HIGHER SCHOOL CERTIFICATE EXAMINATION TRIAL PAPER

2001

MATHEMATICS

Time Allowed – Three Hours
(Plus 5 minutes reading time)

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Directions to Candidates

- Attempt ALL questions.
- All questions are of equal value.
- All necessary working should be shown in every question.
- Board-approved calculators may be used.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \quad \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

YEAR 12 – TRIAL 2001 – Mathematics

QUESTION 1 MARKS

a) Find the value of e^2 correct to two decimal places. 1

b) Factorise $49x^2 - 4$. 1

c) Find the exact value of $\tan \frac{\pi}{3}$. 1

d) Express $3\sqrt{18} - 4\sqrt{8}$ in its simplest surd form. 2

e) Solve $1 - 2x < 5$ and graph the solution on the number line. 3

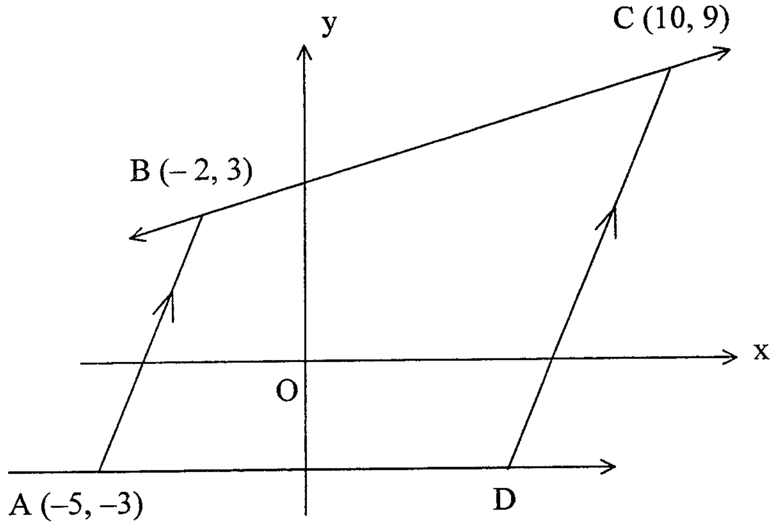
f) Solve the simultaneous equations : 2

$$x - 2y = 8$$
$$5x + 3y = 1$$

g) Graph the parabola $y = 4 - x^2$ on the number plane. 2

QUESTION 2 **MARKS**

On a number plane the points A (– 5, – 3), B (– 2, 3), C (10, 9) and D form a trapezium, in which AB is parallel to DC. AD is parallel to the x axis.



NOT TO SCALE

a) Show that the gradient of the line DC is 2. 1

b) Find the equation of the line DC. 2

c) Show that the coordinates of the point D are (4, –3). 2

d) Find the distance BC. 1

e) Find the equation of the circle centred at C with radius BC. 2

f) Show that the point D lies on the circle. 1

g) Find the coordinates of the midpoint of BD. 1

h) The point E is on the line BA produced such that BCDE is a rhombus. Find the coordinate of E. 2

QUESTION 3 **MARKS**

a) Differentiate the following functions:

i) $\log_e(3x+1)$ 1

ii) $e^x \cos x$ 2

b) Find $\int \frac{4}{2x+1}\, dx$. 2

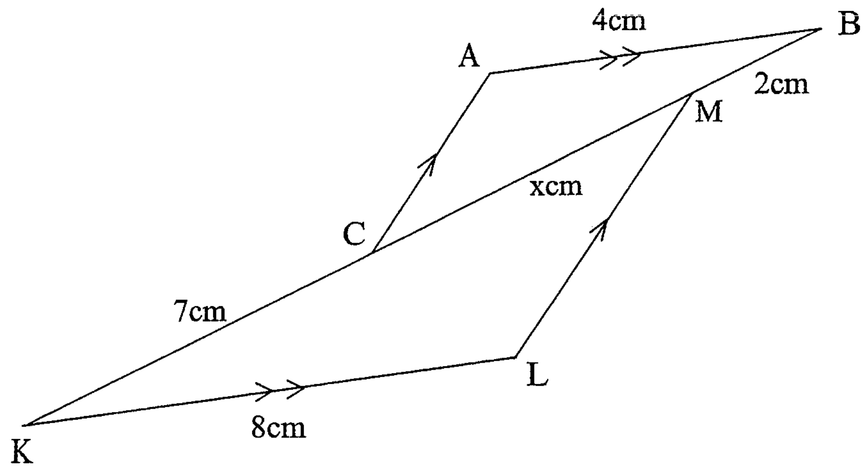
c) Evaluate $\int_0^{\frac{\pi}{6}} 2\sec^2 2x\, dx$. 2

d) Evaluate $\int_0^1 (x^2+1)^2\, dx$. 2

e) Find the equation of the tangent to the curve $y = e^{2x}$ at the point $P\left(\frac{1}{2}, e\right)$. 3

