

Exercise 3.1

1. In each of the sequences below, write down the next two terms and state the n^{th} term.

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| (a) 5, 10, 15, 20, ... | (b) 4, 7, 10, 13, ... |
| (c) 2, 5, 8, 11, ... | (d) 2, 4, 8, 16, ... |
| (e) $3, -\frac{3}{5}, \frac{3}{25}, -\frac{3}{125}, \dots$ | (f) $\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \dots$ |
| (g) $1, \frac{5}{2}, \frac{25}{4}, \frac{125}{8}, \dots$ | (h) $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \dots$ |
| (i) 1, -2, 3, -4, ... | (j) 11, 8, 13, 6, 15, ... |

2. Write down the first four terms of each of the following sequences.

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| (a) $\left\{\frac{\sqrt{n}}{n+1}\right\}$ | (b) $\left\{\frac{(-1)^{n+1}}{n!}\right\}$ | (c) $\left\{\frac{(2x)^{n-1}}{(2n-1)^2}\right\}$ |
| (d) $\left\{\frac{(-1)^n x^{2n-1}}{(2n-1)(2n+1)}\right\}$ | (e) $\left\{\frac{\cos nx}{nx^n}\right\}$ | |

3. Find the n^{th} term of each of the following sequences.

- (a) $-\frac{1}{5}, \frac{3}{8}, -\frac{5}{11}, \frac{7}{14}, -\frac{9}{17}, \dots$
 (b) 1, 0, 1, 0, 1, ...
 (c) $\frac{2}{3}, 0, \frac{3}{4}, 0, \frac{4}{5}, \dots$

4. Find the next five terms of each of the sequences below, using the recursion formula given.

- (a) $u_{n+1} = nu_n - n, u_1 = 3.$
 (b) $u_{n+1} = 2u_n - (-1)^n, u_1 = 1.$
 (c) $u_{n+2} = 3u_{n+1} - 2u_n, u_1 = 1, u_2 = 3.$

5. Determine if each of the following sequences are convergent or divergent. For those that are convergent, find their limits.

- (a) $1 + \frac{1}{2}, 1 + \frac{1}{3}, 1 + \frac{1}{4}, \dots$ (b) $1 + \frac{1}{2}, 2 + \frac{1}{3}, 3 + \frac{1}{4}, \dots$
 (c) $\frac{2}{1 + \frac{1}{2}}, \frac{2}{1 + \frac{1}{3}}, \frac{2}{1 + \frac{1}{4}}, \dots$ (d) $3 - \frac{1}{2n}, 3 - \frac{2}{3n}, 3 - \frac{3}{4n}, \dots$

6. Evaluate each of the following.

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| (a) $\lim_{n \rightarrow \infty} \left(\frac{3}{n}\right)$ | (b) $\lim_{n \rightarrow \infty} \left(\frac{2}{n-1}\right)$ | (c) $\lim_{n \rightarrow \infty} \left(\frac{n^2 + 2}{n}\right)$ |
| (d) $\lim_{n \rightarrow \infty} \left(\frac{n+2}{n-1}\right)$ | (e) $\lim_{n \rightarrow \infty} \left(\frac{2n^2 + 2n}{5n^2 - n}\right)$ | (f) $\lim_{n \rightarrow \infty} \left(\frac{1 + 2 \times 5^n}{4 + 3 \times 5^n}\right)$ |

7. Write each of the following series by using the Σ notation.

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| (a) $1 + 8 + 27 + 64 + \dots + 1000$ | (b) $1 + 3 + 5 + 7 + \dots + 99$ |
| (c) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{512}$ | (d) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} - \frac{1}{243}$ |
| (e) $11 + 8 + 5 + 2 - 1 - 4 - 7$ | (f) $5 + 7 + 11 + 19 + 35 + 67$ |
| (g) $-x + 2x^2 - 3x^3 + \dots + 10x^{10}$ | (h) $\frac{1}{2 \cdot 3} + \frac{2}{3 \cdot 4} + \frac{3}{4 \cdot 5} + \dots + \frac{8}{9 \cdot 10}$ |

