The Language of Sets

A set is a collection of objects, numbers, people, things, etc

A set can be specified by:
(i) listing the elements (members) of the set inside a pair of braces { }
(ii) writing a description within braces
(iii) using set builder notation eg $\{x: \ldots\ldots\}$
– after the colon all the conditions which must be satisfied for something to be a member of the set are written

Example: (i) $S = \{2, 4, 6, 8\}$ which is read as "S is the set whose members are the numerals 2, 4, 6 and 8"

(ii) $S = \{\text{even integers between 1 and 9}\}$ which is read as "S is the set of even integers between 1 and 9"

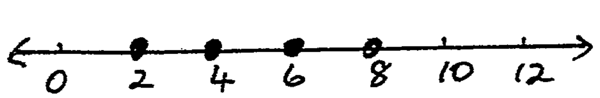
(iii) $S = \{x : 1 < x < 9 \text{ and } x \text{ is even}\}$ which is read as "S is the set of all x such that 1 is less than x which is less than 9 and x is even"

These are all descriptions of the same set.

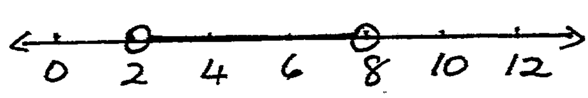
Note: (a) $T = \{x : 1 < x < 9\}$ is the set of all real numbers between 1 and 9. It is not the same set as S ← different as condition is different, doesn't state not even.

The graphs of S and T are:

S:



T



(4) If the type of number is not specified then real numbers are intended.

Two sets are equal if they have exactly the same members.

Notation: The symbol $\in$ means "is a membe of" or "is an element of" and $\notin$ means "is not a member of".

Examples: ① If $X = \{-1, 0, 1, 2, 3\}$ then $0 \in X$ but $1.5 \notin X$

② If $Y = \{y : -1 \leq y \leq 3\}$ then $1.5 \in Y$, $-1 \in Y$ but $3.1 \notin Y$

Size of a Set

A set may be finite or infinite.

Notation: If A is a finite set (know begining & end.) then $|A|$ is used to denote the number of elements of A

$n(A)$ is also use to denote $|A|$.

Examples: ① $A = \{0, 1, 2, 3, \ldots, 19, 20\}$ is a finite set and $|A| = 21$ = 20 no. + 0

② $B = \{$ even numbers $\}$ is an infinite set.

③ $C = \{2\}$ $|C| = 1$

Note that $2 \in \{2\}$ but $2 \neq \{2\}$

2 is an element of the set 2

The Empty Set

The empty set is the set with no members and is denoted by ϕ

$|\phi| = 0$ or $n(\phi) = 0$

There is only 1 empty set.

Subsets

A set X is a subset of a set Y if every element of X is an element of Y

Notation: $X \subseteq Y$ is read as "X is a subset of Y"

Every set is a subset of itself.

The empty set is a subset of every set.

Examples:

① $\{1,2,3\} \subset \{0,1,2,3,4,5\}$

② $\{5\} \subset \{x : 3 \leq x \leq 6\}$

③ $\phi \subset \{1,2,3\}$

④ $\{1,4,9\} \not\subset \{1,2,3,4\}$

Union and Intersection

The union of two sets A and B is denoted by $A \cup B$ and is the set of everything in set A or in set B.

The intersection of two sets A and B is denoted by $A \cap B$ and is the set of everything which belongs to both set A and set B (ie those elements which are common to sets A and B)

Examples: ① $A = \{2, 3, 5, 7, 11\}$ $\quad B = \{2, 4, 6, 8, 10\}$

pitihai $\rightarrow$ $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 10, 11\}$

$A \cap B = \{2\}$

② $C = \{x : x > 3\}$ $\quad D = \{x : x \leq 2\}$

$C \cup D = \{x : x \leq 2 \text{ or } x > 3\}$

$C \cap D = \phi$

Two sets are said to be disjoint if they have no elements in common, eg C and D in ② are disjoint

Set builder notation using the words "or" and "and" can be used to define union and intersection

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Note: The word "or" in mathematics means "and/or."

The Universal Set and the Complement of a Set

A universal set is the set of everything under discussion in a particular context.

The complement of a set A is the set of all members of the universal set which are not in A.

Notation: $\overline{A}$ or $\widetilde{A}$ means "the complement of A."

Examples: ① Let $E = \{0, 1, 2, 3, \ldots, 99, 100\}$ be the universal set.