QUESTION 4 **MARKS**

a)



NOT TO SCALE

In the diagram AB is parallel to KL and AC is parallel to ML.
AB = 4cm, MB = 2cm and KC = 7cm.

i) Show that ΔABC is similar to ΔKLM. 3

ii) Find the length of CM. 2

b) The discriminant of $x^2 - 2x + p = 0$ is $4 - 4p$. 2
For what values of p does this equation have real roots?

c) In their first month of operation the 'Computer Experts' sold 200 computers and they increased their sales by 50 computers each month in the first three years of operation.

i) How many computers did they sell in the last month of the third year? 2

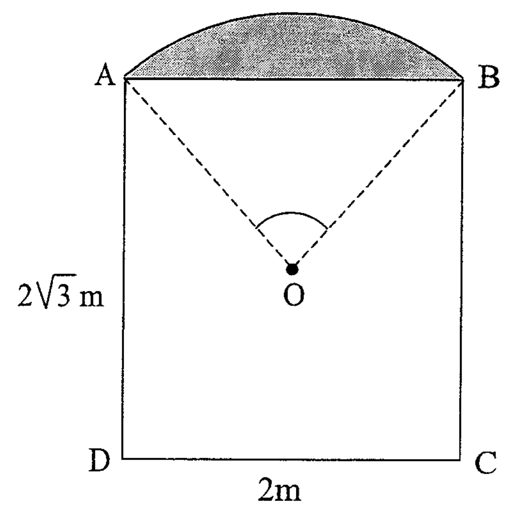
ii) How many computers did they sell over the entire period of three years? 3

QUESTION 5 **MARKS**

a) Solve $\sin \alpha = 0.6$ for $0 \le \alpha \le 2\pi$. Express your answer in radians correct to four significant figures. 2

b) The diagram shows an ancient window which consists of a rectangle ABCD with height $2\sqrt{3}$ m and width 2m surmounted by a minor segment of a circle which is gold stained glass.

The centre of the circle is at O, the point of intersection of the diagonals of the rectangle.



i) Explain why $\angle AOB = \frac{\pi}{3}$ 2

ii) Find the area of the golden minor segment. 3

c) The number of copies released of a home video of a famous movie increased exponentially according to the formula:

$N = Ae^{kt}$, where t is the time in weeks after the movie was first released.

Initially 10 000 copies were released and the number doubled after two weeks.

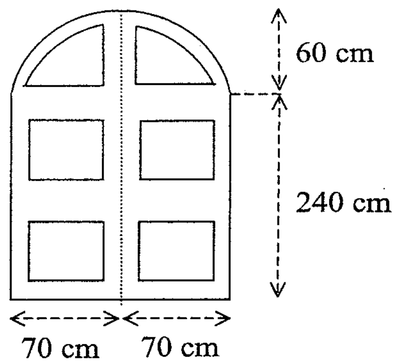
i) Calculate the values of A and k. 2

ii) How many copies were released after 10 weeks? 2

iii) At what rate was the number of copies increasing after 4 weeks? (Answer correct to the nearest whole number.) 1

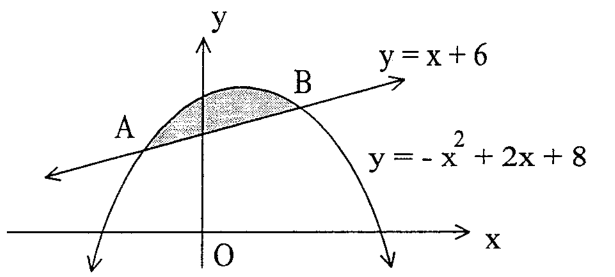
QUESTION 6 **MARKS**

a) The diagram shows a door of an old building which consists of a rectangle surmounted by a parabolic arch. 3



Use Simpson's rule with 3 function values to find an estimate for the area of the door.

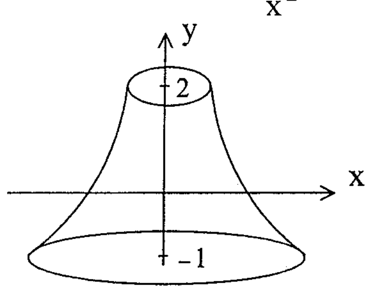
b) The diagram below shows the graphs of
$y = -x^2 + 2x + 8$ and $y = x + 6$.



i) Find the x values of the points of intersection, A and B. 2

ii) Find the shaded area bounded by the curves and the straight line. 3

c) A liquor bottle is obtained by rotating about the y axis the part of the curve $y = \frac{1}{x^2} - 2$ between $y = -1$ and $y = 2$. 4



Find the exact volume of the bottle.

