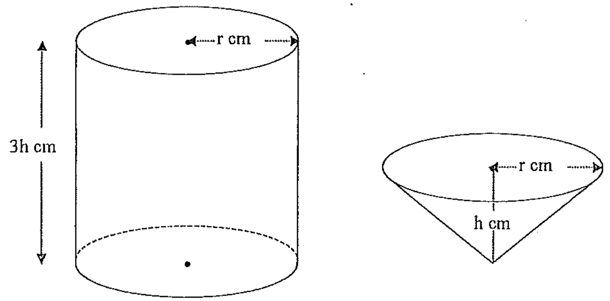
A) $(a-3)(a^2-3a+9)$

B) $(a-3)(a^2-6a+9)$

C) $(a-3)(a^2+3a+9)$

D) $(a-3)(a^2+6a+9)$

QUESTION 3



How many times must the cone be filled with water and emptied into the cylinder in order to fill the cylinder?

A) 3

B) 6

C) 9

D) 27

QUESTION 4

In the circle below, O is the centre and points A, B , C, D and T lie on the circumference. A tangent is drawn from point P touching the circle at T. If ∠PTB = 120°, then the value of $x + y + z$ would equal

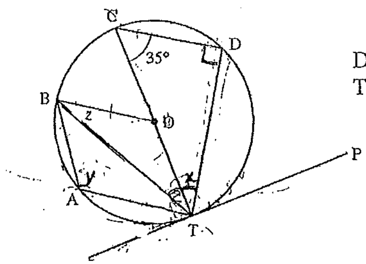


DIAGRAM NOT TO SCALE

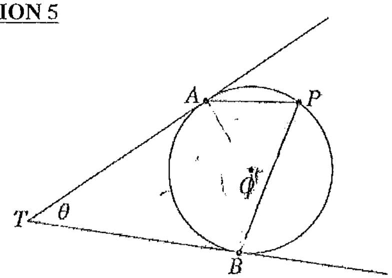
A) 155°

B) 180°

C) 205°

D) 265°

QUESTION 5



The points A, B and P lie on a circle centred at O. The tangents to the circle at A and B meet at the point T, and $\angle ATB = \theta$.

What is $\angle APB$ in terms of θ ?

A) $\dfrac{\theta}{2}$

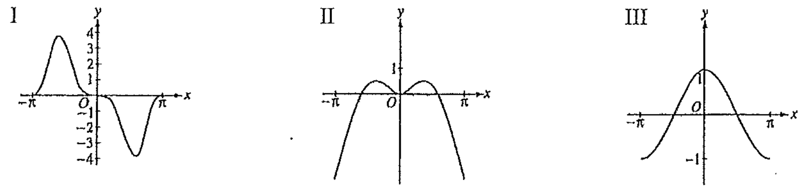
B) $90^\circ - \dfrac{\theta}{2}$

C) θ

D) $180^\circ - \theta$

QUESTION 6

The graphs of three functions are shown below. Which graph represents an odd function?



A) I Only

B) II Only

C) III Only

D) None of the above

END OF SECTION I

SECTION II

Total marks (36)
Attempt Questions 7 – 9
Allow about 1 hour 15 minutes for this section.

Answer all questions
Start each question in a new booklet.
Write your name and question number on the front of every booklet

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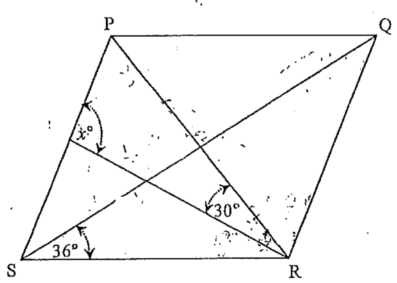
QUESTION 7 **Marks**

a) Simplify $2\sqrt{50}-2\sqrt{18}$ **1**

b) Use mathematical working to show that $0\cdot3\dot{4}\dot{5}=\frac{19}{55}$ **2**

c) PQRS is a rhombus. Find the value of x. You do not have to give reasons. **1**



d) Fully factorise :

i) p^3-2p^2-p+2 **1**

ii) $x^2+xy-6y^2$ **1**

Question 7 continues on the next page...

