Name:_

Sydney Girls High School
Mathematics Department

3 Unit Mathematics – Preliminary Course
Topic Test – Geometry

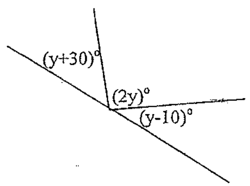
Time Allowed: 75 minutes   Total : 57.

Instructions:
- All answers are to be handed in on **your own paper**.
- Full marks may not be awarded for careless or incomplete work.
- Proofs must be set out clearly.
- Diagrams should be done in pencil and using a ruler.
- The marks allocated for each question are given.

Question 1:

In each of the questions below, find the value of the pronumerals, giving reasons.

a)



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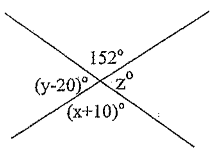
b)



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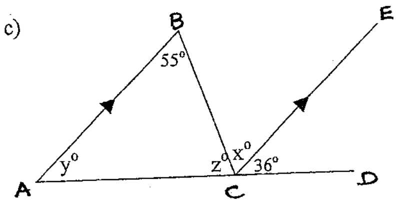
c)



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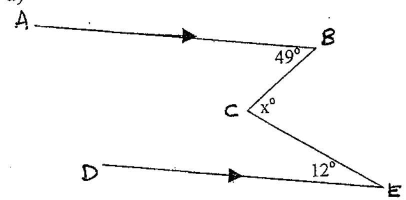
d)



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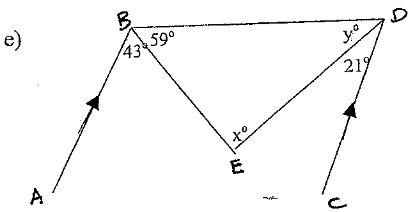
e)



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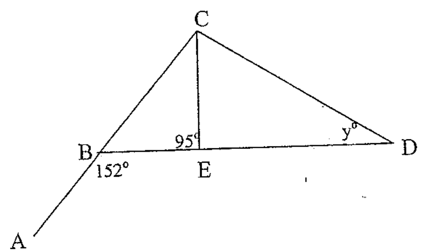
Question 2:

Line CE bisects $\angle BCD$. Find the value of y, giving reasons.



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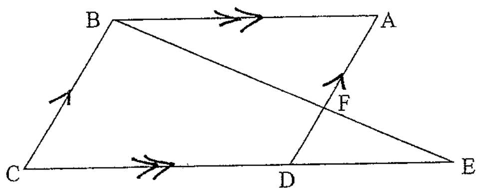
Question 3:

ABCD is a parallelogram with CD produced to E. Prove that $\Delta ABF ||| \Delta CEB$.



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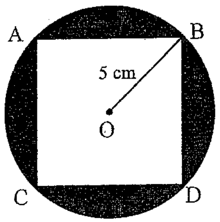
Question 4:

O is the centre of the circle. ABCD is a square. Find the shaded area below, to 1 decimal place.



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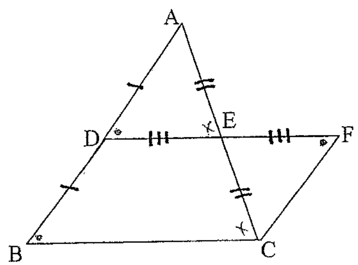
Question 5:

D and E are the midpoints, respectively, of sides AB, AC of a triangle. DE is produced to F, so that DE = EF. Show that BDFC is a parallelogram.



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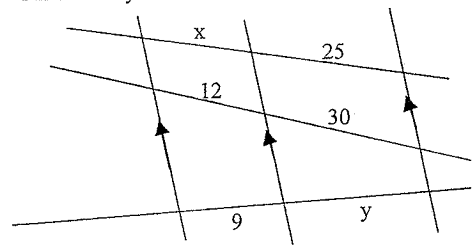
Question 6:

Find x and y.



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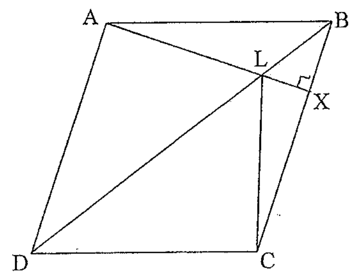
Question 7



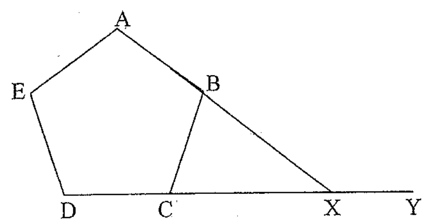
ABCD is a rhombus.

AX is perpendicular to BC and intersects BD at L.

i. State why $\angle ADB = \angle CDB$. 1
ii. Prove $\Delta \cancel{ADB}$ ALD $\equiv \Delta CLD$. 3
iii. Show, giving reasons, that $\angle DAL = 90°$. 3
iv. Hence or otherwise find the size of $\angle LCD$. 1

Question 8:

ABCDE is a regular pentagon and AB and DC produced meet at X. The point Y lies on DCX produced.