QUESTION 7

MARKS

a) Solve $\log_e x = \log_e (2x-1)$. 2

b) The velocity V in m/s of a particle moving in a straight line is given by:

$V = 36 - 4e^{2t}$, where t is the time in seconds.

i) Find the initial velocity of the particle. 1

ii) Determine the exact time at which the particle comes to rest. 2

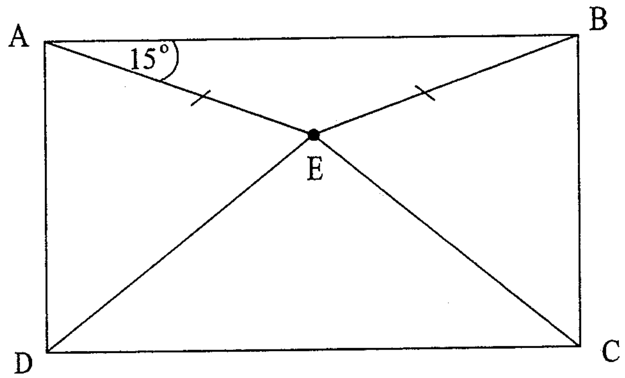
iii) Sketch the graph of the velocity V of the particle as a function of time t. 3

iv) Find the distance travelled by the particle between $t = 0$ and $t = 2\log_e 3$ 2

v) Find an expression for the acceleration a in terms of V. 2

QUESTION 8 **MARKS**

a) E is a point inside the rectangle ABCD such that $AE = BE$ and $\angle EAB = 15°$.



NOT TO SCALE

i) Explain why $\angle DAE = \angle CBE$. 2

ii) Prove that Δ ADE is congruent to Δ BCE. 3

iii) Hence, prove that Δ DEC is isosceles. 1

b) Consider the function :

$y = 4\log_e x + 6 - x$ for $1 \le x \le 10$.

i) Find the stationary point and determine its nature. 2

ii) Show that the curve is concave down throughout the domain. 1

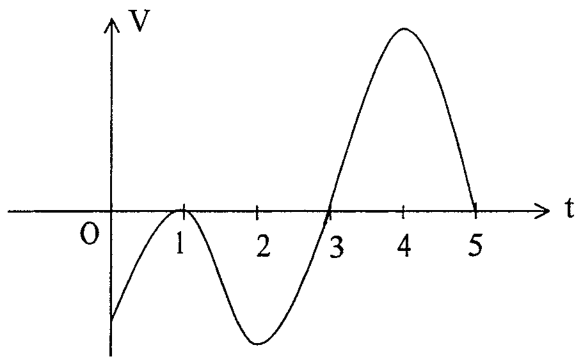
iii) Using an appropriate scale, neatly sketch the curve for the domain $1 \le x \le 10$. 3

QUESTION 9 **MARKS**

a) i) Sketch the graph $y = 2e^x$. 1

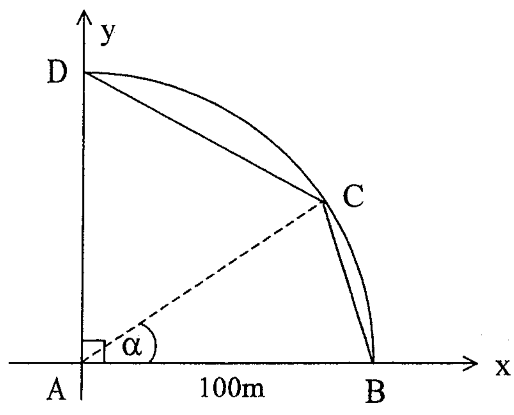
ii) Use your graph to find the number of solutions of the equation $2e^x - x = 2$. 2

b) The graph below shows the velocity V in m/s of a particle moving along the x axis for 5 sec. 3



Sketch the graph of the displacement x metres as a function of time t seconds, given that the particle was at x = 0 when t = 1sec and t = 4sec.

c) ABCD is a quadrilateral inscribed in a quarter of a circle centred at A with radius 100m. The points B and D lie on the x and y axes and the point C moves on the circle such that $\angle CAB = \alpha$ as shown in the diagram below.



