THE SCOTS COLLEGE

HSC TRIAL EXAMINATION

Mathematics Extension 1

AUGUST 2012

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen
- Approved calculators may be used
- Section I-(10 marks) Question 1-10, should be completed on the separate multiple choice answer sheet provided.
- Section II-(60 marks) Questions 11-14 should be completed in separate answer booklets showing all necessary working.
- A table of standard integrals is provided at the back of this paper.

Total marks -70

- Attempt Questions 1-14

Section I

10 Marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section.

Section II

60 Marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a \neq 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Section I - (10 Marks)

(Answer A, B, C or D on your multiple choice answer sheet).

1. The solution to the inequality $\dfrac{x^2-9}{x} \geq 0$ is:

(A) $x \geq 3$ (B) $0 \leq x \leq 3$ or $x \geq -3$ (C) $x \leq -3$ (D) $-3 \leq x \leq 0$ or $x \geq 3$

2. The $\lim\limits_{x \to 0} \dfrac{\sin(\pi x)}{3x}$ is :

(A) $\dfrac{1}{3}$ (B) $\dfrac{\pi}{3}$ (C) π (D) 1

3. Given the angle between the lines $y = mx$ and $y = \dfrac{1}{3}x$ is 45°.

The exact value of m is :

(A) 2 (B) $\dfrac{2}{3}$ (C) -2 (D) $-\dfrac{2}{3}$

4. The derivative of $\ln(\sin^{-1}3x)$ is:

(A) $\sin^{-1}3x\left(\dfrac{3}{\sqrt{1-9x^2}}\right)$ (B) $\dfrac{3}{\sin^{-1}3x}$ (C) $\dfrac{3}{\cos^{-1}3x}$ (D) $\dfrac{1}{\sin^{-1}3x}\left(\dfrac{3}{\sqrt{1-9x^2}}\right)$

5. The inverse function $f^{-1}(x)$ of $f(x) = (x+1)^2 - 2$ is:

(A) $f^{-1}(x) = \sqrt{(y+2)} - 1$ (B) $f^{-1}(x) = \sqrt{(y-2)} + 2$

(C) $f^{-1}(x) = \sqrt{(x+2)} - 1$ (D) $f^{-1}(x) = \sqrt{(x-2)} + 2$

6. Given that $0 < x < \dfrac{\pi}{2}$ the value of the infinite series

$1 - \tan^2 x + \tan^4 x - \tan^6 x + \ldots\ldots$ is equal to :

(A) $\cos^2 x$ (B) $\tan^2 x$ (C) $\sin^2 x$ (D) $\cot^2 x$

7. Given the roots of $x^2 - 2x - 1 = 0$ are $\tan\alpha$ and $\tan\beta$ where α and β are acute.

The value of $(\alpha + \beta)$ is equal to :

(A) $-\dfrac{\pi}{2}$ (B) $\dfrac{\pi}{4}$ (C) $\dfrac{\pi}{2}$ (D) $-\dfrac{\pi}{4}$

8. $\int (\cos^2 x + 2)dx =$

(A) $\dfrac{1}{4}\sin 2x + C$ (B) $\dfrac{x}{2} + \dfrac{1}{2}\sin 2x + C$ (C) $\dfrac{1}{2}\cos 2x + C$ (D) $\dfrac{5x}{2} + \dfrac{1}{4}\sin 2x + C$

9. Given that $x^3 - 3x^2 + 1 = 0$ has a root between 0 and 1. Taking $x = 0.3$ as the first approximation and using one application of Newton's method, a better approximation, correct to two decimal places would be:

(A) 0.80 (B) 0.79 (C) 0.81 (D) 0.78

10. If $\cos x = \dfrac{3}{4}$ and $\sin x < 0$ the exact value of $\sin 2x$ is equal to:

(A) $\dfrac{-3\sqrt{7}}{8}$ (B) $\dfrac{\sqrt{7}}{4}$ (C) $\dfrac{-\sqrt{7}}{4}$ (D) $\dfrac{3\sqrt{7}}{8}$

End of Multiple Choice Section I

Section II- (60 marks)

Complete all questions in SEPERATE answer booklets showing ALL necessary working.

