YEAR 12 – TRIAL 2005 – MATHEMATICS

QUESTION 1

MARKS

a) Find the value of $\log_e 100$ correct to three significant figures. 1

b) Completely factorise $x^3 + 2x^2 - 8x$ 2

c) Differentiate $\dfrac{3x}{5x+4}$ 2

d) Rationalise the denominator of $\dfrac{5-\sqrt{3}}{2\sqrt{3}-4}$ 2

e) Solve $|2x-3| < 13$ 2

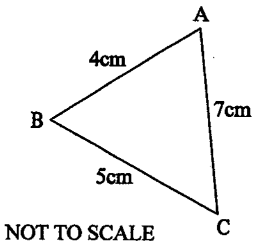
f) i) Differentiate $\ln(\sin x)$ 1

ii) Hence write the primitive of $2\cot x$ 2

QUESTION 2

MARKS

a) In ΔABC, AB is 4cm long, BC is 5cm and CA is 7cm long



NOT TO SCALE

i) Calculate the size of the largest angle correct to the nearest minute. 2

ii) Calculate the area of ΔABC correct to two decimal places. 2

b) $D(0, -2)$, $E(4, 0)$ and $F(2, 4)$ are three points on the number plane

i) Draw a diagram to represent this information. 1

ii) Calculate the length of the interval DF. 1

iii) Calculate the gradient of DF. 1

iv) Write the equation of the line DF in general form. 1

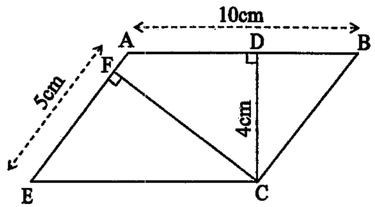
v) Calculate the perpendicular distance from E to the line DF. 2

vi) Calculate the area of the ΔDEF. 2

QUESTION 3 **MARKS**

a) Write down the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 1$. 2

b) ABCE is a parallelogram. $CD \perp AB$ and $CF \perp AE$. AB is 10cm long, CD is 4cm long and AE is 5cm long.



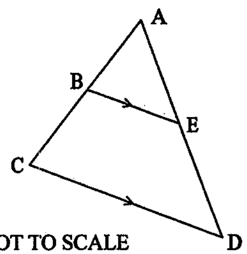
NOT TO SCALE

i) Calculate the area of the parallelogram ABCE. 1

ii) Calculate the length of CF. 2

c) Evaluate $\int_2^4 \frac{3x}{x^2-1}\,dx$ leaving your answer exact and in its simplest form. 3

d) In this diagram BE is parallel to CD.



NOT TO SCALE

i) Prove that ΔABE is similar to ΔACD 2

ii) Given that AE = 3m, ED = 5m and BC = 4m, calculate the length of AB. 2

QUESTION 4 **MARKS**

a) Solve $(x-2)^2 < 9$ and graph your solution on the number line. 2

b) The quadratic equation $2x^2 + 9x - 4 = 0$ has roots of α and β. Use this information to evaluate the following expressions.

i) $\alpha + \beta$ 1

ii) $\frac{1}{\alpha} + \frac{1}{\beta}$ 2

iii) $\alpha^3 + \beta^3$ 2

c) The first three terms of a sequence are $20, 15, 11\frac{1}{4}$

i) State why this sequence is geometric, providing evidence to support your answer. 1

ii) Find the 8^{th} term in this sequence. 1

iii) Write an expression, in terms of n, for the sum of the first n terms of this particular sequence. 1

iv) Explain why the limiting sum of this sequence is 80. 2

QUESTION 5 **MARKS**

a) For the curve $y = 3x^4 - 16x^3 + 24x^2$

i) Show that the curve cuts the x axis only at the origin. 1

ii) Find the turning points and determine their nature. 3

iii) Find the points of inflexion. 2

iv) Sketch the curve, showing the intercepts with axes, turning points and points of inflexion. 2

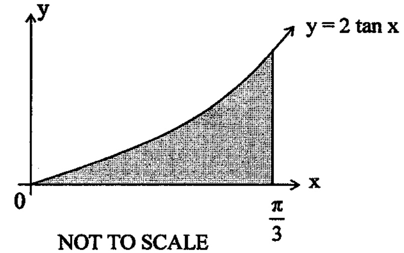
b) A bag contains 5 red marbles, 3 blue marbles and 1 green marble. A marble is selected from the bag at random and not replaced, and a second marble is then selected.