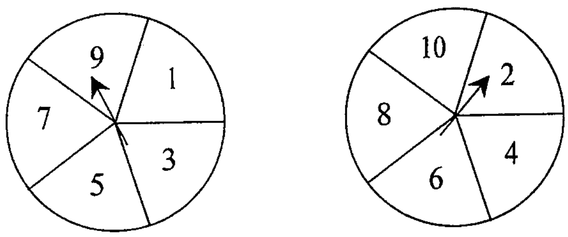
i) Show that the area of the quadrilateral ABCD can be expressed as $A = 5000(\sin\alpha + \cos\alpha)$. 3

ii) Show that the maximum area of this quadrilateral is $5000\sqrt{2}\ m^2$. 3

QUESTION 10 **MARKS**

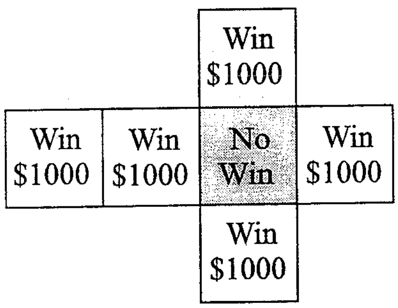
a) A game of chance has two stages.

In stage 1, a player spins the two spinners shown below.



If the sum of the two numbers indicated by the spinners is greater than 11, the player has the chance to move to stage 2.

In stage 2, the player throws the die whose net is shown.



John is to play the game once.

i) What is the probability that he will move to stage 2? 2

ii) What is the probability that he will win $1000 in the game? 1

iii) Vanessa is to play the game twice. What is the probability that she will win at least $1000? 2

QUESTION 10 (Continued) **MARKS**

b) When Grace was young she decided to set up a superannuation fund that would grow quickly.

On her 18^{th} birthday she invested \$M,
on her 19^{th} birthday she invested \$2M,
on her 20^{th} birthday she invested \$4M,
- and so on, doubling the amount invested each year.

All these investments were compounded yearly at the rate of 10% p.a.

Grace made her last investment on her 27^{th} birthday, leaving all her investments to accumulate and earn interest until her 58^{th} birthday.

i) Find an expression in terms of M for the amount she invested on her 27^{th} birthday. 1

ii) Find an expression in terms of M for the value of her first investment on her 58^{th} birthday 2

iii) If the value of her superannuation fund on her 58^{th} birthday was \$1 089 179, find her first investment M to the nearest dollar. 4

Year 12

Trial Mathematics - 2001

Solutions and Mapping grid.

QUESTION 1

(a) $e^2 \approx 7.39$ (correct to 2 d.p.)

(1 mark)

(b) $49x^2 - 4 = (7x-2)(7x+2)$ (1 mark)

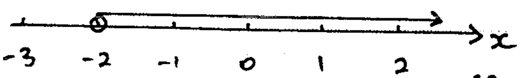
(c) $\tan\frac{\pi}{3} = \sqrt{3}$ (1 mark)

(d) $3\sqrt{18} - 4\sqrt{8} = 9\sqrt{2} - 8\sqrt{2}$

$= \sqrt{2}$ (2 marks)

(e) $1 - 2x < 5$ $\therefore -2x < 4$

$\therefore x > -2$



(3 marks)

(f) $x - 2y = 8$ $\therefore -5x + 10y = -40$

$5x + 3y = 1$ $\quad 5x + 3y = 1$ $\oplus$

$13y = -39$

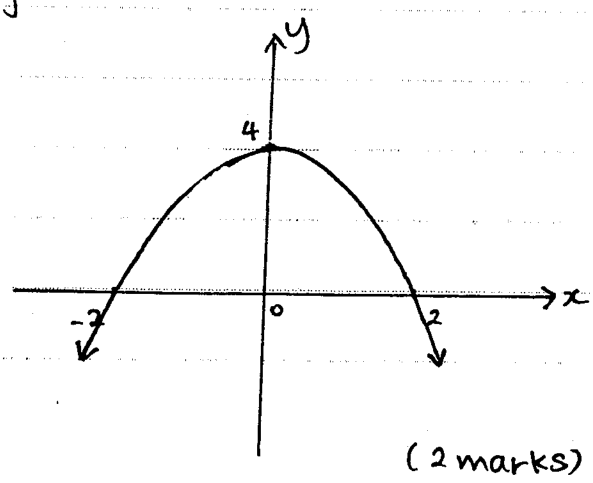
$\therefore y = -3$

$\therefore x = 2$

$\therefore$ The solutions are $x = 2, y = -3$

(2 marks)

(g) $y = 4 - x^2$



(2 marks)

QUESTION 2

(a) $m_{AB} = m_{DC}$ ($AB \parallel DC$)

$\therefore m_{AB} = \frac{3-(-3)}{-2+5} = \frac{6}{3} = 2$

$\therefore$ Gradient of line DC is 2. (1 mark)