Question 11 (15 Marks) Start a new booklet. **Marks**

(a) Use the process of mathematical induction to show that: **3**

$$1+3+9+\ldots\ldots\ldots\ldots\ldots+3^{n-1}=\frac{1}{2}\left(3^n-1\right)$$

(b) Prove that $\dfrac{\sin 3\theta}{\sin\theta}-\dfrac{\cos 3\theta}{\cos\theta}$ is independent of θ. **2**

(c) Given $\sin x+\sqrt{3}\cos x\equiv A\sin(x+\theta)$

(i) Find A and θ in exact form where $0<\theta<\dfrac{\pi}{2}$ **2**

(ii) Hence solve $\sin x+\sqrt{3}\cos x=\sqrt{2}$ where $0\le x\le\pi$ **2**

(d) From the top of a cliff an observer spots two ships out at sea. One is north east with an angle of depression of 8° while the other is south east with an angle of depression of 6°. If the two ships are 400 metres apart, find the height of the cliff, to the nearest metre. **3**

(e) Consider the function $f(x)=\dfrac{1}{2}\cos^{-1}(1-3x)$.

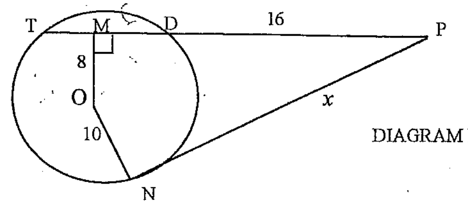
(i) State the domain and range of $f(x)$. **2**

(ii) Hence sketch the graph of $y=f(x)$. **1**

End of Question 11

Question 12 (15 Marks) Start a new booklet. **Marks**

(a)



PN is a tangent to the circle, centre O. OM is perpendicular to the chord TD. DP = 16 units, OM = 8 units and ON = 10 units.

(i) Find the length of PT. **2**

(ii) Hence or otherwise find x. (answer to 1 decimal place) **1**

(b) Let P($2ap$, $2ap^2$) and Q($2aq$, $2aq^2$) be points on the parabola $y=\dfrac{x^2}{2a}$.

(i) Find the equation of the chord PQ. **2**

(ii) If PQ is a focal chord, show that $pq=-\dfrac{1}{4}$ **2**

(iii) Hence show that the locus of the midpoint of PQ is a parabola. **2**

(c) Newton's Law of Cooling states that the rate at which a body cools is proportional to the difference between the temperature T of the body and the room temperature R.

(i) Show that $T=R+Ae^{-kt}$ where A is a constant and t is time, satisfies the equation $\dfrac{dT}{dt}=-k(T-R)$ **1**

(ii) Given that a bowl of chicken soup cools from 80°Celsius to 65°Celsius in 10 minutes in a room with a constant temperature of 25°Celsius.

(α) Find the value of constants A and k. (Round off k to 5 decimal places) **2**

(β) Find the temperature of the chicken soup after half an hour.(To the nearest °C) **1**

(d) Use the substitution $u=\tan x$ to evaluate $\displaystyle\int_{x=0}^{x=\frac{\pi}{6}}(\tan^2 x\sec^2 x)dx$ **2**

End of Question 12

Question 13 (15 Marks) **Start a new booklet.** **Marks**

(a) A particle is moving in a straight line. At time t seconds it has displacement x metres from the origin 0. The velocity v ms^{-1} is given by $v = 2 - x$ and acceleration is a ms^{-2}. Initially the particle is 4 metres to the left of the origin.

(i) Find an expression for a in terms of x. **1**

(ii) Use integration to show that $x = 2 - 6e^{-t}$. **3**

(iii) Sketch the graph of $x = 2 - 6e^{-t}$ showing the x and y intercepts and the horizontal asymptote. **2**

(b) A particle moves so that its distance x centimetres from a fixed point O at time t seconds is $x = 6\sin 2t$.

