$\left(\frac{81}{16}\right)^{-1\cdot25} = \left(\frac{3^4}{2^4}\right)^{-5/4} = \frac{3^{-5}}{2^{-5}} = \frac{1}{3^5} \times \frac{2^5}{1} = \frac{32}{243}$

Write in index form with positive indices $\frac{\sqrt[3]{a^2b^{-1}}}{\sqrt[3]{c^4}}$. Here the expression is $\frac{a^{2/3}\, b^{-1/3}}{c^{4/3}} = \frac{a^{2/3}}{b^{1/3}\, c^{4/3}}$

## SET 5A

1. Write each of the following with positive indices.

(a) $g\, h^{-2}$ (b) $\frac{a^{-1}}{b^{-2}}$ (c) $\frac{2^{-3}\, t^5}{3^{-1}\, v^{-2}}$ (d) $(5y^{-3}\, z)^{-1}$

2. Express in surd form.

(a) $a^{\frac{1}{2}}$ (b) $5^{-1}\, \ell^{-4/3}$ (c) $p^{1/5}\, q^{-3/5}$

(d) $a^{\frac{1}{4}}\, b^{-2/3}$ (e) $a^{2/3}\, (a-b)^{-3/4}$.

3. Express the following without root signs and with positive indices.

(a) $\sqrt[5]{c^2}$ (b) $\sqrt[3]{8h^{-1}k^2}$ (c) $\frac{\sqrt[4]{g^{-1}h^3}}{j^{-5}}$ (d) $\frac{\sqrt{(x-3y)^{-2}}}{x^{-2/3}\, y^3}$

4. Find the numerical values of the following.

(a) $9^{-2}$; $7^0$; $\frac{1}{4^{-3}}$; $\frac{5^{-1}}{3^{-2}}$; $27^{2/3}$; $16^{1\cdot25}$

(b) $4^{-3/2}$; $8^{-2/3}$; $100^{-1\cdot5}$; $243^{0\cdot4}$; $(256)^{-3/8}$

(c) $(11\frac{1}{9})^{\frac{1}{2}}$; $(6\frac{1}{4})^{-1\cdot5}$; $(1\frac{9}{16})^{-3/2}$; $(.001)^{-2/3}$

(d) $729^{-5/6} \div 128^{-3/7}$; $(\frac{9}{16})^{3/2} \div (\frac{8}{27})^{-5/3}$

5. (a) If $x = 16$, $y = 25$ evaluate

(i) $x^{1/2} + y^{1/2}$ (ii) $x^{-1/2} + y^{-1/2}$

(iii) $(y-x)^{1/2}$ (iv) $(\frac{x}{y})^{-1\cdot5}$

(b) If $x = 116$, $y = 100$ find the value of

(i) $\frac{(x-y)^{3/2}}{(x+y)^{2/3}}$ (ii) $\left[\frac{x^2-y^2}{2}\right]^{1/3}$

6. Simplify the following

(i) $y^{3/5} \times y^{-2/5} \times y^{4/5}$ (ii) $\left[6x^{-1/2} \div 2x^{2/3}\right] \times 3x^{5/6}$

(iii) $\frac{9b^{-3/4} \times 2b^{1/2}}{6b^{-1/4}}$ (iv) $\frac{(\sqrt[5]{a})^3 \times a^{-1/2}}{a^{-4/5} \div \sqrt{a^{-3}}}$

7. Study the following example

$$\left[\frac{9^{3n+1} \times 3^{1-n}}{27^{n-2} \times 243^{2-n}} = \frac{(3^2)^{3n+1} \times 3^{1-n}}{(3^3)^{n-2} \times (3^5)^{2-n}} = \frac{3^{6n+2} \times 3^{1-n}}{3^{3n-6} \times 3^{10-5n}}\right.$$
$$\left. = 3^{(6n+2)+(1-n)-(3n-6)-(10-5n)} = \underline{\underline{3^{7n-1}}}\right]$$