i) What is the probability that one marble is green and the other blue ? 2

ii) What is the probability that the two marbles are of different colours ? 2

QUESTION 6 **MARKS**

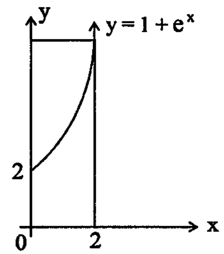
a) The shaded area between the curve $y = 2 \tan x$, the x-axis and the line $x = \frac{\pi}{3}$ is rotated about the x-axis.



NOT TO SCALE

Calculate the volume of the solid formed. 4

b) The graph of $y = 1 + e^x$ is shown. A rectangle has been drawn through the point $(2, 1+e^2)$.



NOT TO SCALE

Show that the curve divides the rectangle into two parts of equal area. 4

c) A radioactive substance decays in such a way that its mass in kg after t days, can be found using the formula $M = M_0e^{kt}$.
Initially the mass of the substance is 80kg, and after 100 days the mass is 60kg.

i) Calculate the value of k correct to 4 significant figures. 2

ii) Using the value of k you found in part (i), calculate the half life (the time it takes for a sample to halve its mass) of this radioactive substance. Give your answer in days and hours, to the nearest hour. 2

QUESTION 7 | **MARKS**

a) The probability of Elise winning a particular game of chance is 20%.

i) If she plays 3 games, what is the probability she will win at least 1 ? — 2

ii) What is the least number of games she must play before the probability of her winning at least one game is 99% ? — 2

b) i) Explain why the sum of the first n terms of the geometric series whose first term is 20, and whose common ratio is -3 will be negative for an even number of terms. Support your answer with calculation. — 2

ii) Calculate the sum of the first 8 terms of this series. — 1

iii) Calculate the least number of terms of the arithmetic series whose first term is 20 and whose common difference is -3 which are required for the sum to be negative. — 2

c) Use Simpson's Rule, with six equal sub-intervals, to find the area between the curve $y = \sin^2 x$, the x axis, and the lines $x = 0$ and $x = \pi$. — 3

Leave your answer in exact form.

QUESTION 8 | **MARKS**

a) When Vanessa began her first job she decided to open an account and deposit \$50 per month into the account. The bank guaranteed an interest rate of 8% p.a. compounding every 6 months if she agreed not to make any withdrawals.

i) How much will be in this account at the end of 12 months ? — 1

ii) How much will be in this account at the end of 30 years ? — 2

iii) How much will be in this account at the end of 30 years if Vanessa doubles the amount of her monthly deposit to \$100 per month at the beginning of the 11^{th} year, and doubles again to \$200 per month at the beginning of the 21^{st} year ? — 3

b) i) Show that the circle $x^2 + y^2 - 8x - 6y = 0$ intersects the lines $3x + 2y = 12$ and $3x - 2y = 12$ at the points $(0, 6)$ and $(8, 6)$ respectively. — 2

ii) Shade the region bounded by $x^2 + y^2 - 8x - 6y = 0$, $3x + 2y = 12$ and $3x - 2y = 12$. — 2

iii) Calculate the area of the region shaded in the previous part. Give your answer correct to 3 decimal places. — 2

QUESTION 9 **MARKS**

a) The equation $x^2 - 6x - 8y - 23 = 0$ represents a parabola.

i) Calculate the co-ordinates of the focus of this parabola. 1

ii) Calculate the equation of the directrix of this parabola. 1

b) A particle moves from the origin at a velocity of $3ms^{-1}$ for 3 seconds. It then moves such that its velocity can be calculated using the formula $v = t^2 - 10t + 24$, where v is calculated in ms^{-1} and t is the time in seconds since the particle first left the origin.

i) At what times is the particle at rest ? 1

ii) Sketch the velocity-time graph for the first 8 seconds of this motion. 1

iii) At what times is the acceleration zero ? 2

iv) Calculate the position of the particle after 6 seconds. 3

v) Calculate the distance travelled by the particle during the first 6 seconds. 3

QUESTION 10 **MARKS**

a) Two different groups of individuals have populations with a rate of change which is proportional to the size of the population.