(b) Using gradient point formula

$\therefore y - 9 = 2(x - 10)$

$\therefore y - 9 = 2x - 20$ $\therefore y = 2x - 11$

$\therefore$ Equation of DC is $y = 2x - 11$ (2 marks)

(c) Equation of AD is $y = -3$

Sub $y = -3$ in $y = 2x - 11$ to find D

$\therefore -3 = 2x - 11$ $\therefore x = 4$ $\therefore y = -3$

$\therefore$ Coordinates of D are $(4, -3)$ (2 marks)

(d) $BC = \sqrt{(10+2)^2 + (9-3)^2} = 6\sqrt{5}$

$\therefore$ Distance BC is $6\sqrt{5}$ units (1 mark)

(e) $(x-10)^2 + (y-9)^2 = (6\sqrt{5})^2$

$\therefore x^2 - 20x + 100 + y^2 - 18y + 81 = 180$

$\therefore x^2 - 20x + y^2 - 18y + 1 = 0$ (equation of circle centred at C, radius BC) (2 marks)

(f) Sub $(4, -3)$ into equation of circle

$\therefore (4-10)^2 + (-3-9)^2 = 180$

$\therefore 180 = 180$

$\therefore$ D lies on circle (1 mark)

(g) $M_{BD} = \left(\frac{-2+4}{2}, \frac{3-3}{2}\right) = (1, 0)$

$\therefore$ Mid-point of BD is $(1, 0)$ (1 mark)

(h) The mid-point of BD, M(1,0), is also the mid-point of CE.

Let E be (x,y)

$\therefore \frac{x+10}{2} = 1 \quad \therefore x = -8$

$\therefore \frac{y+9}{2} = 0 \quad \therefore y = -9$

$\therefore$ E is $(-8,-9)$ (2 marks)

QUESTION 3

(a)(i) $y = \log_e(3x+1)$

$\therefore \frac{dy}{dx} = \frac{3}{3x+1}$ (1 mark)

(ii) $y = e^x \cos x$ (using product rule)

$\therefore \frac{dy}{dx} = e^x \cos x - e^x \sin x$

$= e^x(\cos x - \sin x)$ (2 marks)

(b) $\int \frac{4}{2x+1}dx = 2\int \frac{2}{2x+1}dx$

$= 2\log_e(2x+1) + c$ (2 marks)

(c) $\int_0^{\pi/6} 2\sec^2 2x\,dx = [\tan 2x]_0^{\pi/6}$

$= \tan\frac{\pi}{3} - \tan 0$

$= \sqrt{3}$ (2 marks)

(d) $\int_0^1 (x^2+1)^2 dx = \int_0^1 (x^4 + 2x^2 + 1)dx$

$= [\frac{x^5}{5} + \frac{2x^3}{3} + x]_0^1$

$= (\frac{1}{5} + \frac{2}{3} + 1) - 0 = 1\frac{13}{15}$ (2 marks)

(e) $y = e^{2x} \quad \therefore \frac{dy}{dx} = 2e^{2x}$ (gradient function)

At $x = \frac{1}{2}$, $m_{tan} = 2e$

Using gradient point formula

$\therefore y - e = 2e(x - \frac{1}{2})$

$\therefore y - e = 2ex - e \quad \therefore y = 2ex$ (3 marks)

QUESTION 4

(a)(i) Considering Δ's ACB & LMK:

∠ACB = ∠KML (alternate angles, AC ∥ ML)

∠ABC = ∠MKL (alternate angles, AB ∥ KL)

∠CAB = ∠KLM (remaining angles)

∴ ΔACB ||| ΔLMK (equiangular) (3 marks)

(ii) $\frac{BC}{KM} = \frac{AB}{KL} \quad \therefore \frac{2+x}{x+7} = \frac{1}{2}$

$\therefore x = 3 \quad \therefore$ CM is 3 cm (2 marks)

(b) For equation to have real roots,

$4 - 4p \geq 0. \quad \therefore -4p \geq -4 \quad \therefore p \leq 1$

∴ For $p \leq 1$, the equation has real roots. (2 marks)

(c)(i) Since the number of computers sold each month increases by the same number ∴ The monthly sales form the consecutive terms of an arithmetic sequence. $\therefore T_{36} = 200 + (50 \times 35)$

$= 1950$ (2 marks)

∴ They sold 1950 computers in the last month of the third year.

