SYDNEY GIRLS HIGH SCHOOL

2000

MATHEMATICS

4 UNIT
November Assessment (No 1)

Time allowed - 1.5 hours

DIRECTIONS TO CANDIDATES

NAME Adeline Chen [redacted]

- Attempt ALL questions.
- Questions are not necessarily of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Assume acceleration due to gravity is 10 ms^{-2}.

* All strings may be regarded as weightless and inextensible
* Except where advised otherwise approximate answers should be written correct to 3 significant figures

Question One (16 marks)

Given $f(x) = (x+2)(4-x)$.

Sketch each of the following showing main features. (Each sketch should be approximately 1/3 of a page):

a) $y = f(x)$
b) $y = f(-x)$
c) $y = -f(x)$
d) $y = -2f(x)$
e) $y = |f(x)|$
f) $y = \dfrac{1}{f(x)}$
g) $y = [f(x)]^3$
h) $y^2 = f(x)$

Question Two (13 marks)

No calculus is required for the following.

a) Sketch the polynomial $y = x^3 - 4x^2 + x + 6$ given that $(x-3)$ is one factor of the polynomial.

b) Show that $\dfrac{x^2+x-2}{x-2} = x+3+\dfrac{4}{x-2}$. Hence sketch $y = \dfrac{x^2+x-2}{x-2}$.

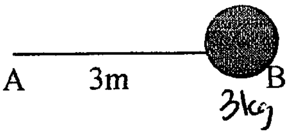
c) Sketch $y = \dfrac{|x+1|}{x+1}$ $\quad -x \quad \dfrac{+\ x+1|}{-\ x+1} =$

Question Three (3marks)

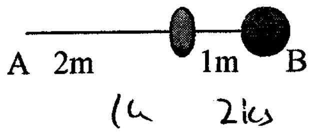
$y = f(x)$ and $y = g(x)$ are shown on a number plane on the last page. Tear that page off and sketch $y = f(x) + g(x)$ on the same number plane. Remember to include that sheet with your answers.

Question Four (5 Marks)

A 3 metre piece of string AB has a mass of 3kg attached at point B. The strings breaks when it reaches a speed of rotation of 60 revolutions per minute.



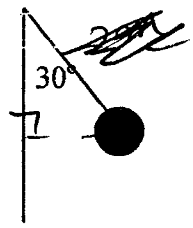
a) Find the breaking strain of the string
b) If the mass at B is replaced by a 2kg mass and an additional 1 kg mass, 2 metres from A is attached, find the new maximum number of revolutions per minute that the string can be rotated.



Question Five (6 Marks)

A mass of 2 kg is rotating in a conical pendulum.

The angle at the vertex is 30^0 and the particle rotates At 20 rpm. Find



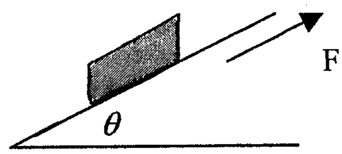
a) tension in the string
b) radius of motion
c) new radius if the angular speed is doubled

Question Six (8 Marks)

A railway line is banked as shown in the adjacent diagram, With the frictional force F being directed up the plane

a) Draw the diagram on your page and label the remaining forces,

mg, N and $\frac{mv^2}{r}$ on the diagram



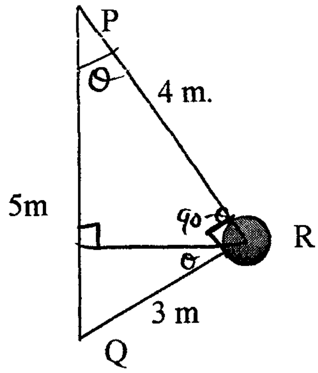
b) Resolve F and N in the vertical and horizontal directions

c) Hence determine expression for F and N in terms of m, g, v, r and θ

d) The railway line is banked due to the radius of curvature of a corner being 400 metres. What should the angle of elevation be for a train travelling at 90 km/hr to exert no lateral force on the rails.