If $A = \{0, 2, 4, 6, \ldots, 98, 100\}$ then
$\overline{A} = \{1, 3, 5, 7, \ldots, 97, 99\}$

Note: $A \cup \overline{A} = E$ and $A \cap \overline{A} = \phi$

② $E = \{$letters of the English alphabet$\}$
$A = \{$consonants in the English alphabet$\}$
$\overline{A} = \{$vowels in the English alphabet$\} = \{a, e, i, o, u\}$

Using set builder notation:

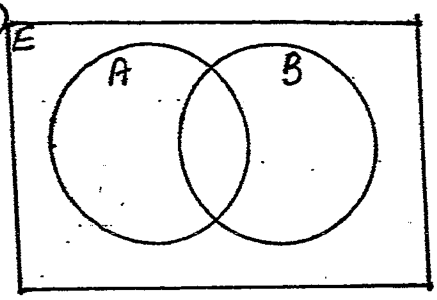
$$\overline{A} = \{x \in A : x \notin A\} \quad (E \text{ is the universal set})$$

Venn Diagrams:

A Venn diagram is a diagram in which sets are represented by circles within a shape (usually a rectangle) which represents the universal set.

Examples :

(1)



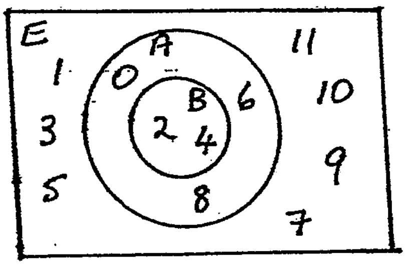
Fill in all the elements in the given Venn diagram.

$E = \{1, 2, 3, 4, \ldots, 15\}$

$A = \{4, 6, 8, 9, 10, 12, 14, 15\}$

$B = \{1, 3, 5, 7, 9, 11, 13, 15\}$

(2)



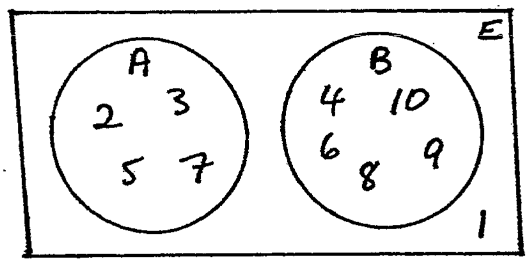
From the diagram given, write down the set for

$E =$

$A =$

$B =$

(3)



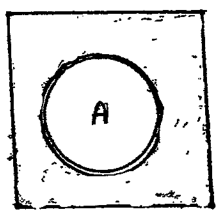
$E =$

$A =$

$B =$

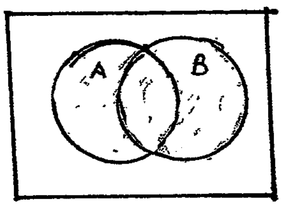
4 Shade the region specified:

(a)



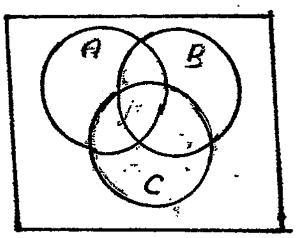
$\overline{A}$

(b)



$A \cup B$

(c)



$(A \cap B) \cup C$

Counting Rule for the Union of Two Sets

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Using Venn Diagrams to Solve Problems

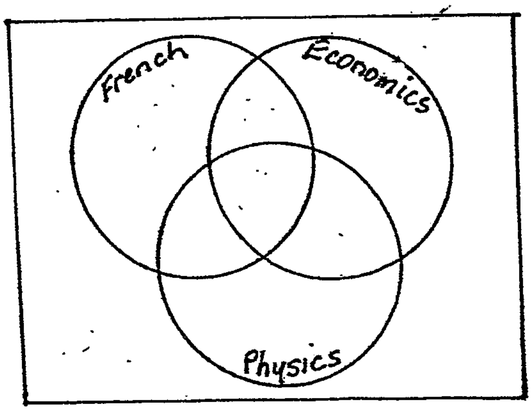
Venn diagrams can be used to solve problems which involve overlapping sets. Usually the number of elements is written in each region rather than the individual elements.

Example: When 150 Year 10 students made their subject selections for Year 11, 15 chose French, 56 chose Economics and 50 chose Physics. Of these 4 chose both French and Physics, 7 chose both Economics and French and 22 chose both Physics and Economics.

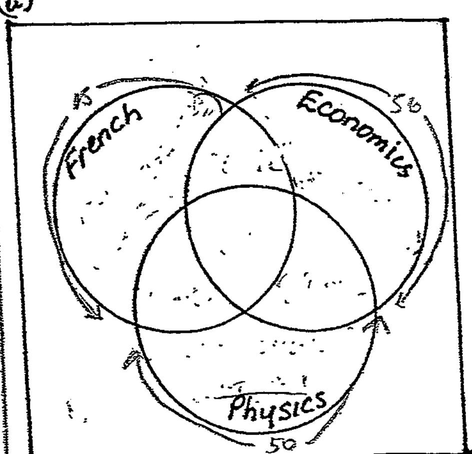
(a) If 3 students chose all three ie French, Economics & ~~French~~ Physics, calculate the number of students who chose none of them.