(ii) $S_{36} = \frac{36}{2}(200+1950) = 38700$

∴ They sold 38700 computers over the three year period. (3 marks)

QUESTION 5

(a) $\sin\alpha = 0.6$ ∴ $\sin\alpha = \sin 0.6435$

∴ $\alpha = 0.6435 + 2K\pi$ or $\alpha = \pi - 0.6435 + 2K\pi$

for $K=0, \alpha = 0.6435$ for $K=0, \alpha = 2.498$

∴ The solution is $\alpha = 0.6435, \alpha = 2.498$ in the domain $0 \le \alpha \le 2\pi$. (2 marks)

(b)(i) $AC = \sqrt{12+4} = 4m$ (using Pythagoras' Theorem)

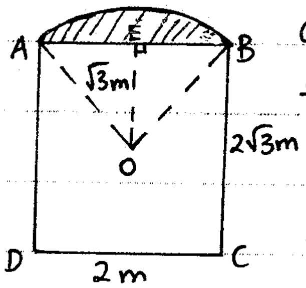
$AO = OC$ ∴ $AO = 2m$

Likewise, $BO = DO$ ∴ $BO = 2m$

∴ $AB = BO = AO = 2m$ ∴ ΔABO is equilateral

∴ $\angle AOB = \frac{\pi}{3}$ (2 marks)

ALTERNATIVE METHOD



Construct OE, such that $AE = EB = 1m$

Let $\angle EOB = \alpha$

∴ $\angle AOE = \alpha$ (EO bisects $\angle AOB$).

∴ $\tan\alpha = \frac{1}{\sqrt{3}}$

∴ $\alpha = \frac{\pi}{6}$ ∴ $\angle AOB = 2\alpha = \frac{\pi}{3}$

(ii) Area of $\Delta ABO = \frac{1}{2} \times 2 \times 2 \times \sin\frac{\pi}{3} = \sqrt{3}$

Area sector $ABO = \frac{1}{2} \times 2^2 \times \frac{\pi}{3} = \frac{2\pi}{3}$

∴ Area of golden segment $= (\frac{2\pi}{3} - \sqrt{3})\ m^2$

$\doteqdot 0.362\ m^2$

(c)(i) When $t=0$, $N=10000$

∴ $10000 = Ae^0$ ∴ $A = 10000$

When $t=2$, $N=20000$

∴ $20000 = 10000e^{2K}$ ∴ $\ln 2 = 2K$

∴ $K = \frac{1}{2}\ln 2 \doteqdot 0.347$ (2 marks)

(ii) When $t=10$, $N = 10000e^{10K}$

∴ $N = 320000$ copies

∴ After 10 weeks there were 320000 copies. (2 marks)

(iii) $N = 10000e^{Kt}$ ∴ $\frac{dN}{dt} = 10000Ke^{Kt}$

when $t=4$, $\frac{dN}{dt} = 10000 \times \frac{1}{2}\ln 2 \times e^{\ln 4}$

$\doteqdot 13863$ (correct to nearest whole number)

∴ Number of copies were increasing at a rate of 13863 copies per week. (1 mark)

QUESTION 6

(a) Area $= \frac{70}{3}[240 + 240 + 4 \times 300]$

$= 39200\ cm^2$ (3 marks)

(b)(i) $y = -x^2 + 2x + 8$ $y = x + 6$

Let $y = y$ to find A & B

∴ $-x^2 + 2x + 8 = x + 6$ ∴ $x^2 - x - 2 = 0$

∴ $(x-2)(x+1) = 0$ ∴ $x = 2$ or -1

∴ $x_A = -1$ & $x_B = 2$. (2 marks)

(ii) Area $= \int_{-1}^{2} [(-x^2+2x+8) - (x+6)]\,dx$

$= \int_{-1}^{2} (-x^2 + x + 2)\,dx$

$= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2}$

$= (-\frac{8}{3} + 2 + 4) - (\frac{1}{3} + \frac{1}{2} - 2) = 4\frac{1}{2}$

$\therefore$ Area bounded is $4\frac{1}{2}$ units^2.
(3 marks)

(c) $V = \pi \int_{-1}^{2} x^2 dy$ where $y + 2 = \frac{1}{x^2}$ $\therefore x^2 = \frac{1}{y+2}$

$= \pi \int_{-1}^{2} \frac{dy}{y+2}$

$= \pi \left[\ln(y+2)\right]_{-1}^{2} = \pi(\ln 4 - \ln 1)$

$= 2\pi \ln 2$

$\therefore$ Exact volume of bottle is $2\pi \ln 2$ units^3.
(4 marks)

QUESTION 7

(a) $\log_e x = \log_e(2x-1)$

$\therefore x = 2x - 1$ $\therefore x = 1$

Checking solution:
for $x = 1$, $\log_e 1 = \log_e(2-1)$ $\therefore 0 = 0$
$\therefore$ Solution valid.
(2 marks)

(b) (i) $v = 36 - 4e^{2t}$

when $t = 0$, $v = 36 - 4e^0 = 32$ m/s

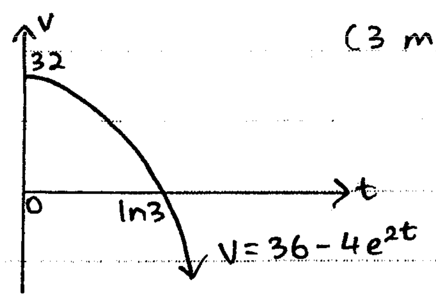
$\therefore$ Initial velocity of particle is 32m/s.
(1 mark)

(ii) The particle comes to rest when $v = 0$.