The first population increased in size by 20% in ten years.

The second population began with exactly twice the number of the first population, and decreased in size by 20% in ten years.

Calculate the time required for the populations to be of equal size. 4

b) Yvonne is arranging a bank loan. The bank offers a reducible interest rate of 8% p.a. She organises for her repayments to be made every 6 months, with the interest calculated on the balance owing immediately prior to her repayment. As she expects her income to rise by about 6% p.a., she decides to make each repayment 3% more than the previous repayment.

Let the amount borrowed be \$P, the size of the first repayment be \$R and the term of the loan be n years.

i) Write an expression, in terms of P and R, for the amount owed at the end of the first year. 2

ii) Write an expression, in terms of P, R and n, for the amount owed at the end of the nth year. 2

iii) Use the formula for the sum of a geometric series to write an expression, in terms of P and n, for R, the size of the first repayment. 4

Question 1

a) $\log_e 100 = 4.605170\ldots$

$\doteqdot 4.61$ (3 signif. figures) (1 mark)

b) $x^3+2x^2-8x = x(x^2+2x-8)$

$= x(x+4)(x-2)$ (2 marks)

c) $\frac{d}{dx}\left(\frac{3x}{5x+4}\right)$ $\quad u' = 3$, $v' = 5$

$= \frac{3(5x+4)-5(3x)}{(5x+4)^2}$

$= \frac{15x+12-15x}{(5x+4)^2}$

$= \frac{12}{(5x+4)^2}$ (2 marks)

d) $\frac{5-\sqrt{3}}{2\sqrt{3}-4} \times \frac{2\sqrt{3}+4}{2\sqrt{3}+4}$

$= \frac{10\sqrt{3}+20-6-4\sqrt{3}}{12-16}$

$= \frac{-6\sqrt{3}-14}{4} = \frac{-3\sqrt{3}-7}{2}$ (2 marks)

e) $-13 < 2x-3 < 13$

$-10 < 2x < 16$

$-5 < x < 8$ (2 marks)

f) i) $\frac{d}{dx}(\ln(\sin x)) = \frac{\cos x}{\sin x} = \cot x$ (1 mark)

ii) Primitive of $2\cot x$ is

$2\ln(\sin x) + c$

or $\ln(\sin^2 x) + c$

(2 marks)

Question 2

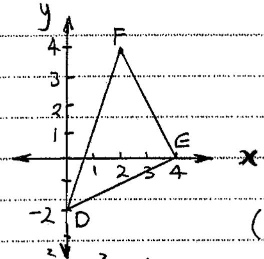
a) i) $\cos B = \frac{4^2+5^2-7^2}{2\times4\times5} = \frac{-8}{10}$

$\therefore B = 101°\,32'$ (2 marks)

(ii) $A = \frac{1}{2}\times4\times5\times\sin 101°32'$

$= 9.80\text{ cm}^2$ (2 decimal places) (2 marks)

b) i)



(1 mark)

ii) $DF^2 = 6^2+2^2 = 40$

$\therefore DF = 2\sqrt{10}$ (1 mark)

(iii) Gradient $= \frac{\text{rise}}{\text{run}} = \frac{6}{2} = 3$ (1 mark)

(iv) $y = mx+b$

$\therefore y = 3x-2$

so $3x-y-2=0$ (1 mark)

v) Perp dist $= \frac{|3\times4-1\times0-2|}{\sqrt{3^2+1^2}}$

$= \frac{10}{\sqrt{10}}\times\frac{\sqrt{10}}{\sqrt{10}} = \sqrt{10}$ (2 marks)

vi) $A = \frac{1}{2}\times b\times h$

$= \frac{1}{2}\times 2\sqrt{10}\times\sqrt{10}$

$= 10\text{ u}^2$. (2 marks)

Question 3

a) $y = e^{2x}$, $y' = 2e^{2x}$

when $x=1$, $y = e^2$ $\quad y' = 2e^2$

$\therefore$ eqn $y - e^2 = 2e^2(x-1)$

$y = 2e^2x - e^2$ (2 marks)

b) i) Area = base × perp height

$= 10\text{cm}\times4\text{cm} = 40\text{cm}^2$ (1 mark)

ii) $40 = 5\text{cm}\times CF$

$\therefore CF = 8\text{ cm}$ (2 marks)

c) $\frac{3}{2}\int_2^4 \frac{2x}{x^2-1}\,dx = \frac{3}{2}\left[\log_e(x^2-1)\right]_2^4$