QUESTION 7 (Continued) **Marks**

e) Solve simultaneously : $2x-y-3=0$, $y=1+6x-3x^2$ **3**

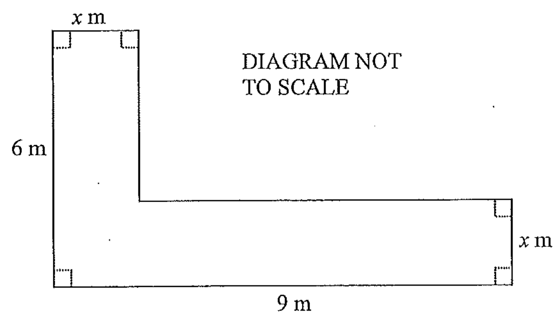
f) Solve $\dfrac{3}{2u^2-7u+3}-\dfrac{2}{u-3}=1$ **3**

End of Question 7

QUESTION 8 Please begin this work in a new booklet. **Marks**

a) Solve $\dfrac{1-4x}{x+2} \leq 5$ **4**

b) Find all the possible values of x given that the area of the figure below is 50m^2. All angles are right angles. **3**

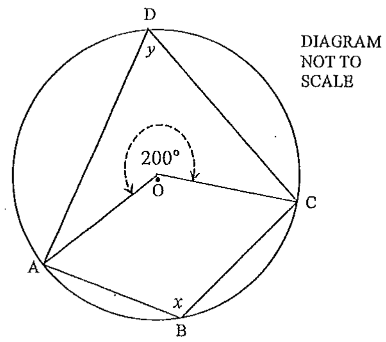


c) Solve $|2n-5|+|n-3|>1$ **5**

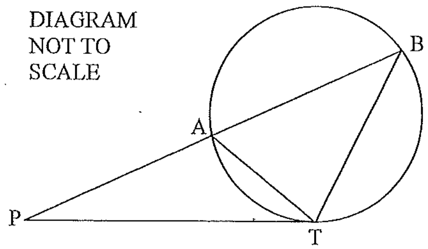
End of Question 8

QUESTION 9 Please begin this work in a new booklet. **Marks**

a) O is the centre of the circle. Reflex $\angle AOC = 240°$ **2**
Find the values of x and y giving reasons.



b)



BA is a chord of the circle produced to P.
PT is a tangent to the circle at T.

Prove that

i) $\Delta PTA \parallel\!| \Delta PBT$ **3**

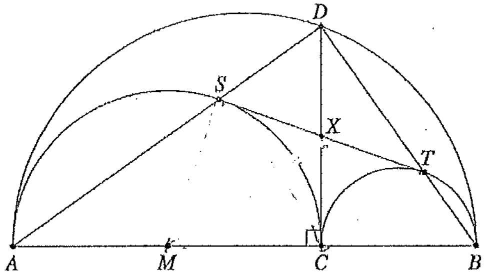
ii) $PT^2 = PA.PB$ **1**

Question 9 continues on the next page...

QUESTION 9 (Continued) **Marks**

c) The diagram shows a large semicircle with diameter AB and two smaller semi-circles with diameters AC and BC, respectively, where C is the point on the diameter AB. The point M is the centre of the semi-circle with diameter AC.

The line perpendicular to AB through C meets the largest semi-circle at the point D. The points S and T are the intersections of the lines AD and BD with the smaller semi-circles. The point X is the intersection of the lines CD and ST.



i) Explain why CTDS is a rectangle. 2

ii) Show that $\triangle MXS$ and $\triangle MXC$ are congruent. 3

iii) Show that the line ST is a tangent to the semicircle with diameter AC. 1

END OF EXAMINATION

Please check your work carefully to eliminate careless mistakes

EAR 11 EXTENSION 1 2014 SEMESTER 1 EXAMINATION SOLUTIONS

1) $\frac{1}{2\sqrt{5}-\sqrt{3}} \times \frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}}$

$= \frac{2\sqrt{5}+\sqrt{3}}{(2\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{2\sqrt{5}+\sqrt{3}}{20-3}$

$= \frac{2\sqrt{5}+\sqrt{3}}{17}$ D

2) $a^3 - 27 = a^3 - 3^3$

$= (a-3)(a^2+a\times 3+3^2)$

$= (a-3)(a^2+3a+9)$ C

3) $V_{cylinder} = \pi r^2 \times 3h = 3\pi r^2 h$

$V_{cone} = \frac{1}{3}\pi r^2 h$

$\frac{V_{cylinder}}{V_{cone}} = \frac{3\pi r^2 h}{\frac{1}{3}\pi r^2 h} = \frac{3}{\frac{1}{3}} =$ C

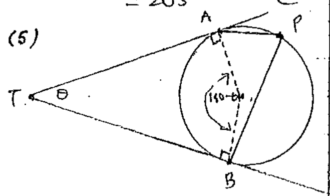
4) $x = 180-90-35 = 55^\circ$

$z = 120-55-35 = 30^\circ$

$y = 120^\circ$

$\therefore x+y+z = 55+120+30 = 205^\circ$ C

(5)