(b) If 61 students did not select at least one of French, Economics or Physics calculate the number who selected all three.

(a)

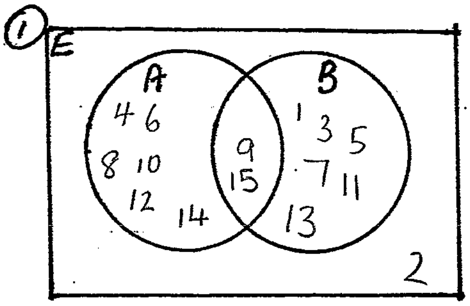


(b)



SOLUTIONS

Examples:

① 

$E = \{1, 2, 3, 4, \ldots, 15\}$
$A = \{4, 6, 8, 9, 10, 12, 14, 15\}$
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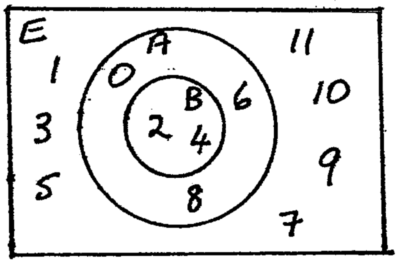
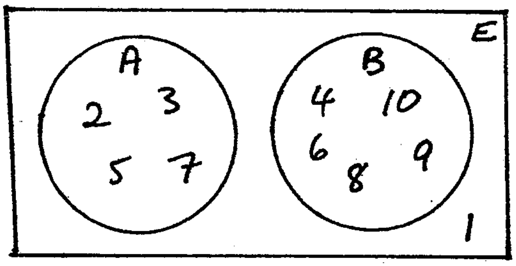
② 

$E = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

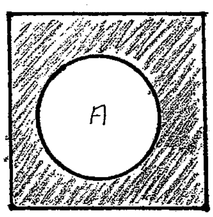
$A = \{0, 2, 4, 6, 8\}$
$B = \{2, 4\}$

③ 

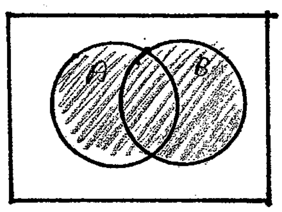
$E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 3, 5, 7\}$
$B = \{4, 6, 8, 9, 10\}$

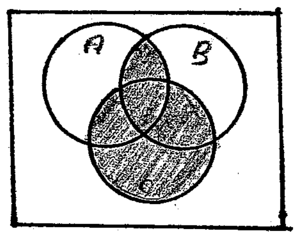
4 Shade the region specified:

(a) 

$\overline{A}$

(b) 

$A \cup B$

(c) 

$(A \cap B) \cup C$

Counting Rule for the Union of Two Sets

Eg. 1).

$$\overset{14}{|A \cup B|} = \overset{8}{|A|} + \overset{8}{|B|} - \overset{2}{|A \cap B|}$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

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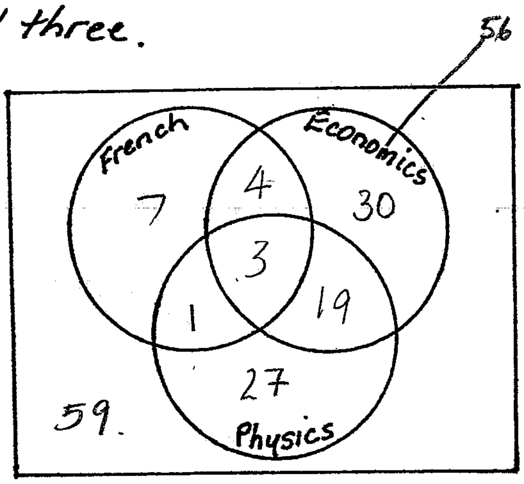
(a)

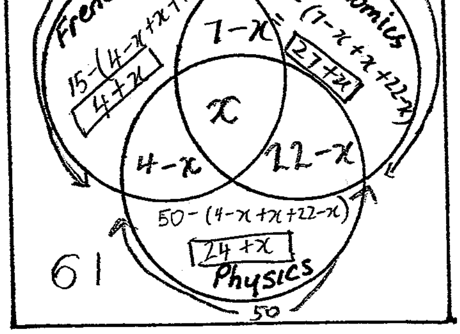


$56+8+27=91$

$150-91=59$ sit outside.

$\therefore$ Number of students who choose none is 59.

(b)



$150-61=89$

$89=(x+4)+(x+27)+(24+x)+(7-x)+(4-x)$
$\quad +(x)+(22-x)$

$89=x+88$

$1=x$

$\therefore x=1$