$= \frac{3}{2}\left[\log_e 15 - \log_e 3\right] = \frac{3}{2}\log_e 5$ (3 marks)

d) i) In ΔABE and ΔACD

$A\hat{B}E = A\hat{C}D$ (corresp $\angle$'s with $BE \parallel CD$)

$\hat{A}$ is common

$\therefore \Delta ABE \,|||\, \Delta ACD$ (equiangular)

(2 marks)

ii) As $\Delta ABE \,|||\, \Delta ACD$,

$\frac{x}{x+4} = \frac{3}{8}$.

so $8x = 3x+12$

$5x = 12$

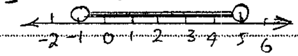
$x = \frac{12}{5} = 2\frac{2}{5}$ m

or $x = 2.4$ m. (2 marks)

Question 4

a) As $(x-2)^2 < 9$ then $|x-2| < 3$. So

$-3 < x-2 < 3$ or $-1 < x < 5$.



(2 marks)

b) i) $\alpha+\beta = \frac{-b}{a} = \frac{-9}{2} = -4\frac{1}{2}$ (1 mark)

ii) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = -\frac{9}{2}\div\frac{-4}{2} = \frac{9}{4} = 2\frac{1}{4}$ (2 marks)

iii) As $(\alpha+\beta)^3 = \alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3$

then $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= \left(\frac{-9}{2}\right)^3 - 3\times\frac{-9}{2}(-2)$

$= -118\frac{1}{8}$ (2 marks)

c) i) $\frac{15}{20} = \frac{3}{4}$ and $\frac{11\frac{1}{4}}{15} = \frac{3}{4}$

$\therefore$ sequence is Geometric (1 mark)

ii) $T_8 = ar^7 = 20\times\left(\frac{3}{4}\right)^7 = 2\frac{10972}{16384}$ (1 mark)

iii) $S_n = \frac{a(1-r^n)}{1-r} = \frac{20\left(1-\left(\frac{3}{4}\right)^n\right)}{1-\frac{3}{4}}$

$\therefore S_n = 80\left(1-\left(\frac{3}{4}\right)^n\right)$ (1 mark)

iv) as $n\to\infty$, $\left(\frac{3}{4}\right)^n \to 0$

$\therefore 1-\left(\frac{3}{4}\right)^n \to 1$

so $80\left(1-\left(\frac{3}{4}\right)^n\right) \to 80$ as $n\to\infty$

(2 marks)

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Question 5

a) i) Solve $x^2(3x^2-16x+24)=0$

For quadratic $\Delta = -32$ ∴ no sol^n

only solution is $x=0$. (1 mark)

ii) $\frac{dy}{dx} = 12x^3-48x^2+48$

$= 12x(x^2-4x+4) = 12x(x-2)^2$

∴ $y'=0$ when $x = 0, 2, 2$

$y = 0, 16, 16$

$y'' = 36x^2-96x+48$ $y'' =$ +ve, 0, 0

test (2,16) for p.o. inflexion

x	1.5	2	2.5
y''	−ve	0	+ve

∴ point of horizontal inflexion at (2,16)

and min T.P. at (0,0) (3 marks)

(iii) points of inflexion when $y''=0$

ie $12(3x^2-8x+4)=0$

ie $12(3x-2)(x-2)=0$

∴ suspected p.o.inf at $x=\frac{2}{3}, 2$

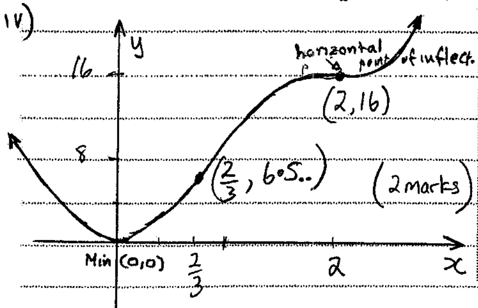
already proved for (2,16)

test $(\frac{2}{3}, 6.5\ldots)$

x	$\frac{1}{3}$	$\frac{2}{3}$	1
y''	+ve	0	−ve

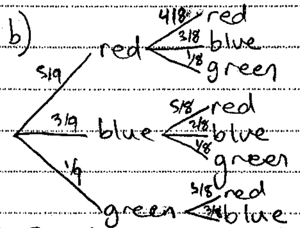
∴ p.o. inflex at $(\frac{2}{3}, 6.5\ldots)$ and (2,16) (2 marks)

iv)