$\therefore 36 - 4e^{2t} = 0$ $\therefore 9 = e^{2t}$ $\therefore \ln 9 = 2t$

$\therefore t = \ln 3$ seconds. (2 marks)

(iii) (3 marks)



(iv) The distance travelled by the particle is the area under the graph. This area is found by integrating between $t=0$ and $t=\ln 3$, and between $t=\ln 3$ and $t=2\ln 3$ and adding the results.

Distance $= \int_0^{\ln 3}(36 - 4e^{2t})dt + \left|\int_{\ln 3}^{2\ln 3}(36 - 4e^{2t})dt\right|$

$= \left[36t - 2e^{2t}\right]_0^{\ln 3} + \left|\left[36t - 2e^{2t}\right]_{\ln 3}^{2\ln 3}\right|$

$= (36\ln 3 - 18 - 0 + 2) +$
$|72\ln 3 - 162 - 36\ln 3 + 18|$

$= 128$ m (2 marks)

(v) $a = \frac{dv}{dt} = -8e^{2t}$ but $v - 36 = -4e^{2t}$

$\therefore a = 2(v - 36)$ (2 marks)

QUESTION 8

(a) (i) $\angle DAB = \angle CBA = 90°$ (angles in a rectangle)

$\angle DAE = 90 - 15 = 75°$ (complementary angles)

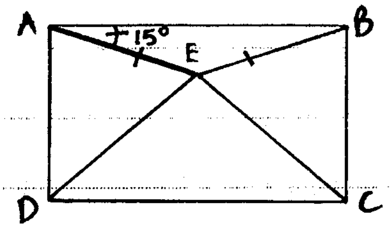
$\angle CBE = 90 - 15 = 75°$ (complementary angles)

$\therefore \angle DAE = \angle CBE$ (2 marks)

(ii) Data: $AE = EB$, $\angle EAB = 15°$

Aim: Prove that $\Delta ADE \equiv \Delta BCE$

Construction: Figure



Proof: Considering Δ's ADE & BCE

AE = EB (data)

∠DAE = ∠CBE (proven)

AD = BC (opposite sides of rectangle ABCD are equal).

∴ ΔADE ≡ ΔBCE (S.A.S.) (3 marks)

(iii) DE = EC (corresponding sides of congruent Δ's ADE & CBE are equal)

∴ ΔDEC is isosceles (2 equal sides) (1 mark)

(b)(i) $y = 4\log_e x + 6 - x$

$\therefore \frac{dy}{dx} = \frac{4}{x} - 1$ (gradient function)

Let $dy/dx = 0$ to find possible stationary turning points $\therefore \frac{4}{x} - 1 = 0 \quad \therefore x = 4$

x		4	
$\frac{dy}{dx}$	+	0	−
y	↗	7·5	↘

For $x<4$, $\frac{dy}{dx} > 0$

For $x>4$, $\frac{dy}{dx} < 0$

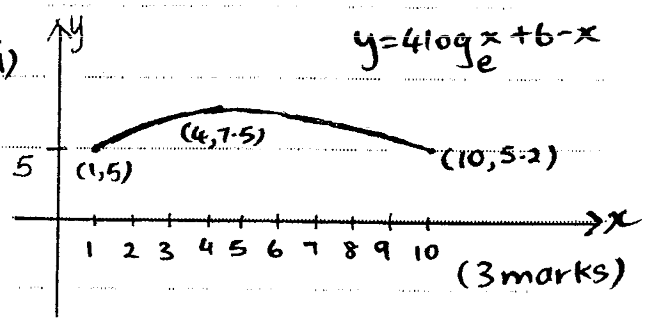
∴ There is a maximum turning point at $(4, 4\ln 4 + 2) \doteqdot (4, 7\cdot5)$ (2 marks)

(ii) $\frac{d^2y}{dx^2} = \frac{-4}{x^2}$

As the second derivative is negative for all values of x,

∴ Curve is concave down throughout the domain $1 \le x \le 10$ (1 mark)