In AOBT

$\angle AOB = 360-90-90-\theta = 180-\theta$

$\angle APB = \frac{1}{2}(180-\theta) = 90-\frac{\theta}{2}$ B

6) odd function ⇒ point symmetry ∴ I only B

7) $2\sqrt{50} - 2\sqrt{18} = 2\times\sqrt{25}\times\sqrt{2} - 2\times\sqrt{9}\times\sqrt{2}$

$= 2\times 5\times\sqrt{2} - 2\times 3\times\sqrt{2}$

$= 10\sqrt{2} - 6\sqrt{2}$

$= 4\sqrt{2}$

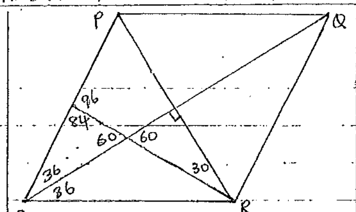
Let $x = 0.3\dot{4}\dot{5} = 0.34545...$

$100x = 34.54545...$

$99x = 34.5 - 0.3 = 34.2$

$x = \frac{34.2}{99} \times \frac{10}{10} = \frac{342}{990} \div 3 = \frac{114}{330} \div 2 = \frac{57}{165} \div 3 = \frac{19}{55}$ $\therefore 0.3\dot{4}\dot{5} = \frac{19}{55}$

c



$x = 96^\circ$

di $p^3-2p^2-p+2 = p^2(p-2)-(p-2)$

$= (p-2)(p^2-1)$

$= (p-2)(p+1)(p-1)$

ii $x^2+xy-6y^2$

$= x^2+3xy-2xy-6y^2$

$= x(x+3y)-2y(x+3y)$

$= (x+3y)(x-2y)$

e $2x-y-3=0$ ①

$y = 1+6x-3x^2$ ②

sub ② into ①

$2x-(1+6x-3x^2)-3=0$

$2x-1-6x+3x^2-3=0$

$3x^2-4x-4=0$ $a+b=-4$, $ab=-12$, $-6, 2$

$3x^2-6x+2x-4=0$

$3x(x-2)+2(x-2)=0$

$(x-2)(3x+2)=0$

$x-2=0$, $x=2$ $3x+2=0$, $3x=-2$, $x=-\frac{2}{3}$

when $x=2$: $y = 1+6\times 2-3\times 2^2 = 1$

when $x=-\frac{2}{3}$: $y = 1+6\times -\frac{2}{3} - 3\times(-\frac{2}{3})^2 = -4\frac{1}{3}$

f. $2u^2-7u+3$ $a+b=-7$, $ab=6$, $-6,-1$

$= 2u^2-6u-u+3$

$= 2u(u-3)-(u-3)$

$= (u-3)(2u-1)$

$\frac{3}{2u^2-7u+3} - \frac{2}{u-3} = 1$

$\frac{3}{(u-3)(2u-1)} - \frac{2}{u-3} = 1$

$\frac{3}{(u-3)(2u-1)} - \frac{2(2u-1)}{(u-3)(2u-1)} = \frac{1(u-3)(2u-1)}{(u-3)(2u-1)}$

$\frac{3-2(2u-1)}{(u-3)(2u-1)} = \frac{(u-3)(2u-1)}{(u-3)(2u-1)}$

$3-2(2u-1) = (u-3)(2u-1)$

$3-4u+2 = 2u^2-u-6u+3$

$0 = 2u^2-3u-2$ $a+b=-3$, $ab=-4$, $-4, 1$

$2u^2-3u-2=0$

$2u^2-4u+u-2=0$

$2u(u-2)+1(u-2)=0$

$(u-2)(2u+1)=0$

$u-2=0$, $u=2$ $2u+1=0$, $2u=-1$, $u=-\frac{1}{2}$

Q8 a $\frac{1-4x}{x+2} \le 5$ $x \ne -2$

$\times (x+2)^2$

$(1-4x)(x+2) \le 5(x+2)^2$

$x+2-4x^2-8x \le 5(x^2+4x+4)$

$-4x^2-7x+2 \le 5x^2+20x+20$

$0 \le 9x^2+27x+18$

$9x^2+27x+18 \ge 0$

$\div 9$ $x^2+3x+2 \ge 0$ $a+b=3$, $ab=2$, $2,1$

$x^2+2x+x+2 \ge 0$

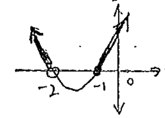
$x(x+2)+(x+2) \ge 0$

$(x+2)(x+1) \ge 0$

$(x+2)(x+\ ) = 0$

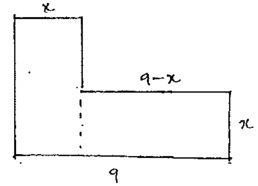
$x+2=0$, $x=-2$ $x+1=0$, $x=-1$

$x \ne -2$ ∵ denominator cannot be zero.