Now simplify the following

(a) $\frac{2^x \cdot 4^{x+1}}{8^{x-2}}$ (b) $\frac{3^{-n} \cdot 9^{2n-2}}{3^{3n-2} \cdot 27}$ (c) $\frac{25^{2n-1} \times 5^{1-n}}{25^{1-n} \times (5^n)^3}$

8. Using the fact that $12 = 2^2.3$, $1\frac{1}{3} = 2^2.3^{-1}$, $18 = 3^2.2$ simplify the following.

(a) $12^x \div 4^x$ (b) $6^{2-y} \times 2^y \div 3^{1-y}$ (c) $18^{2x} \div 12^{3x}$

(d) $\frac{12^{3x} \cdot 8^{-1}}{9^{-x} \cdot 4^{-2x} \cdot 6^x}$

9. (a) If $2^q = \frac{4^{3x+1}}{2^2 \cdot 8^{2x}}$ find q.

(b) If $2^x \cdot 3^y = \frac{8^{1/2} \cdot 6^{-3/2}}{\sqrt{3}}$ find x and y.

10. Study the following example carefully

$5^{n+1} + 5^{n-1} = 5^{n-1}[5^2 + 1] = 5^{n-1}.(26)$

Hence $\frac{5^{n+1} + 5^{n-1}}{5^{n-2} + 5^n} = \frac{5^{n-1}(5^2+1)}{5^{n-2}(5^2+1)} = \frac{5^{n-1}}{5^{n-2}} = 5.$

Now do the following exercises

(a) $\frac{3^{n+1} + 3^{n-1}}{3^{n+2} + 3^n}$ (b) $\frac{5^n + 5^{n+2}}{5^n}$ (c) $\frac{4^n - 2^{n-1}}{2^n - \frac{1}{2}}$

(d) $\frac{6^n + 3^n}{2^{n+1} + 2}$ (e) $\frac{6^{2a} - 3^a}{12^a - 1}$ (f) $\frac{10^n + 15^n}{2^4.3^n + 2^{n+4}}$

11. If $7^{2x-5} = 1$ show that $2x - 5 = 0$ (using $7^0 = 1$), hence find x. Use this approach to find x in each case below.

(a) $25^x = \frac{1}{\sqrt{125}}$ (b) $(\frac{1}{9})^{2x-1} = 3.(27^{-x})$

(c) $(.125)^x = \sqrt{0.5}$ (d) $(\frac{1}{5})^{1+x^2} = (\sqrt{5})^{8x+4}$

12. Solve the following equations.

(a) $9^x = 27$ (b) $9^x = \sqrt{3}$ (c) $5^{3x-4} = 1$

(d) $9^x.27^{x-2} = 3^{-x}$ (e) $8^{1-x}.2^{x-3} = 4$ (f) $(.01)^x = 100(10^{-x})$

13. Solve (a) $y^{4/3} = 81$ (b) $x^{-2} = 64$ (c) $x^{-2/3} = 16^{-1/6} \cdot 9^{1/3}$

14. Show that $2^{1/2}.3^{1/3} > 5^{1/4}.6^{1/6}$ by raising both sides to the twelfth power. (12 is the L.C.M. of 2, 3, 4, 6 the denominators of the indices). Use this approach to show

(a) $2^{2/3}.9^{1/3} > 2^{5/3}$ (b) $2.3^{1/4} < 7^{1/2}$

(c) $2^{1/2}.5^{1/3} > 3^{1/2}.7^{1/6}$ (d) $2^{-1/4}.243^{-1/12} > 2^{-5/12}.3^{-1/6}.7^{-1/12}$

15. Arrange in *ascending* order of magnitude.

(a) $3^{1/2}$, $2^{5/6}$, $(3/11)^{-1/3}$ (b) $\sqrt{2}$, $\sqrt[3]{\pi}$, $(1/4)^{-2/3}$

16. Study the following example carefully

If $125^x.5^y = \frac{1}{5}$ and $2^x = 4^y \div 32$

then $5^{3x+y} = 5^{-1}$ and $2^x = 2^{2y-5}$

solving $3x + y = -1$ and $x = 2y - 5$ simultaneously, we have $x = -1$ and $y = 2$.