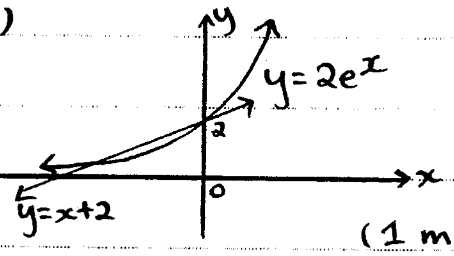
(iii)



(3 marks)

QUESTION 9

(a)(i)



(1 mark)

(ii) $2e^x - x = 2 \quad \therefore 2e^x = x + 2$

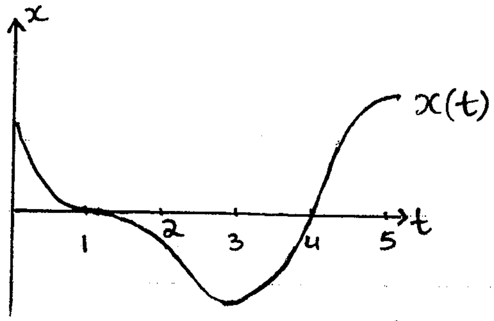
∴ The solutions of the equation are the x-coordinates of the points of intersection of the 2 curves $y = 2e^x$ and $y = x + 2$. From the graph, we could see that the line intersects the curve twice.

∴ There are 2 solutions for the given equation. (2 marks)

(b)

t		1		3		5
v	−	0	−	0	+	0
x	↘	$\underset{0}{-}$	↘	—	↗	—

Note: When integrating the curve, a turning point becomes a point of inflexion and an intercept becomes a turning point.



(3 marks)

(c)(i) Area $\Delta CAB = \frac{1}{2} \times 100^2 \times \sin\alpha = 5000 \sin\alpha$

Area $\Delta CAD = \frac{1}{2} \times 100 \times 100 \times \sin(90-\alpha)$

$= 5000 \cos\alpha$

$\therefore$ Area $ABCD = 5000 \sin\alpha + 5000 \cos\alpha$

$= 5000 (\sin\alpha + \cos\alpha)$

(3 marks)

(ii) $A = 5000 \ (\sin\alpha + \cos\alpha)$

$\therefore \frac{dA}{d\alpha} = 5000 \ (\cos\alpha - \sin\alpha)$ (gradient function)

Let $dA/d\alpha = 0$ to find turning points.

$\therefore \cos\alpha = \sin\alpha \quad \therefore \tan\alpha = 1 = \tan\frac{\pi}{4}$

$\therefore \alpha = \frac{\pi}{4}$

α		$\pi/4$	
$\frac{dA}{d\alpha}$	+	0	−
A	↗	—	↘

For $\alpha < \frac{\pi}{4}$, $\frac{dA}{d\alpha} > 0$ $\quad \therefore \alpha = \frac{\pi}{4}$ is a
For $\alpha > \frac{\pi}{4}$, $\frac{dA}{d\alpha} < 0$ maximum turning point.

For $\alpha = \frac{\pi}{4}$, $A = 5000\sqrt{2}\ m^2$

$\therefore$ Maximum area of quadrilateral is $5000\sqrt{2}\ m^2$.

QUESTION 10

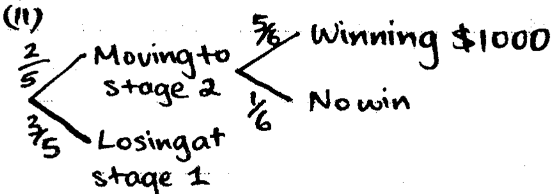
(a)(i)

10		x	x	x	x
8			x	x	x
6				x	x
4					x
2					
	1	3	5	7	9

$\therefore$ Probability of moving to stage 2

$= \frac{10}{25} = \frac{2}{5}$ (2 marks)

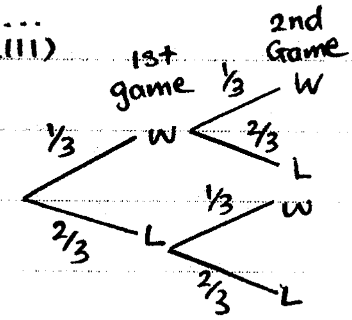
(ii)



$\therefore$ Probability of winning \$1000 $= \frac{2}{5} \times \frac{5}{6} = \frac{1}{3}$

(1 mark)

(iii)



$\therefore$ Probability $= 1 -$ losing twice $= 1 - \frac{4}{9} = \frac{5}{9}$

$\therefore$ Probability that she wins at least \$1000 is $\frac{5}{9}$. (2 marks)

(b)(i) The amounts invested form consecutive terms of a geometric sequence which has first term M and common ratio $r = 2$.

$\therefore$ Amount invested on 27^{th} birthday is: