chapter • 3

Numbers and Sets III

- Sets

KEY NOTES

1 **Sets**
A **set** is a collection of objects that are clearly defined.

The objects in set A are the **elements** of set A.

If p is an element of set A, we write $p \in A$.

If p is not an element of set A, we write $p \notin A$.

2 **Subsets**
If each element of set A is also an element of set B, then set A is called a **subset** of set B.

We write this as $A \subset B$.

If A is not a subset of B, we write $A \not\subset B$.

3 **Equal sets**
Two sets are equal if both contain the same elements.

We say that $A = B$ if, and only if, $A \subset B$. and $B \subset A$.

If A is not equal to B, we write this as $A \neq B$.

4 **Number sets**
(a) $\mathbb{N}$ represents the set of positive integers.
$\mathbb{N} = \{1, 2, 3, ...\}$

(b) $\mathbb{Z}$ represents the set of integers.
$\mathbb{Z} = \{......, -3, -2, -1, 0, 1, 2, 3, ...\}$

(c) $\mathbb{Q}$ represents the set of rational numbers.

$$\mathbb{Q} = \left\{x \middle| x = \frac{p}{q}, p \in Z, q \in Z, q \neq 0\right\}$$

(d) $\mathbb{R}$ is the set of real numbers. It comprises all rational and irrational numbers.

(e) $\mathbb{C}$ is the set of complex numbers.
$\mathbb{C} = \{x|x \in a + ib, a \in R, b \in R, i^2 = -1\}$

These sets of numbers are connected by this relationship:

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$$

5 **Empty set**
An empty set is a set that does not contain any elements.

It is represented by ϕ.

An empty set is regarded as a subset of any set X, that is $\phi \subset X$.

6 **Universal set**
The universal set is the set that contains all the elements of the sets in a discussion. The symbol for the universal set is $\mathscr{E}$.

7 **Operations of set**
(a) **Union of sets**
The union of set A and set B, is represented by $A \cup B$. This is the set that contains all the elements belonging to set A or set B, or both sets.

$$A \cup B = \{x|x \in A \text{ atau } x \in B\}.$$

(b) **Intersection of sets**
The intersection of set A and set B, is represented by $A \cap B$. This is the set that contains all the elements belonging to both set A or set B.

$$A \cap B = \{x | x \in A \text{ and } x \in B\}.$$

If $A \cap B = \phi$, then A and B are said to be mutually exclusive.

(c) **Difference between two sets**
The difference between set A and set B is the set that contains all the elements of set A but not the elements of set B.

$$A - B = \{x | x \in A \text{ but } x \notin B\}.$$

Note: $(A - B) \cap B = \phi$.

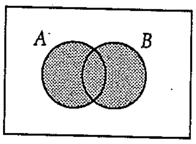
(d) **Complement of a set**
The complement of set A is written as A' (atau A^c atau $\bar{A}$). This is the set that contains all the elements that does not belong to set A.

$$A' = \{x | x \in \mathscr{E}, x \notin A\}$$

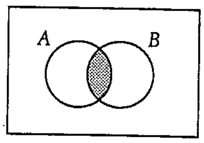
Note: (a) A' is the difference between the universal set $\mathscr{E}$ with set A.
(b) $(A')' = A$

(e) **Venn diagram**
A Venn diagram is a geometrical representation of sets using shaded areas. A Venn diagram shows the overall relationship between the sets.

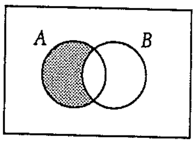
Example:



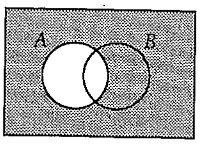
$A \cup B$ is shaded



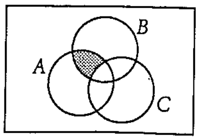
$A \cap B$ is shaded



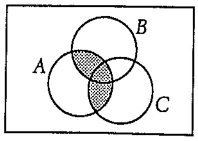
$A - B$ is shaded



A' is shaded



$A \cap (B - C)$ is shaded



$A \cap (B \cup C)$ is shaded

8 **Algebraic laws of sets**

(a) $A \cup A = A, A \cap A = A$
(b) $(A \cup B) \cup C = A \cup (B \cup C)$, $(A \cap B) \cap C = A \cap (B \cap C)$
(c) $A \cup B = B \cup A; A \cap B = B \cap A$
(d) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(e) $A \cup \phi = A \,;\, A \cap \mathscr{E} = A$
(f) $A \cup \mathscr{E} = \mathscr{E} \,;\, A \cap \phi = \phi$
(g) $A \cup A' = \mathscr{E} \,;\, A \cap A' = \phi$
(h) $(A')' = A \,;\, \mathscr{E}' = \phi \,;\, \phi' = \mathscr{E}$
(i) $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

9 **General rules**

(a) $A - B = A \cap B'$
(b) $A - (B \cup C) = (A - B) \cap (A - C)$
(c) $A - (B \cap C) = (A - B) \cup (A - C)$

WORKED EXAMPLES

Example 1

Given $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$A = \{3, 4, 5\}$
$B = \{2, 4, 6, 8\}$
$C = \{1, 3, 5, 7\}$

(i) List all the subsets of A.

