SYDNEY GIRLS H. S.

11m3 Integration Test Oct 31, 2003

1. Evaluate:

i) $\int_1^2 x\sqrt{x}.dx$ ii) $\int_{-1}^1 (2x^2 - 6).dx$

iii) $\int_1^2 \frac{x^3 - 2x^2 + 6}{x^2}.dx$ iv) $\int_1^2 3x^2(x^3 - 1).dx$

2. Find the following indefinite integrals:

i) $\int \frac{x}{2}.dx$ ii) $\int \frac{1}{2x^2}.dx$ iii) $\int \frac{dx}{(4x-1)^2}$

3. Find the area enclosed by $y = x^2 - 3x$ and the X axis.

4. Find the area enclosed by the curve $y = x^3 - x$, the X axis and the ordinates $x = -1$ and $x = 2$.

5. Find the area enclosed by the curves $y = x^3 + 1$, $y = 4 - 2x$ and the X axis

6. Find the volume of the solid of revolution formed by rotating the region bounded by the $y = x^3$, $y = 8$ and the Y axis is rotated about:
   a) the Y axis
   b) the X axis.

7. a) Evaluate $\int_0^4 2^{x-2}.dx$ using:
   i) Simpsons Rule with 5 ordinates
   ii) Trapezoidal Rule with 5 ordinates

   b) If the answer correct to three decimal places is 4.869, determine the percentage error using Simpsons Rule

Integration Test

Q1

i) $\int_1^2 x \cdot \sqrt{x}\, dx = \int_1^2 x^1 \cdot x^{1/2}\, dx$

$$= \int_1^2 x^{3/2}\, dx$$

$$= \int_1^2 \frac{2x^{5/2}}{5}$$

$$= \left[\frac{2x^{5/2}}{5}\right]_1^2$$

$$= \frac{2\sqrt{32}}{5} - \frac{2}{5}$$

$$= \frac{8\sqrt{2}-2}{5}$$

ii) $\int_{-1}^1 (2x^2-6)\, dx = \left[\frac{2x^3}{3} - 6x\right]_{-1}^1$

$$= \left(\frac{2}{3}-6\right) - \left(-\frac{2}{3}+6\right)$$

$$= -10\frac{2}{3}$$

iii) $\int_1^2 \frac{x^3-2x^2+6}{x^2}\, dx = \int_1^2 x - 2 + \frac{6}{x^2}\, dx$

$$= \left[\frac{x^2}{2} - 2x - \frac{6x^{-1}}{1}\right]_1^2$$

$$= (2-4-3)-\left(\frac{1}{2}-2-6\right)$$

$$= 2\frac{1}{2}$$

iv) $\int_1^2 3x^2(x^3-1)\, dx = \int_1^2 3x^5 - 3x^2\, dx$

$$= \left[\frac{\cancel{3}x^6}{\cancel{6}2} - \frac{\cancel{3}x^3}{\cancel{3}}\right]_1^2$$

$$= \left[(32-8)-\left(\frac{1}{2}-1\right)\right]$$

$$= 24\frac{1}{2}$$

Q2 i) $\int \frac{x}{2}\,dx = \frac{1}{2}\int x\,dx$

$= \frac{1}{2}\int \frac{x^2}{2}\,dx$

$= \frac{x^2}{4} + C$

ii) $\int \frac{1}{2x^2}\,dx = \frac{1}{2}\int x^{-2}\,dx$

$= \frac{1}{2}\int -\frac{x^{-1}}{1}\,dx$

$= -\frac{1}{2x} + C$

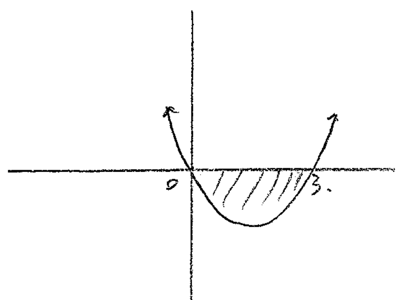
iii) $\int \frac{dx}{(4x-1)^2} = \int (4x-1)^{-2}\,dx$

$= \frac{(4x-1)^{-1}}{-4} + C$

$= -\frac{1}{4(4x-1)} + C$

3. $y = x^2 - 3x$

$= x(x-3)$



$\left|\int_0^3 (x^2 - 3x)\,dx\right|$

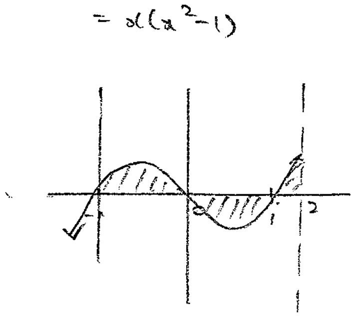
$\left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_0^3$

$= (9 - 13.5)$

$= 4\frac{1}{2}\ u^2$

4. $y = x^3 - x$

$= x(x^2-1)$



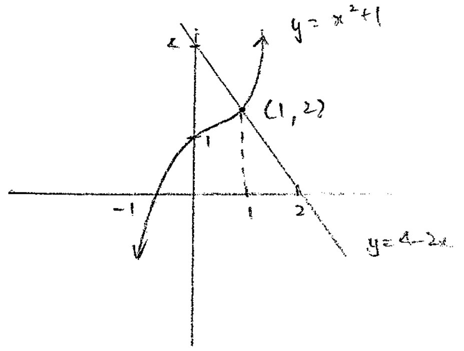
$\int_1^2 x^3 - x\,dx + \left|\int_0^1 x^3 - x\,dx\right| + \int_{-1}^0 x^3 - x\,dx$

$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_1^2 + \left|\left[\frac{x^4}{4} - \frac{x^2}{2}\right]_0^1\right| + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^0$

$= (2 + \frac{1}{4}) + \left|(\frac{1}{4} - \frac{1}{2})\right| + (0 - (\frac{1}{4} - \frac{1}{2}))$

$= 2\frac{3}{4}\ u^2$

5 $y = x^3+1$, $y = 4-2x$.



$4-2x = x^2+1$

$x^2+2x-3=0$.

$(x+3)(x-1)=0$ ∴ $x=-3$ or 1.

(1, 2).

$\int_{-1}^{1} x^3+1 \; dx$

$= \left[\frac{x^4}{4} + x \right]_{-1}^{1}$

$= (1\frac{1}{4}) - (\frac{1}{4} - 1)$

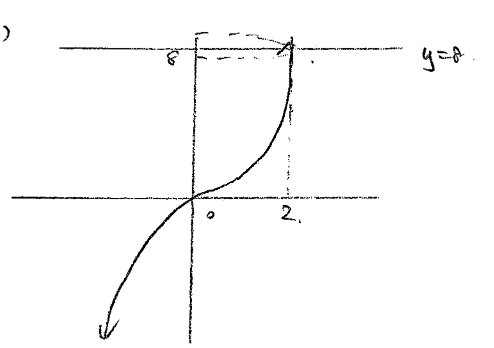
$= 2u^2$.

Area of $\Delta = \frac{1 \times 2}{2}$

$= 1u^2$

∴ Total area $= 3u^2$

6a)



$y = x^3$ ∴ $x = \sqrt[3]{y} = y^{1/3}$

$V = \pi \int_0^8 (y^{1/3})^2 \, dx$

$= \pi \int_0^8 y^{2/3} \, dx$

$= \pi \left[\frac{3y^{5/3}}{5} \right]_0^8$

$= \frac{96\pi}{5} u^3$.

b) $8 = x^3$

∴ $x = 2$

$V = \pi \int_0^2 (x^3)^2 \, dx$

$= \pi \left[\frac{x^7}{7} \right]_0^2$

$= \pi \frac{128}{7}$

Area of cylinder $= \pi r^2 h$

$= \pi \times 8^2 \times 2$

$= 128\pi$

∴ Area $= 128\pi - \frac{128}{7}\pi$

$= 109\frac{5}{7} \pi u^3$.

7a i) $\int_0^4 2^{x-2}$

0	1	2	3	4
1/4	1/2	1	2	4

$A \doteqdot \frac{h}{3}\{(y_1 + y_{n+1}) + 2(y_3 + y_5 + \dots + y_{n-1}) + 4(y_2 + y_4 + \dots + y_n)\}$

$\doteqdot \frac{1}{3}\{(1/4 + 4) + 2(1) + 4(1/2 + 2)\}$

$\doteqdot 5\frac{5}{12} \quad \doteqdot 5.417$ (3dp).

ii) $A \doteqdot \frac{h}{2}\{f(a) + 2f(a+h) + 2f(a+2h) + \dots + f(b)\}$

$\doteqdot \frac{1}{2}\{(\frac{1}{4} + 4) + 2(\frac{1}{2} + 1 + 2)\}$

$\doteqdot 5\frac{5}{8}$.

b) $5.417 - 4.869 = 0.548$

$0.548 \div 4.869 = 11.25\%$.