$\therefore x < -2$ or $x \ge -1$

b



$A = 6\times x + (9-x)\times x$

$50 = 6x+9x-x^2$

$50 = 15x-x^2$

$x^2-15x+50=0$ $a+b=$, $ab=$, $-5,-10$

$x^2-5x-10x+50=0$

$x(x-5)-10(x-5)=0$

$(x-5)(x-10)=0$

$x-5=0$, $x=5$ $x-10=0$, $x=10$

$x \ne 10$m since this longer than the longest side of 9m.

$\therefore x = 5$m only

c $|2n-5|+|n-3| > 1$

(+/+) $2n-5+n-3 > 1$

$3n-8 > 1$

$3n > 9$

$n > 3$

(+/−) $2n-5-(n-3) > 1$

$2n-5-n+3 > 1$

$n-2 > 1$

$n > 3$

(−/+) $-(2n-5)+n-3 > 1$

$-2n+5+n-3 > 1$

$-n+2 > 1$

$-n > -1$

$n < 1$

(−/−)

$-(2n-5)-(n-3)>1$

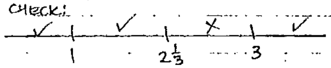
$-2n+5-n+3>1$

$-3n+8>1$
$\quad\; -8 \quad -8$

$-3n>-7$
$\div -3 \quad \div -3$

$n<2\frac{1}{3}$

CHECK:



$n=0$ LHS $=|2\times 0-5|+|0-3|$
$=|-5|+|-3|$
$=5+3$
$=8>1$ ✓

$n=2$ LHS $=|2\times 2-5|+|2-3|$
$=|-1|+|-1|$
$=1+1$
$=2>1$ ✓

$n=2\frac{1}{2}$ LHS $=|2\times 2\frac{1}{2}-5|+|2\frac{1}{2}-3|$
$=|0|+|-\frac{1}{2}|$
$=0+\frac{1}{2}$
$=\frac{1}{2}\not>1$ ✗

$n=4$ LHS $=|2\times 4-5|+|4-3|$
$=|3|+|1|$
$=3+1$
$=4>1$ ✓

∴ final solution is $n<2\frac{1}{3}$ or $n>3$

$x=200°\div 2$ (∠ at circumference $=\frac{1}{2}$ ∠ at centre standing on the same arc)
$=100°$

$y=180-x$ (opposite ∠'s of a cyclic quadrilateral are supplementary)
$=180-100$
$=80°$

Data: BA is a chord of a circle produced to P. PT is a tangent to the circle, touching it at T.
TA and BT are joined.

Aim: To prove i) $\Delta PTA \,|||\, \Delta PBT$
ii) $PT^2=PA.PB$

Proof: i) In ΔPTA and ΔPBT
$\angle PTA=\angle PBT$ (∠ in the alternate segment)
$\angle P$ is common
$\angle PAT=180°-\angle P-\angle PTA$ (∠ sum of Δ PTA)
$=180°-\angle P-\angle PBT$ (∠ sum of Δ PBT)
$=\angle PTB$
∴ $\Delta PTA \,|||\, \Delta PBT$ (equiangular)

corresponding sides of similar Δ's are in the same ratio

∴ $\frac{PT}{PB} = \frac{PA}{PT}$

$PT^2=PA.PB$

$\angle ADB=90°$ (∠ in semicircle with diameter AB)

$\angle ASC=90°$ (∠ in semicircle with diameter AC)

∴ $\angle DSC=90°$ ($\angle DSA=180°$ as it is a straight line)

$\angle CTB=90°$ (∠ in semicircle with diameter CB)

∴ $\angle CTD=90°$ ($\angle DTB=180°$ as it is a straight line)

$\angle SCT=360°-\angle ADB-\angle DSC-\angle CTD$ (∠ sum of quadrilateral CSDT)
$=360°-90°-90°-90°$
$=90°$

∴ CTDS is a rectangle as all ∠'s are right ∠'s.

ii In ΔMXS and ΔMXC
$MS=MC$ (radii of circle, centre M)
$SX=XC$ (diagonals of a rectangle are equal and bisect one another)
MX is common
∴ $\Delta MXS \equiv \Delta MXC$ (SSS)

NOTE: You cannot assume at this stage that SX is a tangent unless you have proved it to be so.
∴ you cannot say they are congruent by RHS.

iii In ΔMSX and ΔMCX
$\angle MSX=\angle MCX=90°$
(corresponding ∠'s of congruent Δ's)
∴ $ST \perp MS$
radius $MS \perp ST$ at point of contact with circle, S. ∴ ST must be a tangent since the tangent ⊥ a radius at point of contact.