(ii) Write down each of the following sets:
(a) B'
(b) $C - A$
(c) $(A \cap B)'$
(d) $(A \cup C) \cap (B \cup C)$

Solution:

(i) The subsets of A are as follows:
ϕ, $\{3\}$, $\{4\}$, $\{5\}$, $\{3, 4\}$, $\{3, 5\}$, $\{4, 5\}$, $\{3, 4, 5\}$

(ii) (a) $B' = \{1, 3, 5, 7\}$

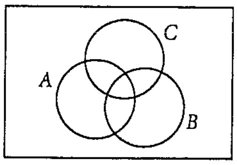
(b) $C - A = \{1, 7\}$

(c) $A \cap B = \{4\}$
$\therefore (A \cap B)' = \{1, 2, 3, 5, 6, 7, 8\}$

(d) $A \cup C = \{1, 3, 4, 5, 7\}$
$B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$\therefore (A \cup C) \cap (B \cup C) = \{1, 3, 4, 5, 7\}$

Example 2

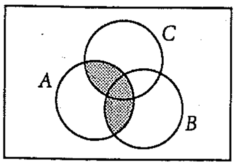
In the Venn diagram below,



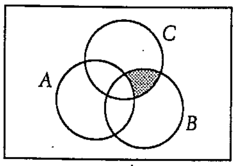
shade the areas for these sets:
(i) $A \cap (B \cup C)$
(ii) $B \cap (C - A)$
(iii) $A \cap (B - C)'$

Solution:

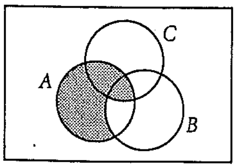
(i)



(ii)



(iii)

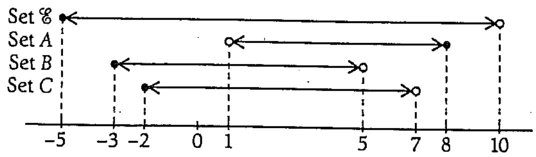


Example 3

Given $\mathscr{E} = \{x \mid -5 \leqslant x < 10, x \in \mathbb{R}\}$
$A = \{x \mid 1 < x \leqslant 8, x \in \mathbb{R}\}$
$B = \{x \mid -3 \leqslant x < 5, x \in \mathbb{R}\}$
$C = \{x \mid -2 \leqslant x < 7, x \in \mathbb{R}\}$

Write down each of the following set:
(a) $A \cup B \cup C$
(b) $A - B$
(c) $(A - B) \cap C$
(d) $(B \cap C) - A$
(e) $(A \cap C) \cap B'$

Solution:



(a) $A \cup B \cup C = \{x \mid -3 \leqslant x \leqslant 8, x \in \mathbb{R}\}$
(b) $A - B = \{x \mid 5 \leqslant x \leqslant 8, x \in \mathbb{R}\}$
(c) $(A - B) \cap C = \{x \mid 5 \leqslant x < 7, x \in \mathbb{R}\}$
(d) $B \cap C = \{x \mid -2 \leqslant x < 5, x \in \mathbb{R}\}$
$\therefore (B \cap C) - A = \{x \mid -2 \leqslant x \leqslant 1, x \in \mathbb{R}\}$
(e) $A \cap C = \{x \mid 1 < x < 7, x \in \mathbb{R}\}$
$B' = \{x \mid -5 \leqslant x < -3 \text{ or } 5 \leqslant x < 10, x \in \mathbb{R}\}$
$\therefore (A \cap C) \cap B' = \{x \mid 5 \leqslant x < 7, x \in \mathbb{R}\}$

Example 4

Using the algebraic laws of sets, show that for any set A and set B,

$$A \cup B = A \cup (B \cap A') = B \cup (A \cap B')$$

Solution:

$A \cup (B \cap A') = (A \cup B) \cap (A \cup A')$
$= (A \cup B) \cap \mathscr{E}$
$= A \cup B$

$B \cup (A \cap B') = (B \cup A) \cap (B \cup B')$
$= (B \cup A) \cap \mathscr{E}$
$= B \cup A$
$= A \cup B$