(i) Find the size of $\angle ABC$. 2

(ii) Find the size of $\angle BXY$ giving reasons. 3

Question 9:

ABC is a triangle. BC is produced to D and BA is produced to Y such that YC bisects $\angle ACD$. X lies on BC such that CX = AC.

i. Draw a diagram to illustrate the above information. 1
ii. Prove that $\angle AXC = \angle YCD$. 4
iii. Prove that $\Delta ABX ||| \Delta BYC$. 3
iv. If AX = 5 cm, YC = 18 cm and BX = 3 cm, find the length of BC. 2

Question 10:

A regular polygon has interior angles, each of size 165°. How many sides are there in the polygon? 3

☺ END OF TEST ☺

Question 1:

a) $(y+30)^\circ + 2y^\circ + (y-10)^\circ = 180^\circ$ (str. ∠)
$4y + 20 = 180^\circ$
$4y = 160^\circ$
$y = 40^\circ$

b) $x + 10^\circ = 152^\circ$ (vert. opp ∠s)
$x = 142^\circ$

$(y-20)^\circ + 152^\circ = 180^\circ$ (adj. supp)
$y + 132^\circ = 180^\circ$
$y = 48^\circ$

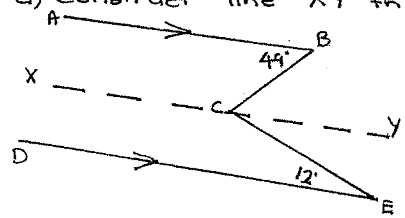
$z = y - 20^\circ$ (vert. opp)
$= 48 - 20$
$= 28^\circ$

c) $x = 55^\circ$ (alt; AB || CE)

$z + 55 + 36 = 180^\circ$ (str. ∠)
$z + 91 = 180^\circ$
$z = 89^\circ$

$y = 36^\circ$ (corr; AB || CE)

d) Construct line XY through C, parallel to AB.



$\angle BCY = 49^\circ$ (alt; AB || XY)
$\angle YCE = 12^\circ$ (alt; XY || DE)
$x = \angle BCY + \angle YCE$ (adj. ∠s)
$= 49 + 12$
$= 61^\circ$

e) $\angle BDC + \angle DBA = 180^\circ$ (co-int ; $AB \parallel CD$)

$(y+21) + (43+59) = 180^\circ$

$y + 123 = 180$

$y = 57^\circ$

$x + 57^\circ + 59^\circ = 180^\circ$ ($\angle$ sum Δ)

$x + 116 = 180$

$x = 64^\circ$

Question 2:

a) $\angle EBC = 180 - 152$ (adj. supp)

$= 28^\circ$

$\angle BCE = 180 - (28+95)$ ($\angle$ sum Δ)

$= 57^\circ$

$\angle DCE = 57^\circ$ (CE bisects $\angle BCD$; given)

$\angle CED = 180 - 95^\circ$ (adj. supp)

$= 85^\circ$

In ΔECD $\quad y + 57 + 85 = 180$ ($\angle$ sum Δ)

$y = 38^\circ$

Question 3:

In ΔABF and ΔCEB

$\angle BAF = \angle BCE$ (opp. $\angle$s of $\parallel$gram ABCD equal)

$\angle ABF = \angle BEC$ (alt ; $AB \parallel CE$)

$\therefore \angle AFB = \angle CBE$ ($\angle$ sum Δ)

$\therefore \Delta AFB \parallel\!| \Delta CEB$ (equiangular)

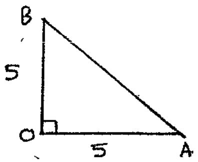
Question 4:

Area circle $= \pi r^2$

$= \pi \times 5^2$

$= 25\pi \text{ cm}^2$

In square ABCD, AD bisects BC at right angles (property of a square).



Area square $= \frac{1}{2} \times 5 \times 5$

$= 12.5 \text{ cm}^2$

$\therefore$ Shaded area $= 25\pi - (12.5 \times 4)$

$= \cancel{66.0}\ 28.5 \text{ cm}^2$ (to 1 dec. pl)

Question 5:

In ΔADE and ΔFEC

$EC = AE$ (E is midpt of AC; given)

$EF = DE$ (given)

$\angle CEF = \angle DEA$ (vert. opp)

$\therefore \Delta ADE \equiv \Delta FEC$ (SAS)

$\therefore FC = AD$ (corr. sides of cong Δs)

$\therefore FC = BD$ (both equal to AD).