(i) Show that the particle is moving in simple harmonic motion. **2**

(ii) What is the period of the motion? **1**

(iii) Find the velocity of the particle when it first reaches 3 centimetres to the right of the origin. (Answer to 2 decimal places) **2**

(c) A cylindrical water tank stands on one of its circular ends which has a radius of 4 metres. Water is pumped into the tank at a rate of $1.8m^3/\min$.

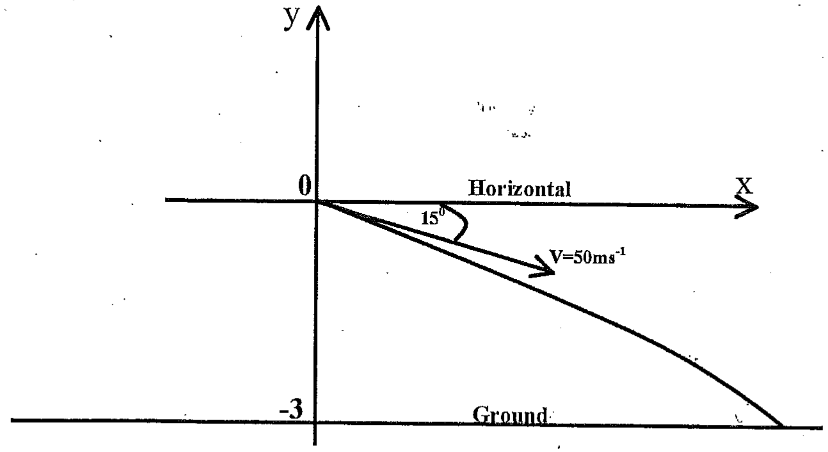
(i) Find the rate at which the level of water in the tank is rising. (Answer to 4 decimal places) **2**

(ii) At what rate in $m^3/\min$ would water need to be pumped into the tank for the water level to rise at $15mm/\sec$? **2**

End of Question 13

Question 14 (15 Marks) **Start a new booklet.** **Marks**

(a) When served from the baseline, a tennis ball leaves the middle of the racket 3 metres above the ground with a velocity of $50ms^{-1}$ at an angle of 15° below the horizontal. The origin 0 is defined as where the ball leaves the racket and the horizontal and vertical equations of motion are $\ddot{x} = 0$ and $\ddot{y} = -10$.



(i) Show that $x = 50t\cos 15$ and $y = -5t^2 - 50t\sin 15$ are the horizontal and vertical coordinates of the ball at time t. **4**

(ii) Find the time at which the ball strikes the ground. (Answer in seconds to 3 decimal places) **2**

(iii) Given that a tennis court is 24 metres from base line to base line, with the net at halfway. Would it be possible for this particular serve to be in play? Discounting net height, justify your answer mathematically. **2**

(iv) Find the acute angle at which the ball hits the ground. (Answer to the nearest degree) **2**

(v) Find the Cartesian equation of the flight of the tennis ball and show that it is a parabola with downward concavity. **2**

(b) Given the equation $x^3 + 3Wx = -H$ has 2 equal roots. Prove that $-4W^3 = H^2$ **3**

End of Examination

- Q11 PA /15
- Q12 RL /15
- Q13 AC /15
- Q14 PR /15

Ext-1 TSC Trial HSC 2012.

Question 1 Name: P. Atkinson

Teacher: Solutions

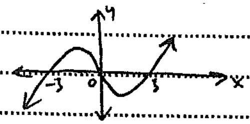
Multiple Choice Solutions

Q1

(a) $\dfrac{x^2-9}{x} \geq 0$

$\dfrac{x^2(x^2-9)}{x} \geq 0$

$x(x-3)(x+3) \geq 0$



(a) D $-3 \leq x \leq 0$ or $x \geq 3$

(b) $\lim\limits_{x\to 0} \dfrac{\sin \pi x}{3x} = \lim\limits_{x \to 0} \dfrac{\pi}{3}\left(\dfrac{\sin \pi x}{\pi x}\right)$

(b) B $= \dfrac{\pi}{3}$

(c) $1 = \dfrac{m - \frac{1}{3}}{1 + \frac{1}{3}m}$

$1 + \frac{1}{3}m = m - \frac{1}{3}$

(c) A $m = 2$.