$\therefore A \cup B = A \cup (B \cap A') = B \cup (A \cap B')$

Example 5

Using the algebraic laws of sets, show that for any set A and set B,

(i) $B \cap (B - A)' = A \cap B$

(ii) $A' \cap (B - A)' = (A \cup B)'$

Solution:

(i)
$$\begin{aligned} B \cap (B - A)' &= B \cap (B \cap A')' \\ &= B \cap (B' \cup A) \\ &= (B \cap B') \cup (B \cap A) \\ &= \phi \cup (B \cap A) \\ &= B \cap A \\ &= A \cap B \end{aligned}$$

(ii)
$$\begin{aligned} A' \cap (B - A)' &= A' \cap (B \cap A')' \\ &= A' \cap (B' \cup A) \\ &= (A' \cap B') \cup (A' \cap A) \\ &= (A' \cap B') \cup \phi \\ &= A' \cap B' \\ &= (A \cup B)' \end{aligned}$$

EXERCISE

1 Given $\mathscr{E} = \{a, b, c, d, e\}$
$A = \{a, b, d\}$
$B = \{b, d, e\}$

Find:
(a) $A \cap B$
(b) B'
(c) $A' \cap B$
(d) $A' \cap B'$
(e) $B - A$
(f) $B' - A'$
(g) $(A \cup B)'$

2 Given $\mathscr{E} = \{a, b, c, d, e, f, g\}$
$A = \{a, b, c, d, e\}$
$B = \{a, c, e, g\}$
$C = \{b, d, f, g\}$

Find:
(a) $B \cap A$
(b) $C - B$
(c) $C' \cap A$
(d) $(A - B')'$
(e) $(A - C)'$
(f) $(A \cap A')'$

3 Given $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$A = \{1, 3, 5, 7\}$
$B = \{1, 2, 3, 4\}$
$C = \{2, 4, 6, 8\}$

List the elements of each of the following sets:
(a) $A \cap B$
(b) $A \cup B$
(c) A'
(d) $(A \cap B) \cup C$
(e) $(A \cup C) \cap (B \cup C)$
(f) $(A - B) \cap (A - C)$

4 Determine if the set
$A = \{x | x \in Q \mid \sqrt{2} < x < \sqrt{3}\}$
is an empty set.

5 If $A = \{1, 0\}$, state if the following statements are true or false.
(a) $0 \in A$
(b) $\{0\} \in A$
(c) $\phi \in A$
(d) $\phi \subset A$
(e) $\{0\} \subset A$
(f) $0 \subset A$

6 (i) List all the subsets of $\{\phi\}$.
(ii) Is the set $\{\phi\}$ an empty set?
(iii) Is the statement $\phi \subseteq \{\phi\}$ true?
(iv) Is the statement $\{\phi\} \subseteq \phi$ true?

7 Given $\mathscr{E} = \{x| -8 \leqslant x < 15, x \in \mathbb{R}\}$
$A = \{x| -3 \leqslant x < 10, x \in \mathbb{R}\}$
$B = \{x| -5 \leqslant x < 6, x \in \mathbb{R}\}$
$C = \{x| -7 \leqslant x < 8, x \in \mathbb{R}\}$

Write down each of the following set:
(a) $A \cup B \cup C$
(b) $(A \cup B) \cap C$
(c) $(A - B) \cap C$
(d) $(A \cap B) - C$
(e) $A' \cap B' \cap C'$

8 Using the algebraic laws of sets, show that, for any set A and set B,
(i) $A \cup (A' \cap B) = A \cup B$
(ii) $(A \cup B) \cap (A \cup B') = A$

9 For any set A and set B, verify the following results using the algebraic laws of sets.
(i) $A \cap (A' \cap B)' = A$
(ii) $(A \cap B) \cup (A \cap B') = A$

10 For any set P and set Q, verify the following results using the algebraic laws of sets.
(i) $P \cap (P \cap Q)' = P \cap Q'$
(ii) $P \cup (P' \cup Q)' = P$

11 For any set A and set B, verify the following results using the algebraic laws of sets.
(i) $A - (B \cup C) = (A - B) \cap (A - C)$
(ii) $A - (B \cap C) = (A - B) \cup (A - C)$

12 Using the algebraic laws of sets, prove that for any set A and set B,
(i) $A - B = B' - A'$
(ii) $A' - B = (A \cup B)'$

13 For any sets A, B, and C, use the algebraic laws of sets to prove that

$$A \cap (B - C) = (A \cap B) - (A \cap C)$$

14 For any sets A and B, use algebraic laws of sets to show that

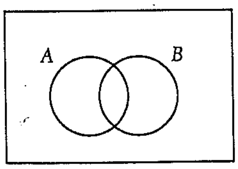
$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$

15 Given $A \cup B = \{1, 2, 3, 4, 5\}$
$A \cup C = \{1, 2, 3, 4, 5, 7\}$
$A \cap B = \{2, 4\}$
$A \cap C = \phi$
$B \cap C = \{1, 3, 5\}$

Write down sets A, B, and C.