Now solve the simultaneous equations below

(a) $\begin{cases} 5^{x+y} = 125 \\ 7^{x-y} = 1 \end{cases}$ (b) $\begin{cases} 8^x = 16 \div 2^y \\ 5^x \cdot 25^{-y} = \frac{1}{5} \end{cases}$

17. Study the following example

If $3^{2x} - 10.3^x + 9 = 0$, then if $u = 3^x$ we have

$u^2 - 10u + 9 = 0$, i.e. $(u-9)(u-1) = 0$

$\therefore$ $u = 3^x = 9$ or $1$ i.e. $\underline{x = 2 \text{ or } 0}$.

Solve for x, each of the following

(a) $2^{2x} - 3.2^x + 2 = 0$ (b) $49^x - 6.7^x = 7$

(c) $9^x = 2.3^{x+1} + 27$ (d) $x^{3/2} - 1728x^{-3/2} + 37 = 0$

18. Use tables to solve the following

(a) $9^x = 7$ (b) $5^{x+2} = 3$ (c) $9^x = 3^{x+2} + 10$

(d) $4^x - 13.2^x + 40 = 0$

19. Show that (i) $(x - x^{-1})^2 = x^2 - 2 + x^{-2}$, and that

$(x - x^{-1})^3 = x^3 - 3x + 3x^{-1} - x^{-3}$

(ii) $(x^{1/3} - y^{1/3})(x^{2/3} + x^{1/3}y^{1/3} + y^{2/3}) = x - y$

on multiplication.

20. Simplify the following

(i) $a^{1/3}(a^{2/3} + 5a^{-1/3})$ (ii) $(x^{1/2} + y^{1/2})(x^{1/2} - y^{1/2})$

(iii) $(x^{-1} - y^{-1})^2$ (iv) $(x^{1/2} + x^{-1/2})^2$

(v) $(x^{1/3} - 1)(x^{2/3} + x^{1/3} + 1)$

(vi) $(x^{3/2} + 1 + x^{-3/2})(x^{3/2} - 1 + x^{-3/2})$

(vii) $(x^2 - x^{-2})^3$

21. (i) If $x = a^{3/2} + a^{-3/2}$, $y = a^{3/2} - a^{-3/2}$, find the values of $x(x+y)$ and $x^2 - y^2$.



25
2 25
12 5
500
625

(ii) If $z = 2 + \sqrt{3}$, find the values of $z + z^{-1}$, $z^2 + z^{-2}$ in simplest surd form.

(iii) Given $x = 2^{1/3} + 4^{1/3}$, find, without tables, the value of $x^3 - 6x$.

22. Show that if $\frac{a^{-1} + b^{-1}}{a^{-2} - b^{-2}} = \frac{a^{-1} + b^{-1}}{(a^{-1}+b^{-1})(a^{-1}-b^{-1})}$ factorising the difference of two squares,

$$= \frac{1}{a^{-1} - b^{-1}} = \frac{1}{\frac{1}{a} - \frac{1}{b}} = \frac{1}{\frac{b-a}{ab}} = \frac{ab}{b-a}$$

[Note that $\frac{1}{a^{-1} - b^{-1}} \neq \frac{1}{a^{-1}} - \frac{1}{b^{-1}}$

23. Show that $a - b = (a^{1/2} - b^{1/2})(a^{1/2} + b^{1/2})$ as a difference of 2 squares, and that $a^{3/2} - b^{3/2} = (a^{1/2} - b^{1/2})(a + a^{1/2}b^{1/2} + b)$ as a difference of 2 cubes. Hence simplify $\frac{a - b}{a^{3/2} - b^{3/2}}$.

