Exercise 3.3

1. For each of the following geometric series, write down the term indicated in brackets and the n^{th} term.

(a) $\frac{1}{2} + 1 + 2 + \ldots$ (8^{th} term)

(b) $162 + 54 + 18 + \ldots$ (6^{th} term)

(c) $200 - 50 + 12\frac{1}{2} + \ldots$ (5^{th} term)

(d) $-\frac{4}{9} - \frac{2}{3} - 1 - \ldots$ (7^{th} term)

2. Find the number of terms in each of the geometric series below, and also the sum of the series.

(a) $\frac{1}{4} + \frac{1}{2} + \ldots + 64$

(b) $9 - 6 + \ldots + \frac{256}{729}$

(c) $100 + 50 + \ldots + \frac{25}{16}$

(d) $2 - \frac{8}{3} + \ldots - 14\frac{2150}{2187}$

3. Find the sum of each of the following geometric series.

(a) $100 + 20 + \ldots$ to 8^{th} term

(b) $4 - 2 + \ldots$ to 10^{th} term

(c) $2 - 6 + \ldots$ to n^{th} term

(d) $a^k + a^{k+2} + \ldots$ to n^{th} term

4. The terms of a geometric series are positive, with the first term 80. If the sum of the first three terms is 185, find the common ratio of the series.

5. If two numbers, m and n, are such that m, n and 10 form an arithmetic progression, whereas n, m and 10 form a geometric progression, find the values of m and n.

6. Find the geometric mean of each of the following numbers.

(a) 3 and 27

(b) $\frac{1}{3}$ and $\frac{1}{27}$

(c) 10^3 and 10^{27}

7. Given that the geometric mean of $4p - 3$ and $9p + 4$ is $6p - 1$, find the values of p.

8. The second and fifth terms of a geometric series are 405 and –120 respectively. Find the seventh term and the sum of the first seven terms of the series.

9. In a geometric progression, the second term exceeds the first term by 20 and the fourth term exceeds the second term by 15. Find the possible values of the first term.

10. Find the sum of the first n terms of the series $\frac{1}{12} + \frac{1}{4} + \frac{3}{4} + \ldots$. Find also the number of terms required such that the sum exceeds 100.

11. A geometric series has first term 16 and common ratio $\frac{3}{4}$. If the sum of the first n terms exceeds 60, find the smallest value of n.

12. A student saves his pocket money with the intention to buy an encyclopaedia set costing RM1200. If he saves RM50 in the bank each month and the bank pays him an interest of $\frac{1}{2}$% per month, how many months has he got to save before he can afford to buy the encyclopaedia?

13. A house buyer borrows RM50,000 from a bank to buy a house which costs RM70,000. The rate of interest charged by the bank is 9% per annum, and is calculated based on the amount outstanding at the beginning of each year. The house buyer is required to repay his loan in monthly instalments for a period of 15 years. Assuming that the rate of interest is fixed for the entire duration of the loan, find the amount per month he has to repay the bank.

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1. (a) $64, 2^{n-2}$

(b) $\frac{2}{3}, 162\left(\frac{1}{3}\right)^{n-1}$

(c) $\frac{25}{32}, 200\left(-\frac{1}{4}\right)^{n-1}$

(d) $-\frac{81}{16}, -\left(\frac{3}{2}\right)^{n-3}$

2. (a) $9, 127\frac{3}{4}$

(b) $9, 5\frac{394}{729}$

(c) $7, 198\frac{7}{16}$

(d) $8, -7\frac{1\,541}{2\,187}$

3. (a) $124\frac{3\,124}{3\,125}$

(b) $2\frac{85}{128}$

(c) $\frac{1}{2}[1 - (-3)^n]$

(d) $\frac{a^k[a^{2n} - 1]}{a^2 - 1}$

4. $\frac{3}{4}$

5. $m = -5, n = 2\frac{1}{2}$

6. (a) 9

(b) $\frac{1}{9}$

(c) 1×10^{15}

7. $p = 13$

8. $-53\frac{1}{3}; -385\frac{5}{6}$

9. –40; –8

10. $\frac{1}{24}(3^n - 1)$; 8

11. 10

12. 23

13. RM516.92