(2 marks)

5 b)



i) Prob (blue + green) $= \frac{1}{9}\times\frac{3}{8} + \frac{3}{9}\times\frac{1}{8} = \frac{1}{12}$ (2 marks)

ii) Prob (different) = 1 − Prob (same)

$= 1 - (5/9 \times 4/8 + 3/9 \times 2/8)$

$= 1 - \frac{26}{72} = \frac{46}{72} = \frac{23}{36}$ (2 marks)

Question 6

a) $V = \pi\int_0^{\frac{\pi}{3}}(2\tan x)^2\,dx$

$= 4\pi\int_0^{\frac{\pi}{3}} \tan^2 x\,dx$

$= 4\pi\int_0^{\frac{\pi}{3}}(\sec^2 x - 1)\,dx$

$= 4\pi\left[\tan x - x\right]_0^{\frac{\pi}{3}}$

$= 4\pi\left[\tan\frac{\pi}{3} - \frac{\pi}{3} - (\tan 0 - 0)\right]$

$= 4\pi\left(\sqrt{3} - \frac{\pi}{3}\right)\,u^3$ (4 marks)

b) Area below curve $= \int_0^2 (1+e^x)\,dx$

$= \left[x + e^x\right]_0^2 = (2+e^2) - (0+1) = e^2+1$

Area of rectangle $= 2\times(1+e^2)$

so area under curve < ½ area of rectangle. (4 marks)

Question 6 (continued)

c) i) $M_0 = 80e^{k0} = 80$

∴ $60 = 80e^{100k}$

$\frac{3}{4} = e^{100k}$ hence $\ln(\frac{3}{4}) = 100k$

so $k = -2.877\times10^{-3}$ (2 marks)

ii) $40 = 80e^{-2.877\times10^{-3}t}$

$.5 = e^{-2.877\times10^{-3}\times t}$

so $t = \ln(.5) \div -2.877\times10^{-3}$

ie $t = 240.92$ days = 240 days 22hrs. (2 marks)

Question 7

a) i) Prob (win at least 1) = 1 − Prob (win none)

$= 1 - .8\times.8\times.8 = .488 = 48.8\%$ (2 marks)

ii) Prob (win at least one in "n" games)

$= 1-(.8)^n$

∴ solve $1-(.8)^n > .99$

$-(.8)^n > -.01$

$(.8)^n < .01$

so $n(\log .8) < \log(.01)$

∴ $n > \log(.01) \div \log(.8)$

$n > 20.63$

∴ Elise needs to play 21 games to have 99% chance of winning at least once (2 marks)

b) i) $S_n = \frac{20(1-(-3)^n)}{1-(-3)}$

$= 5(1-(-3)^n)$

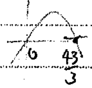
If n is even $(-3)^n$ is positive and greater than 1. Hence $5(1-(-3)^n) < 0$. (2 marks)

b) ii) $S_8 = 5(1-(-3)^8) = -32800$ (1 mark)

iii) $S_n = \frac{n}{2}(2a+(n-1)d)$

Solve $\frac{n}{2}(40+(n-1)\times-3) < 0$

$43n - 3n^2 < 0$



∴ $n < 0$ or $n > \frac{43}{3}$

∴ 15 terms will be needed (2 marks)

c) Each sub-interval $= \frac{\pi}{6}$ units

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$\sin^2 x$	0	$\frac{1}{4}$	$\frac{3}{4}$	1	$\frac{3}{4}$	$\frac{1}{4}$	0

Area $= \frac{\frac{\pi}{6}}{3}\left(0+0+2(\frac{3}{4}+\frac{3}{4})+4(\frac{1}{4}+1+\frac{1}{4})\right)$

$= \frac{\pi}{18}(0+3+6) = \frac{\pi}{2}u^2$ (3 marks)

Question 8

a) i) Every 6 months deposit \$300, 4% int.