24. Given that $(xy^{-1} - yx^{-1}) \div (x^2y^{-2} - y^2x^{-2})$, show that the expression equals

$$\left(\frac{x}{y} - \frac{y}{x}\right) \div \left(\frac{x^2}{y^2} - \frac{y^2}{x^2}\right) = \left(\frac{x^2 - y^2}{xy}\right)\left(\frac{x^2y^2}{x^4 - y^4}\right) = \frac{xy}{x^2 + y^2}$$

25. Simplify

(a) $x^{-1} - y^{-1}$ (b) $\frac{1}{x^{-1} - y^{-1}}$ (c) $\frac{x^{-1} - y^{-1}}{x^{-2} - y^{-2}}$

(d) $\frac{x^{-2} - y^{-2}}{x^{-3} - y^{-3}}$ (e) $\frac{a^{\frac{1}{2}} b^{-\frac{1}{2}} - a^{-\frac{1}{2}} b^{\frac{1}{2}}}{a^{-1} - b^{-1}}$

## SEQUENCES

When we deal with the set of all positive odd numbers, i.e. $\{1, 3, 5, 7, \ldots\ldots, (2n-1), \ldots\}$ where n is a positive integer, we are dealing with a set of numbers whose members are arranged in some definite order (or obeys some rule). Such a set is an example of a *sequence* of numbers. Consider the following sets of numbers

(i) $\{1, 4, 9, 16, 25, \ldots\ldots, n^2, \ldots\}$

(ii)

| position of term | 1 | 2 | 3 | 4 | ..... | n | ..... |
|---|---|---|---|---|---|---|---|
| value of term | $\frac{1}{2}$ | $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{4}{5}$ | | $\frac{n}{n+1}$ | |

(iii) $\{\frac{1}{3}, \frac{1}{7}, \frac{1}{11}, \frac{1}{15}, \frac{1}{19}\}$

Each of the above are examples of sequences. (iii) is called a finite sequence while (i) and (ii) are infinite

# CHAPTER 5.

## SET 5A

1. (a) $\frac{g}{h^2}$ (b) $\frac{b^2}{a}$ (c) $\frac{3v^2t^5}{8}$ (d) $\frac{y^3}{5z}$ 2. (a) $\sqrt{a}$ (b) $\frac{1}{5.\sqrt[3]{\ell^4}}$

(c) $\sqrt[5]{\frac{p}{q^3}}$ (d) $\frac{\sqrt[4]{a}}{\sqrt[3]{b^2}}$ (e) $\frac{\sqrt[3]{a^2}}{\sqrt[4]{(a-b)^3}}$ 3. (a) $c^{2/5}$ (b) $\frac{2k^{2/3}}{h^{1/3}}$

(c) $\frac{h^{3/4}}{g^{1/4}}$ (d) $\frac{x^{2/3}}{y^3(x-3y)}$ 4. (a) $\frac{1}{81}$, 1, 64, $\frac{9}{5}$, 9, 32

(b) $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{1000}$, 9, $\frac{1}{8}$. (c) $\frac{10}{3}$, $\frac{8}{125}$, $\frac{64}{125}$, 100 (d) $\frac{8}{243}$, $\frac{1}{18}$

5. (a) (i) 9 (ii) $\frac{9}{20}$ (iii) 3 (iv) $\frac{125}{64}$ (b) (i) $\frac{16}{9}$ (ii) 12

6. (i) y (ii) $9x^{-1/3}$ (iii) 3 (iv) $a^{-3/5}$ 7. (a) 256 (b) $\frac{1}{243}$

(c) $5^{2n-3}$ 8. (a) $3^x$ (b) 12 (c) $3^x2^{-4x}$ (d) $2^{9x-3}3^{4x}$

9. (a) q = 0 (b) x = 0, y = -2 10. (a) $\frac{1}{3}$ (b) 26 (c) $2^n$

(d) $\frac{3^n}{2}$ (e) $3^a$ (f) $\frac{5^n}{16}$ 11. (a) $x = \frac{-3}{4}$ (b) x = 1 (c) $x = \frac{1}{6}$

(d) x = -1 or -3 12. (a) x = 1.5 (b) $x = \frac{1}{4}$ (c) $x = \frac{4}{3}$

(d) x = 1 (e) x = -1 (f) x = -2 13. (a) y = 27 (b) $x = \frac{1}{8}$

(c) $x = \frac{2}{3}$ 15. (a) $2^{5/6} > 3^{1/2} > (\frac{3}{11})^{-1/3}$ (b) $(\frac{1}{4})^{-2/3} > \sqrt[3]{\pi} > \sqrt{2}$

16. (a) $x = y = 1\frac{1}{2}$ (b) x = y = 1 17. (a) x = 0 or 1

(b) x = 1 (c) x = 2 (d) x = 9 18. (a) x = .89 (b) x = -1.32

(c) x = 2.1 (d) x = 3 or 2.3 20. (i) a + 5 (ii) x - y

(iii) $x^{-2}+y^{-2}-2x^{-1}y^{-1}$ (iv) $x + \frac{1}{x} + 2$ (v) x - 1 (vi) $x^3+x^{-3}+1$

(vii) $x^6-3x^2+3x^{-2}-x^{-6}$ 21. (i) $2a^3+2$ and 4 (ii) 4 and 14 (iii) 6

23. $\frac{\sqrt{a}+\sqrt{b}}{a+b+\sqrt{ab}}$ 24. (a) $\frac{y-x}{xy}$ (b) $\frac{xy}{y-x}$

(c) $\frac{xy}{x+y}$ (d) $\frac{xy(y+x)}{y^2+xy+x^2}$ (e) $-a^{1/2}b^{1/2}$

## SET 5B

1. (a) 1,3,5,7,9 (b) 1,4,9,16,25 (c) 2,4,8,16,32 (d) $\frac{1}{3},\frac{1}{9},\frac{1}{27},\frac{1}{81},\frac{1}{243}$

(e) $2\frac{1}{3},2\frac{5}{9},2\frac{19}{27},2\frac{65}{81},2\frac{211}{243}$ (f) $\frac{5}{4}$, 1, $\frac{9}{10}$, $\frac{11}{13}$, $\frac{13}{16}$ (g) $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{11}$, $\frac{1}{15}$, $\frac{1}{19}$

(h) 2,6,12,20,30 (i) 1, $\frac{1}{8}$, $\frac{1}{27}$, $\frac{1}{64}$, $\frac{1}{125}$ (j) 1,5,19,65,211

(k) -1,2,-3,4,-5 (ℓ) -1, $\frac{1}{2},\frac{-1}{3}$, $\frac{1}{4},\frac{-1}{5}$ (m) 1,-2,4,-8,16

(n) 4,-8,16,-32,64 (o) -3,8,-13,18,-23 (p) $1,2\frac{1}{5},2\frac{17}{25},2\frac{109}{125},2\frac{593}{625}$

(q) x, $\frac{x^2}{2}$, $\frac{x^3}{3}$, $\frac{x^4}{4}$, $\frac{x^5}{5}$ (r) -1, $\frac{x}{4},\frac{-x^2}{9},\frac{x^3}{16},\frac{-x^4}{25}$ (s) $\frac{x}{3}$, $\frac{4x}{3}$, $\frac{9x}{3}$, $\frac{16x}{3}$, $\frac{25x}{3}$

(t) a,a+d,a+2d,a+3d,a+4d (u) a,ar,$ar^2$,$ar^3$,$ar^4$ (v) -1,1,-1,1,-1

(w) 1,0,-1,0,1 (x) $\frac{1}{a+3}$, $\frac{2}{a+6}$, $\frac{3}{a+9}$, $\frac{4}{a+12}$, $\frac{5}{a+15}$

(y) a, a(r+1), $a(r^2+r+1)$, $a(r^3+r^2+r+1)$, $a(r^4+r^3+r^2+r+1)$

(z) $\frac{x-1}{y+1}$, $\frac{2-x}{y+2}$, $\frac{x-3}{y+3}$, $\frac{4-x}{y+4}$, $\frac{x-5}{y+5}$ 2. (a) $\frac{49}{48}$, (b) 12 (c) $4\frac{7}{8}$ (d) $\frac{13}{14}$

3. (i) $u_{127}$ = 505, 255 is not a member (ii) $u_{133}$ = 382, $u_{32}$=79, $u_6$=1

4. (i) 20th, 15th (ii) 91 is a member 5. 96, $\frac{3}{4}$ are members