(d) $\dfrac{d}{dx} \ln(\sin^{-1} 3x)$

$= \dfrac{\frac{d}{dx}(\sin^{-1} 3x)}{\sin^{-1} 3x}$

(d) D $= \dfrac{1}{\sin^{-1} 3x} \times \dfrac{3}{\sqrt{1-9x^2}}$

(e) $x = (y+1)^2 - 2$

$y+1 = \sqrt{(x+2)}$

$y = \sqrt{x+2} - 1$.

(e) C $f^{-1}(x) = \sqrt{(x+2)} - 1$.

(f) $S_\infty = \dfrac{1}{1-(-\tan^2 x)}$

$= \dfrac{1}{1+\tan^2 x}$

$= \dfrac{1}{\sec^2 x}$

(f) A $= \cos^2 x$.

(g) $\tan\alpha + \tan\beta = 2$

$\tan\alpha \tan\beta = -1$.

$\tan(\alpha+\beta) = \dfrac{2}{1 - -1}$

(g) B $= 1$.

(h) $\int (\cos^2 x + 2)\, dx$

$= \int \left(\frac{1}{2}(\cos 2x + 1) + 2\right) dx$

(h) D $= \frac{5x}{2} + \frac{1}{4}\sin 2x + C$.

(i) $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

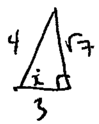
$\therefore x_2 = 0.3 - \dfrac{[(0.3)^3 - 3(0.3)^2 + 1]}{(3(0.3)^2 - 6(0.3))}$

(i) B $\therefore x_2 = 0.79$ (2dp)

(j) $\sin 2x = 2\sin x \cos x$

$= 2 \times -\dfrac{\sqrt{7}}{4} \times \dfrac{3}{4}$

(j) A $= -\dfrac{3\sqrt{7}}{8}$



4th Quad

$\sin x < 0$

$\cos x > 0$

Q1	
(a)	D
(b)	B
(c)	A
(d)	D
(e)	C
(f)	A
(g)	B
(h)	D
(i)	B
(j)	A

TSC yr 12 Trial HSC 2012

Mathematics Ext-1

Question 1/2 Name: P. Atkinson

Teacher: Solutions

Section I - (10 Marks) MC

1. D
2. B
3. A
4. D
5. C
6. A
7. B
8. D
9. B
10. A

Section II - (60 marks)

(15 Marks) Q11 (a) $1 + 3 + 9 + \ldots + 3^{n-1} = \frac{1}{2}(3^n - 1)$

$T_n = 3^{n-1}$ $\quad S_n = \frac{1}{2}(3^n - 1)$

When $n=1$ $\quad 3^{1-1} = \frac{1}{2}(3^1 - 1)$

$3^0 = \frac{1}{2}(2)$ (1)

$1 = 1$ $\therefore$ true for $n=1$

Assume true for $n=k$

$\therefore 1 + 3 + 9 + \ldots + 3^{k-1} = \frac{1}{2}(3^k - 1)$

Prove true for $n = k+1$

ie $S_{k+1} = S_k + T_{k+1}$

$= \frac{1}{2}(3^k - 1) + 3^{(k+1)-1}$

$= \frac{1}{2}(3^k - 1) + 3^k$

$= \frac{1}{2}(3^k - 1 + 2 \times 3^k)$

$= \frac{1}{2}(3 \times 3^k - 1)$ (1)

$S_{k+1} = \frac{1}{2}(3^{k+1} - 1)$

which is in the form $S_n = \frac{1}{2}(3^n - 1)$

where $n = k+1$

$\therefore$ true for $n = k+1$ when true for $n = k$ (1)

Hence true for all n.

(b) Prove that $\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta}$ is independent of θ.

$= \dfrac{\cos\theta \sin 3\theta - \cos 3\theta \sin\theta}{\sin\theta\cos\theta}$ (1)

$= \dfrac{\sin(3\theta - \theta)}{\frac{1}{2} \times 2\sin\theta\cos\theta}$

$= \dfrac{\sin 2\theta}{\frac{1}{2}\sin 2\theta}$ (1)

$= 2$.

(c) (i) $\sin x + \sqrt{3}\cos x \equiv A\sin(x+\theta)$

$= A(\sin x\cos\theta + \cos x \sin\theta)$

$\therefore A\cos\theta = 1$ and $A\sin\theta = \sqrt{3}$

$A^2 = (\sqrt{3})^2 + 1^2$

$A = 2$. (1)

Also $\sin\theta = \sqrt{3}$

$\therefore \sin\theta = \dfrac{\sqrt{3}}{2}$ A

$\theta = \dfrac{\pi}{3}$ (1)

(ii) Solve $\sin x + \sqrt{3}\cos x = \sqrt{2}$, $0 \leq x \leq \pi$

hence from (i)

$2\sin(x + \frac{\pi}{3}) = \sqrt{2}$ (1)

$\sin(x + \frac{\pi}{3}) = \dfrac{\sqrt{2}}{2}$

$x + \frac{\pi}{3} = \frac{\pi}{4}, \frac{3\pi}{4}, 2\pi + \frac{\pi}{4}, \ldots$

$x = \dfrac{3\pi}{4} - \dfrac{\pi}{3}$

$x = \dfrac{9\pi - 4\pi}{12}$

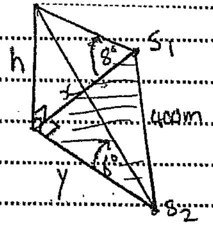
$x = \dfrac{5\pi}{12}$ $\quad (0 \leq x \leq \pi)$ (1)

Question 2|3

Name: P. Atkinson

Teacher: Solutions

(d)



① – diagram

$\tan 6° = \frac{h}{y}$ $\tan 8° = \frac{h}{x}$

$y = \frac{h}{\tan 6°}$ $x = \frac{h}{\tan 8°}$

by pythag

$400^2 = \left(\frac{h}{\tan 6}\right)^2 + \left(\frac{h}{\tan 8}\right)^2$ ① – working

$400^2 = h^2\left(\frac{1}{\tan^2 6} + \frac{1}{\tan^2 8}\right)$

$h = \sqrt{400^2 \div \left(\frac{1}{\tan^2 6} + \frac{1}{\tan^2 8}\right)}$

$h = \sqrt{160\,000 \div (141 \cdot 1516173)}$

$h = 34\text{ m}$ ① correct – answer.

(e) $f(x) = \frac{1}{2}\cos^{-1}(1-3x)$

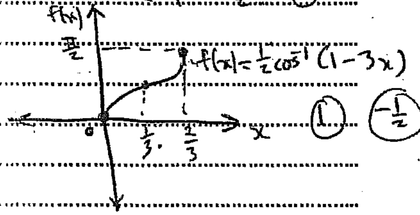
(i) D: $-1 \le 1-3x \le 1$

$-2 \le -3x \le 0$

$\frac{2}{3} \ge x \ge 0$

hence D: $0 \le x \le \frac{2}{3}$ ①

R: $0 \le y \le \frac{\pi}{2}$ ①



① $\left(-\frac{1}{2}\right)$ each mistake

Q12 (15 Marks)

(a) $TM = \sqrt{10^2 - 8^2}$ ① by pythag

(i) $= 6$ units

$TD = 12$ units (OM perp bisector of TD)

$\therefore PT = 12 + 16$

$= 28$ units ①

(ii) $x^2 = PD \times PT$

$x^2 = 16 \times 28$ ①

$x = 21.2$ units

(b) $m_{PQ} = \frac{2aq^2 - 2ap^2}{2aq - 2ap}$

(i) $= \frac{q^2 - p^2}{q - p}$

$= \frac{\cancel{(q-p)}(q+p)}{\cancel{(q-p)}}$

$\therefore m_{PQ} = p + q$ ①

Eqn of PQ

$y - 2ap^2 = (p+q)(x - 2ap)$

$\therefore y = px + qx - 2apq$ ①

(ii) $y = \frac{x^2}{2a}$

$\therefore x^2 = 2ay$

$\therefore 4A = 2a$

$A = \frac{a}{2}$

Focus is $S\left(0, \frac{a}{2}\right)$ ①

Focus satisfies focal eqn.

$\therefore \frac{a}{2} = 0 + 0 - 2apq$

$a = -4apq$

$\therefore pq = \frac{a}{-4a}$

$\therefore pq = -\frac{1}{4}$ as req ①

Question 3

Name: P. Atkinson

Teacher: Solutions

(iii) Midpoint PQ is $\left(\frac{2ap + 2aq}{2}, \frac{2ap^2 + 2aq^2}{2}\right)$ ①

Midpt PQ is $\left(\frac{\cancel{2}a(p+q)}{\cancel{2}}, \frac{\cancel{2}a(p^2+q^2)}{\cancel{2}}\right)$

$x = a(p+q)$ $y = a(p^2 + q^2)$

and $pq = -\frac{1}{4}$

$y = a\left[(p+q)^2 - 2pq\right]$

$y = a(p+q)^2 - 2apq$

$y = \frac{x^2}{a} - \left(2a \times -\frac{1}{4}\right)$

$y = \frac{x^2}{a} + \frac{a}{2}$ $ay = x^2 + \frac{a^2}{2}$

$ay - \frac{a^2}{2} = x^2$ ①

$x^2 = a\left(y - \frac{a}{2}\right)$

which is parabola $V\left(0, \frac{a}{2}\right)$ focal length $= \frac{a}{4}$

$x^2 = 4\left(\frac{a}{4}\right)\left(y - \frac{a}{2}\right) \to x^2 = 4a(y-k)$

(c) (i) $T = R + Ae^{-kt}$

$\frac{dT}{dt} = \frac{d}{dt}(T)$

$= \frac{d}{dt}\left(R + Ae^{-kt}\right)$

$\therefore \frac{dT}{dt} = -kAe^{-kt}$

but $Ae^{-kt} = T - R$

$\therefore \frac{dT}{dt} = -k(T - R)$ ①

hence $T = R + Ae^{-kt}$ is a solution of $\frac{dT}{dt} = -k(T - R)$

(ii)

(α) As $t \to \infty$ $T \to 25$

$T = 25 + Ae^{-kt}$

When $t = 0$ $T = 80$

$80 = 25 + Ae^0$

$\therefore A = 55$ ①

$T = 25 + 55e^{-kt}$

when $t = 10$ $T = 65$

$65 = 25 + 55e^{-10k}$ ①

$e^{-10k} = \frac{40}{55}$

$k = \frac{\ln\left(\frac{8}{11}\right)}{-10}$ $\therefore k = 0.03185$

(β) Find T when $t = 30$

$T = 25 + 55e^{-0.03185(30)}$

$T = 25 + 21.15$

$T = 46.15°C$ ①

$T \doteqdot 46°C$

(d) $u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x\, dx$ ①

$\int_{x=0}^{x=\frac{\pi}{6}} \tan^2 x \sec^2 x\, dx$

$= \int_0^{\frac{1}{\sqrt{3}}} u^2\, du$

$= \left[\frac{u^3}{3}\right]_0^{\frac{1}{\sqrt{3}}}$

$= \frac{\left(\frac{1}{\sqrt{3}}\right)^3}{3} - 0$

$= \frac{1}{3\sqrt{3}} \times \frac{1}{3}$

$= \frac{1}{9\sqrt{3}}$

$= \frac{\sqrt{3}}{27}$ ①

Question 4

Name: P. Atkinson

Teacher: Solutions

Q13 (a) (15 marks)

$v = 2 - x$

When $t=0$ $x = -4$.

(i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = a = \frac{d^2x}{dt^2}$

$\therefore a = \frac{d}{dx}\left(\frac{1}{2}(2-x)^2\right)$

$a = \frac{1}{2}\frac{d}{dx}(4-4x+x^2)$

$a = \frac{1}{2}(-4+2x)$ (1)

$a = x-2$

(ii) Show that $x = 2 - 6e^{-t}$

$\frac{dx}{dt} = 2 - x$

$\frac{dt}{dx} = \frac{1}{(2-x)}$

$t = \int \frac{1}{2-x}\,dx$ (1)

$t = -\ln(2-x) + c$.

When $t=0$ $x = -4$

$0 = -\ln 6 + c$

$\therefore c = \ln 6$

$t = -\ln(2-x) + \ln 6$ (1)

make x-subject

$\ln(2-x) = -t + \ln 6$

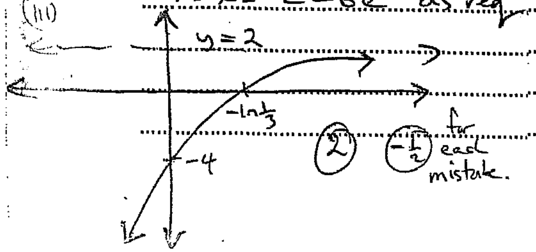
$e^{(-t+\ln 6)} = 2-x$

$e^{-t} \times e^{\ln 6} = 2-x$

$e^{-t} \times 6 = 2-x$ (1)

$\therefore x = 2 - 6e^{-t}$ as req.

(iii)



(2) ($-\frac{1}{2}$ for each mistake)

(b) $x = 6\sin 2t$

(i) $\dot{x} = 12\cos 2t$ (1)

$\ddot{x} = -24\sin 2t$

$\ddot{x} = -4(6)\sin 2t$

$\ddot{x} = -(2^2)(6\sin 2t)$ (1)

$\ddot{x} = -(2^2)x$ $\therefore$ as req.

(ii) $T = \frac{2\pi}{n}$

$T = \frac{2\pi}{2}$ (1)

$T = \pi$

(iii) $x = 6\sin 2t$

When $x = 3$

$3 = 6\sin 2t$

$\sin 2t = \frac{1}{2}$

$2t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6} \ldots$

$t = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12} \ldots$ (1)

$\dot{x} = 12\cos 2t$

$\dot{x} = 12\cos\frac{2\pi}{12}$

$\dot{x} = 12\cos\frac{\pi}{6}$

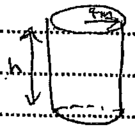
$\dot{x} = 10.39\ \text{cms}^{-1}$ (1)

Question 4/5

Name: P. Atkinson

Teacher: Solutions

(c)



Let radius = r

height = h

(i) $V = \pi r^2 h$ $V = 16\pi h$

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ (1)

$\frac{dV}{dh} = 16\pi$ $\frac{dV}{dt} = 1.8$

$1.8 = 16\pi \times \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{1.8}{16\pi}$ (1)

$\frac{dh}{dt} = 0.0358\ \text{m/min}$

(ii) $\frac{dh}{dt} = 15\ \text{mm/sec}$.

$= \frac{15 \times 60}{1000}\ \text{m/min}$

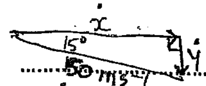
$\frac{dh}{dt} = 0.9\ \text{m/min}$ (1)

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$= 16\pi \times 0.9$

$\frac{dV}{dt} = 45.2\ \text{m}^3/\text{min}$ (1)

Q14 (i)



$\sin 15° = \frac{\dot{y}}{50}$

$\dot{y} = 50\sin 15°$

$\cos 15° = \frac{\dot{x}}{50}$

$\dot{x} = 50\cos 15$.

$\ddot{x} = 0$ $\ddot{y} = -10$

$\dot{x} = c_1$ $\dot{y} = -10t + c_1$ (1)

$\dot{x} = 50\cos 15$ When $t=0$ $\dot{y} = -50\sin 15$

$x = \int 50\cos 15\,dt$ (1) $-50\sin 15 = -10(0) + c_1$

$x = 50t\cos 15 + c_2$ $c_1 = -50\sin 15$

When $t=0$ $x=0$ $\dot{y} = -10t - 50\sin 15$.

$\therefore c_2 = 0$ $y = \int -10t - 50\sin 15$

$x = 50t\cos 15$ $y = -5t^2 - 50t\sin 15 + c$

When $t=0$ $y=0$

(1) $c_2 = 0$ (1)

$x = 50t\cos 15$ $y = -5t^2 - 50t\sin 15$

as req.

(ii) The ball strikes the ground when $y = -3$.

$-3 = -5t^2 - 50t\sin 15$ (1)

$5t^2 + 50t\sin 15 - 3 = 0$

$t = \frac{-50\sin 15 + \sqrt{(2500\sin^2 15 + 60)}}{10}$

$t = 0.2141098177$ sec.

$t = 0.214$ sec. (1)

(iii) When $t = 0.214$

$x = 50(0.214)\cos 15$

$x = 10.34$ m. (1)

Which is less than the 12m halfway (1)

Hence the serve would not be in play.

Question 5

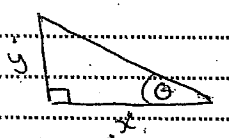
Name: P. Atkinson

Teacher: Solutions

(iv) When $t = 0.214$ sec

$\dot{x} = |50\cos 15|$ 1

$\dot{y} = |-10(0.214) - 50\sin 15|$



$\tan\theta = \dfrac{\dot{y}}{\dot{x}}$

$\tan\theta = 0.312259$ 1

$\theta \doteqdot 17°20'$

$\theta \doteqdot 17°$

(v) $x = 50t\cos 15$

$t = \dfrac{x}{50\cos 15}$ $y = -5t^2 - 50t\sin 15$

$$y = -5\left(\frac{x}{50\cos 15}\right)^2 - 50\left(\frac{x}{50\cos 15}\right)\times\frac{\sin 15}{1}$$

$$y = \frac{-5x^2}{2500\cos^2 15} - \frac{\cancel{50}\,x\sin 15}{\cancel{50}\cos 15}$$

$$y = -\left(\frac{1}{500\cos^2 15}\right)x^2 - x\tan 15$$ 1

which is in the form

$y = ax^2 + bx + c$

where $a < 0$ 1

∴ parabola concave down

$x^3 + 3Wx = -H$

(b) $x^3 + 3Wx + H = 0$

2 equal roots

Prove $-4W^3 = H^2$

$\therefore \alpha + \alpha + \beta = 0$ $2\alpha + \beta = 0$

$\alpha^2\beta = -H.$

$\alpha^2(-2\alpha) = -H$ 1

$\alpha^3 = \frac{H}{2}$ 1

α solves $x^3 + 3Wx + H = 0$

$\therefore \alpha^3 + 3W\alpha + H = 0$

$\frac{H}{2} + 3W\alpha + H = 0$

$3W\alpha = -H - \frac{H}{2}$

$\cancel{3}W\alpha = -\frac{\cancel{3}H}{2}$

$\alpha = -\frac{H}{2W}$

$\therefore \alpha^3 = -\frac{H^3}{8W^3}$

but $\alpha^3 = \frac{H}{2}$ 1

$\therefore \frac{H}{2} = \frac{-H^3}{8W^3}$

$\frac{8HW^3}{2} = -H^3$

$4HW^3 = -H^3$

$4W^3 = -H^2$ 1

$\therefore H^2 = -4W^3$

as req