∴ 1 yr = \$300 × $(1.04)^2$ + 300 × 1.04 = \$636.48 (1 mark)

ii) 30 years means 60 interest periods

∴ Total = \$300 $(1.04 + 1.04^2 + \ldots + 1.04^{60})$

= \$300 × $\frac{1.04(1.04^{60}-1)}{1.04-1}$

= \$74 253.09 (2 marks)

iii) First 10 years = $300(1.04^{60} + 1.04^{59} + \ldots + 1.04^{41})$

next 10 years = $600(1.04^{40} + 1.04^{39} + \ldots + 1.04^{21})$

Final 10 years = $1200(1.04^{20} + 1.04^{21} + \ldots + 1.04)$

ie Total $= 300\times1.04^{41}\frac{(1.04^{20}-1)}{.04} + 600\times1.04^{21}\frac{(1.04^{20}-1)}{.04} + 1200\times1.04\frac{(1.04^{20}-1)}{.04}$

= \$122482.57 (3 marks)

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Question 8 continued

b) i) test (0,6) : $0+36-0-36=0$ - true

test (8,6) : $64+36-64-36=0$ - true

∴ both points lie on the circle

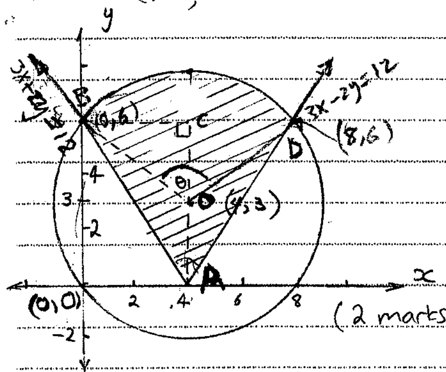
test (0,6) : $0+12=12$ - true

test (8,6) : $24-12=12$ - true

∴ (0,6) lies on $x^2+y^2-8x-6y=0$ and $3x+2y=12$ and (8,6) lies on $x^2+y^2-8x-6y=0$ and $3x-2y=12$. (2 marks)

ii) $x^2-8x+16+y^2-6y+9=25$

$(x-4)^2+(y-3)^2=25$ is a circle centre (4,3), radius 5 units



iii) In diagram, ΔOBC has BC ⊥ OC

$\sin C\hat{O}B = 4/5$ ∴ $C\hat{O}B = 0.927...$

$\theta = 2C\hat{O}B = 1.8549...$

∴ Area of minor segment with Chord $BD = \frac{1}{2}\times 25\times(\theta-\sin\theta)$

$=\frac{1}{2}\times 25\times(1.85..-\sin 1.85..)$

$=12.0548$

Area of triangle $ABD = \frac{1}{2}\times 6\times 8 = 24$

Area of shaded region $= 24+12.0548$

$= 36.055\ u^2$

(2 marks)

Question 9

a) i) $x^2-6x+9 = 8y+23+9$

$(x-3)^2 = 8(y+4)$

∴ Vertex at (3,-4), focal length 2

so focus at (3,-2) (1 mark)

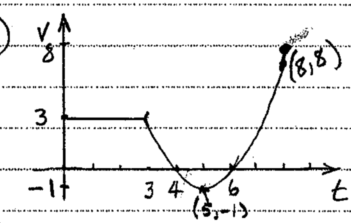
ii) directrix is $y=-6$ (1 mark)

b) i) Particle at rest means $v=0$

so $t^2-10t+24=0$

$(t-6)(t-4)=0$ ∴ $t=4$ or 6 sec (1 mark)

ii)



(1 mark)

iii) From the graph $a=0$ when $\frac{dv}{dt}=0$ so when $t<3$ seconds and when $t=5$ sec.

iv) when $t=3$, $x=9$

and $x=\frac{t^3}{3}-5t^2+24t+C$

so $9=9-45+72+C$

$-27=C$ ∴ $x=\frac{t^3}{3}-5t^2+24t-27$

so when $t=6$, $x=72-180+144-27$

∴ particle is 9m to the right of the origin when $t=6$ seconds (3 marks)

Alternatively, Area $=\int_0^3 3\,dt + \int_3^6 v\,dt$

$=9+\int_3^6 (t^2-10t+24)\,dt$

$=9+\left[\frac{t^3}{3}-5t^2+24t\right]_3^6 = 9+(36-36) = 9$

Question 9 continued

v) Distance travelled is equal to area between velocity curve and the "time" axis, or work out distances between stops.