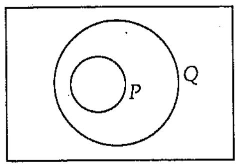
16 Using a Venn diagram, show that the result $A \cup (B - C) = (A \cup B) - (A \cup C)$ is not true.

17



In the Venn diagram above, shade the areas that represent:
(a) $B - A$
(b) $A' \cup B$
(c) $A \cap B'$
(d) $A' - B'$

18



In the Venn diagram above, shade the areas that represent:
(a) $Q - P$
(b) $P' \cup Q$
(c) $P \cap Q'$
(d) $P' - Q'$

19 In a class of 40 pupils, 17 will sit for the Mathematics paper, 18 will take the Physics paper, 6 will sit for Physics and Chemistry, 14 will take Mathematics but not Chemistry, 8 will take Mathematics only, 7 will take Mathematics and Physics, and 2 will not take Mathematics, Physics and Chemistry.

Find the number of pupils who will take Chemistry.

20 You have three sets A, B and C.

If $A \cap B = \{c, e\}$
$B \cap C = \{a, d\}$
$A \cap C = \phi$
$B - C = \{c, e, f, h\}$
$C - B = \{b\}$
$A \cup C = \{a, b, c, d, e, g\}$

write the sets A, B, and C.

ANSWERS

3 Numbers and Sets III

1 (a) $\{b, d\}$ (b) $\{a, c\}$
(c) $\{e\}$ (d) $\{c\}$
(e) $\{e\}$ (f) $\{a\}$
(g) $\{c\}$

2 (a) $\{a, c, e\}$ (b) $\{b, d, f\}$
(c) $\{a, c, e\}$ (d) $\{b, d, f, g\}$
(e) $\{b, d, f, g\}$ (f) ℰ

3 (a) $\{1, 3\}$
(b) $\{1, 2, 3, 4, 5, 7\}$
(c) $\{2, 4, 6, 8\}$
(d) $\{1, 2, 3, 4, 6, 8\}$
(e) $\{1, 2, 3, 4, 6, 8\}$
(f) $\{5, 7\}$

4 No

5 (a) True (b) False
(c) False (d) True
(e) True (f) False

6 (i) ϕ, $\{\phi\}$ (ii) No
(iii) Yes (iv) No

7 (a) $\{x \mid -7 \leqslant x < 10, x \in \mathbb{R}\}$
(b) $\{x \mid -5 \leqslant x < 8, x \in \mathbb{R}\}$
(c) $\{x \mid 6 \leqslant x < 8, x \in \mathbb{R}\}$
(d) ϕ
(e) $\{x \mid -8 \leqslant x < -7$ or $10 \leqslant x < 15, x \in \mathbb{R}\}$

8 –

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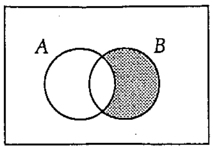
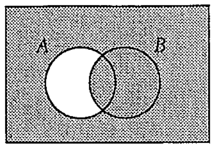
15 $A = \{2, 4\}$
$B = \{1, 2, 3, 4, 5\}$
$C = \{1, 3, 5, 7\}$

16 –

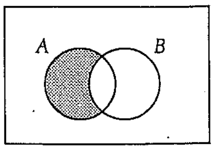
17



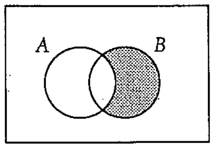
(a) $B - A$



(b) $A' \cup B$

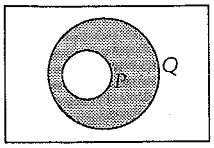


(c) $A \cap B'$

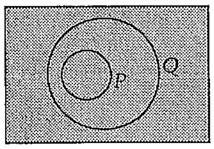


(d) $A' - B'$

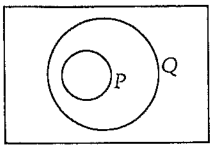
18



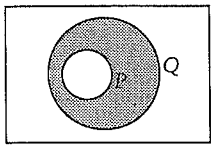
(a) $Q - P$



(b) $P' \cup Q$



(c) $P \cap Q'$



(d) $P' - Q'$

19 18 people

20 $A = \{c, e, g\}$
$B = \{a, c, d, e, f, h\}$
$C = \{a, b, d\}$