

Series and Sequences Test

September 2003

Instructions

- Show all necessary working
- Use separate paper

Name _____

Question One (14 marks)

- a) For the sequence 9, 15, 21,
- Find the thirty - first term
 - Find the sum of the first thirty one terms
 - Is 261 a term of the sequence? (give reasons for your answer)
 - How many terms are less than 500?
- b) Find the first term and common ratio of a geometric sequence in which the second term is 19 and the fifth term is 152.
- c) Find the sum of the first 22 terms of the series $8+12+18+27+.....$
- d) $(4 + x)$, $(3 - 2x)$, and $(7 + 4x)$ are successive terms of an arithmetic progression. Find the value of x .

Question Two (2 marks)

Find the compound interest earned if \$10 000 is invested at 4%pa for five years.

Question Three (6 marks)

- a) The sum of the first three terms of an arithmetic progression is 15. The seventh term is 20 more than the fourth term. Find the first term and the common difference.
- b) Evaluate $\sum_{n=4}^{16} 5 \times 2^{n-1}$

Question Four (3 marks)

If $1 + 2x + 4x^2 + = \frac{3}{4}$, find the value of x .

Question Five (3 marks)

The sum of n terms of an arithmetic sequence is give by $S_n = 3n + 2n^2$.

- Find the first term.
- Find the common difference.

Question Six (5 marks)

- A woman invests \$5000 at the beginning of each year into a superannuation scheme. If interest is paid at the rate of 8% pa on the investment (compounded annually), how much will the investment be worth after 10 years?
- How much would the above investment be worth if the interest is compounded every 6 months? (Hint: 8%pa = 4% every six months)

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Q1. a) 9, 15, 21, ...

$a = 9, d = 6$

i) $T_n = a + (n-1)d$

$T_{31} = 9 + (30)6$
 $= 189$ (2)

ii) $S_n = \frac{n}{2}(a+d)$
 $= \frac{31}{2}(9+189)$
 $= 3069$ (2)

iii) $261 = 9 + (n-1)6$

$252 = 6n - 6$
 $6n = 258$
 $n = 43$ (2)

Yes as n an integer

iv) $9 + (n-1)6 < 500$

$6n - 6 < 491$
 $6n < 497$
 $n < 82.83$

ie 82 terms (2)

b) $ar = 19$ (1)

$ar^4 = 152$ (1)

$\frac{(2)}{(1)} \Rightarrow r^3 = 8$
 $r = 2$
 $a = 19/2$ (2)

c) $a = 8, r = \frac{12}{8}$
 $= 1.5$

$n = 22$
 $S_n = \frac{a(r^n - 1)}{r - 1}$
 $= \frac{8(1.5^{22} - 1)}{0.5}$
 $= 119,693.2425$ (2)

d) $(3-2x) - (4+x) = (7+9x) - (3-2x)$

$3 - 2x - 4 - x = 7 + 4x - 3 + 2x$

$-1 - 3x = 4 + 6x$

$-5 = 9x$

$x = -5/9$ (2)

Q2

$I = \$10000(1.04)^5 - \10000

$= \$2166.53$ (2)

Q3

a) $a + a + d + a + 2d = 15$

$3a + 3d = 15$

$a + d = 5$ (1)

$a + 3d + 20 = a + 6d$

$3d = 20$

$d = \frac{20}{3}$ (1)

$a = \frac{20}{3} = 5$
 $a = -5/3$ (3)

b) $\sum_{n=4}^{16} 5 \times 2^{n-1}$
 leave out 3 terms

ie $a = 40, n = 13, r = 2$ (1)

$S_n = \frac{40(2^{13} - 1)}{2 - 1}$ (1)

$= 327640$ (3)

Q4

$1 + 2x + 4x^2 + \dots + \frac{3}{4}$

$a = 1, r = 2x, S_{\infty} = \frac{3}{4}$

$S_{\infty} = \frac{a}{1-r}$

$\frac{3}{4} = \frac{1}{1-2x}$

$3 - 6x = 4$

$-6x = 1$

$x = -1/6$ (3)

Question 5

a) $S_1 = 3 + 2$
 $= 5$

ie $a = 5$

✓

b) $S_2 = 6 + 8$
 $= 14$

Now $T_2 = S_2 - a$

$= 14 - 5$

$= 9$ ✓

∴ $d = 9 - 5$

$= 4$ ✓

③

Question 6

a) The first \$5000 amounts to $\$5000(1.08)^{10}$

" second " " " $\$5000(1.08)^9$

" third " " " $\$5000(1.08)^8$ ✓

until the last " " " $\$5000(1.08)^1$

After ten years value of investments

$V = \$5000(1 + 1.08 + 1.08^2 + \dots + 1.08^{10})$ ③
 $= \$5000 \frac{(1.08^{10} - 1)1.08}{1.08 - 1}$ [geometric ✓ $r = 1.08, n = 10$
 $a = 1.08$

$= \$78227.44$ ✓

b) The first \$5000 amounts to $\$5000(1.04)^{20}$

" second " " " $\$5000(1.04)^{18}$

third " " " $\$5000(1.04)^{16}$ ✓

last " " " $\$5000(1.04)^2$

Value = $\frac{\$5000 \times 1.04^2 ((1.04^2)^{10} - 1)}{1.04^2 - 1}$ ②

$= \$78941.10$ ✓