First 3 seconds $- 3\times 3 = 9$m

From $t=3$ to $t=4$, $A=\int_3^4 t^2-10t+24\,dt$

$A=\left[\frac{t^3}{3}-5t^2+24t\right]_3^4 = \left(\frac{64}{3}-80+96\right)-(9-45+72)$

$A=37\frac{1}{3}-36 = 1\frac{1}{3}$ m

From $t=4$ to $t=6 = \left|\left[\frac{t^3}{3}-5t^2+24t\right]_4^6\right|$

$=\left|(72-180+144)-\left(37\frac{1}{3}\right)\right| = \left|-1\frac{1}{3}\right| = 1\frac{1}{3}$

∴ Total dist travelled $= 11\frac{2}{3}$ metres

Question 10

a) Let initial population be A

∴ For Group 1: $P=Ae^{kt}$

$1.2A = Ae^{k\times 10}$, so $1.2=e^{10k}$

∴ $k=(\log_e 1.2)\div 10 = 0.018232...$

For group 2 $P=2Ae^{kt}$

so $.8\times 2A = 2Ae^{k\times 10}$ so $.8=e^{10k}$

hence $k=(\log_e .8)\div 10 = -0.02231...$

so solve $2Ae^{-.022..t}=Ae^{.018...t}$

∴ $2=e^{.018..t}\div e^{-.022..t}$

$2=e^{.018t+.022t}$

∴ $\ln 2 = (.018...+.022...)t$

hence $t=\ln 2\div .04054....$

so $t=17.098...$ yrs.

≑ 17 yrs 1 month

(4 marks)

b) 8% pa = 4% each 6 months.

First repayment $=\$R$, second $=1.03R$ etc

i) Amount owed end 6 months $=\$P\times 1.04 - R$

end 1st year $=\$(P\times 1.04 - R)\times 1.04 - 1.03R$

$=\$P\times 1.04^2 - 1.04R - 1.03R$ (2 marks)

ii) nth year means 2n repayments.

3rd period $=(P\times 1.04^2-1.04R-1.03R)\times 1.04 - 1.03^2$

$=P\times 1.04^3 - 1.04^2R - 1.03\times 1.04R - 1.03^2R$

end n yrs (2n periods) $=P\times 1.04^{2n} - 1.04^{2n-1}R - 1.03\times 1.04^{2n-2}R - 1.03^2\times 1.04^{2n-3}R \ldots - 1.03^{2n-2}\times 1.04R - 1.03^{2n-1}R$

$=P\times 1.04^{2n} - R(1.04^{2n-1}+1.04^{2n-2}\times 1.03 + \ldots + 1.04\times 1.03^{2n-2} + 1.03^{2n-1})$

(2 marks)

iii) Prove Geometric Series and find "r"

$r=\frac{T_2}{T_1}=\frac{1.04^{2n-2}\times 1.03}{1.04^{2n-1}} = \frac{1.03}{1.04}$

$r=\frac{T_3}{T_2}=\frac{1.04^{2n-3}\times 1.03^2}{1.04^{2n-2}\times 1.03} = \frac{1.03}{1.04}$ ∴ GS.

After n years amount owed = 0

solve $0 = P\times 1.04^{2n} - R\left(\frac{1.04^{2n-1}\left(1-\left(\frac{1.03}{1.04}\right)^{2n}\right)}{1-\frac{1.03}{1.04}}\right)$

∴ $R = P\times 1.04^{2n} \div \frac{1.04^{2n-1}\left(1-\left(\frac{1.03}{1.04}\right)^{2n}\right)}{1-\frac{1.03}{1.04}}$

$R = P\times 1.04^{2n}\times\frac{.01}{1.04}\times\frac{1}{1.04^{2n-1}\left(1-\left(\frac{1.03}{1.04}\right)^{2n}\right)}$

$R=\frac{P\times 1.04^{2n}\times .01}{1.04^{2n}\left(1-\left(\frac{1.03}{1.04}\right)^{2n}\right)}$

$R=\frac{.01P}{1-\left(\frac{1.03}{1.04}\right)^{2n}}$ or $\frac{P}{100\left(1-\left(\frac{1.03}{1.04}\right)^{2n}\right)}$

(4 marks)