8. Write down the terms of each of the following series without the Σ notation.

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| (a) $\sum_{r=1}^{10} (r^2 - r)$ | (b) $\sum_{r=1}^8 \frac{1}{r^2}$ | (c) $\sum_{r=1}^5 (3r + 2)$ |
| (d) $\sum_{r=1}^9 (-1)^{r-1} (3r)$ | (e) $\sum_{r=1}^{10} (8 - r)$ | (f) $\sum_{r=1}^7 (r + 2)(r + 4)$ |
| (g) $\sum_{r=1}^5 (6 - r)^2$ | (h) $\sum_{r=1}^5 (2r^2 - 3r - 5)$ | |

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1. (a) 25, 30; $5n$ (b) 16, 19; $3n + 1$
 (c) 14, 17; $3n - 1$ (d) 32, 64; 2^n
 (e) $\frac{3}{625}, -\frac{3}{3125}; (-1)^{n-1} \frac{3}{5^{n-1}}$
 (f) $\frac{9}{11}, \frac{11}{13}; \frac{2n-1}{2n+1}$
 (g) $\frac{625}{16}, \frac{3125}{32}; (\frac{5}{2})^{n-1}$
 (h) $\frac{1}{30}, \frac{1}{42}; \frac{1}{n(n+1)}$
 (i) 5, -6; $(-1)^{n-1}n$
 (j) 4, 17; $10 + (-1)^{n+1}n$
2. (a) $\frac{\sqrt{1}}{2}, \frac{\sqrt{2}}{3}, \frac{\sqrt{3}}{4}, \frac{\sqrt{4}}{5}$
 (b) $\frac{1}{1!}, -\frac{1}{2!}, \frac{1}{3!}, -\frac{1}{4!}$
 (c) $\frac{1}{1^2}, \frac{2x}{3^2}, \frac{4x^2}{5^2}, \frac{8x^3}{7^2}$
 (d) $-\frac{x}{1.3}, \frac{x^3}{3.5}, -\frac{x^5}{5.7}, \frac{x^7}{7.9}$
 (e) $\frac{\cos x}{x}, \frac{\cos 2x}{2x^2}, \frac{\cos 3x}{3x^3}, \frac{\cos 4x}{4x^4}$
3. (a) $\frac{(-1)^n(2n-1)}{3n+2}$
 (b) $\frac{1 - (-1)^n}{2}$
 (c) $\frac{n+3}{n+5} \cdot \frac{1 - (-1)^n}{2}$
4. (a) 3, 2, 2, 3, 8, 35, ...
 (b) 1, 3, 5, 11, 21, 43, ...
 (c) 1, 3, 7, 15, 31, 63, 127, ...
5. (a) Convergent, 1 (b) Divergent
 (c) Convergent, 2 (d) Convergent, 3
6. (a) 0 (b) 0 (c) ∞
 (d) 1 (e) $\frac{2}{5}$ (f) $\frac{2}{3}$
7. (a) $\sum_{r=1}^{10} r^3$ (b) $\sum_{r=1}^{50} (2r - 1)$
 (c) $\sum_{r=0}^9 \frac{1}{2^r}$ (d) $\sum_{r=1}^6 (-1)^{r-1} \left(\frac{1}{3}\right)^{r-1}$
 (e) $\sum_{r=1}^7 (14 - 3r)$ (f) $\sum_{r=1}^6 (3 + 2^r)$
 (g) $\sum_{r=1}^{10} (-1)^r r x^r$ (h) $\sum_{r=1}^6 \frac{r}{(r+1)(r+2)}$
8. (a) $0 + 2 + 6 + 12 + 20 + 30 + 42 + 56 + 72 + 90$
 (b) $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \frac{1}{49} + \frac{1}{64}$
 (c) $5 + 8 + 11 + 14 + 17$
 (d) $3 - 6 + 9 - 12 + 15 - 18 + 21 - 24 + 27$
 (e) $7 + 6 + 5 + 4 + 3 + 2 + 1 + 0 - 1 - 2$
 (f) $15 + 24 + 35 + 48 + 63 + 80 + 99$
 (g) $25 + 16 + 9 + 4 + 1$
 (h) $-6 - 3 + 4 + 15 + 30$