Also $\angle DAE = \angle ECF$ (corr. $\angle$ of cong Δs)

$\therefore AD \parallel FC$ (equal alternate angles $\angle DAE$ and $\angle ECF$)

i.e. $BD \parallel FC$ (~~BD is part of~~

$\therefore$ BDFC is a parallelogram (one pair of equal, ~~opp~~ parallel opp. sides BD and FC)

Question 6:

$\frac{x}{25} = \frac{12}{30}$ (ratio of intercepts)

$30x = 300$

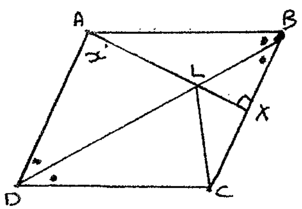
$x = 10$

$\frac{9}{y} = \frac{12}{30}$ (ratio of intercepts)

$12y = 270$

$y = 22.5$

Question 7:



(i) $\angle ADB = \angle CDB$ (diagonals of a rhombus bisect the angles through which they pass)

(ii) In ΔALD and ΔCLD

$AD = CD$ (sides of a rhombus are equal)

$\angle ADL = \angle CDL$ (proven in i)

DL is common

$\therefore \Delta ADL \equiv \Delta CLD$ (SAS)

iii) Let $\angle DAL = x$

$\angle BDC = y$.

$BC = CD$ (sides of a rhombus are equal)

$\therefore \Delta BCD$ is isos. (two sides equal)

$\therefore \angle CBD = \angle BDC$ (base $\angle$s isos Δ =)

$= y^\circ$.

In ΔBLX

$\angle BLX = 180^\circ - 90^\circ - y$ ($\angle$ sum Δ)

$= 90 - y$

$\therefore \angle ALD = 90 - y$ (vert opp.)

$\angle ADL = \angle CDL$ (proven in i)

$= y$

$\therefore$ In ΔALD

$\angle DAL = 180 - (90 - y) - y$ ($\angle$ sum Δ)

$= 90 + y - y$

$= 90^\circ$

iv) $\angle LCD = \angle DAL$ (corres. $\angle$s of cong Δs)

$= 90^\circ$

Question 8:

(i) Angle sum $= 180(n-2)$

$= 180(5-2)$

$= 180 \times 3$

$= 540^\circ$

$\therefore \angle ABC = \frac{540^\circ}{5}$

$= 108^\circ$

(ii) $\angle CBX = 180 - 108^\circ$

$= 72^\circ$ (supp. adj)

Similarly $\angle BCX = 72^\circ$

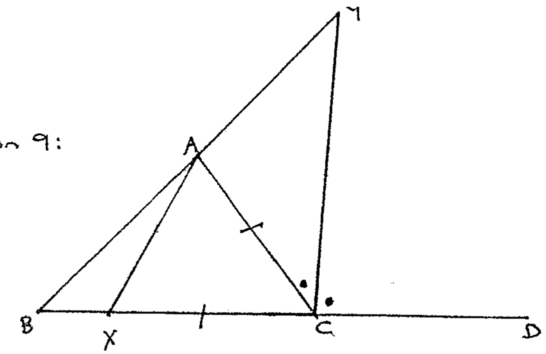
$\therefore \angle BXY = \angle BCX + \angle CBX$ (ext. $\angle$ thm)

$= 72 + 72$

$= 144^\circ$

Question 9:

(i)



(ii) Let $\angle YCD = x$.

$\angle AXC = y$.

In ΔAXC

$AC = XC$ (given)

$\therefore \Delta AXC$ is isos. (two sides equal)

$\therefore \angle XAC = \angle AXC$ (base $\angle$s isos Δ =)

$= y$.

Now $\angle ACD = \angle AXC + \angle XAC$ (ext. $\angle$ thm)

$= y + y$

$= 2y$. ①

But $\angle ACY = \angle YCD$ (YC bisects $\angle ACD$)

$= x$.

$\therefore \angle ACD = \angle ACY + \angle YCD$

$= x + x$

$= 2x$ ②

As ① = ② $2y = 2x$

$\therefore y = x$

$\therefore \angle AXC = \angle YCD$

iii) $\angle AXC = \angle YCD$ (proven)

$\therefore AX \parallel YC$ (equal corresponding angles)

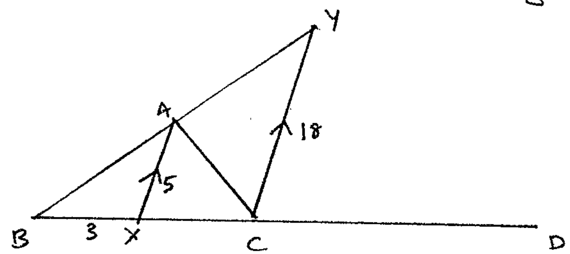
$\therefore \angle BXA = \angle BCY$ (corres; $AX \parallel YC$)

$\angle BAX = \angle BYC$ (")

$\angle XBA = \angle CBY$ (common)

$\therefore \Delta ABX \parallel\!\mid \Delta CBY$ (equiangular)

iv)



$\frac{BX}{BC} = \frac{AX}{YC}$ (corr. sides in same ratio)

$\frac{3}{BC} = \frac{5}{18}$

$5BC = 54$

$BC = 10.8\text{cm}$

Question 10:

$165n = 180(n-2)$

$165n = 180n - 360$

$360 = 15n$

$n = 24$

$\therefore$ 24 sides