e) What should the angle of elevation be if a 20 tonne train travelling at 108 km/hr is to exert as much force up the track as a 50 tonne train travelling at 72 km/hr exerts down the track

Question Seven (7 Marks)



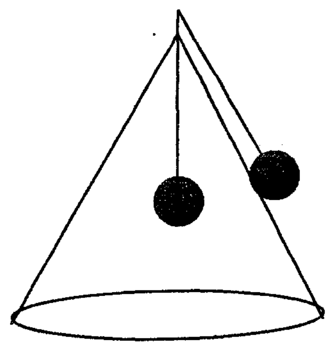
In the adjacent diagram, point Q is 5 metres directly below point P

A mass of 2kg is attached to P by a 4 metre string, and to point Q by a 3 metre string.

a) If the mass at R rotates about PQ at 5 rad/sec, find the tension in both parts of the string

b) What is the minimum angular speed of rotation for there to be some tension in the string QR

Question Eight (6 Marks)



String of length 2 metres is threaded through the top of a cone and masses of 2 kg are placed at both ends. The mass inside the cone remains stationary while the mass on the outside of the cone moves around the outside of the cone at 2 radians per second. If the semi-vertical angle of the cone is 30^0, find how far (to the nearest centimetre) the stationary mass is below the top of the cone if the system is in equilibrium.

(You may assume the small section of the string above the top of the cone to be insignificant)

4U Task 1 Nov 2000

Curve Sketching & Circular Motion

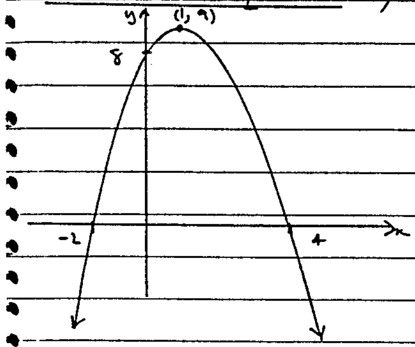
Question One (16 marks)

Marks 2 each part
$-\frac{1}{2}$ mark for leaving of coordinates of intercepts & vertex on first occurrence only

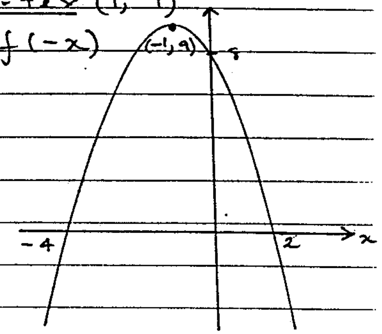
a) $f(x) = (x+2)(4-x)$

x incepts -2, 4, y incept 8

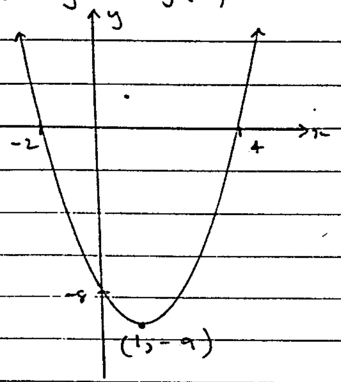
Axis of Symmetry $x = \frac{-2+4}{2} = 1$, Vertex $(1, 9)$



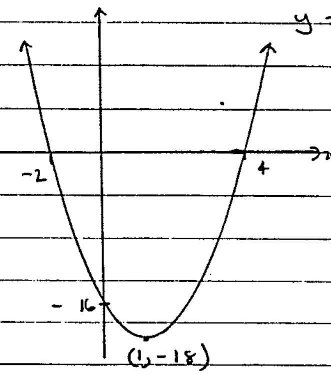
b) $y = f(-x)$



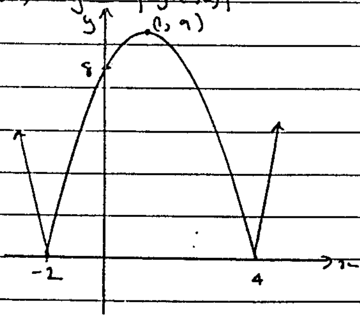
c) $y = -f(x)$



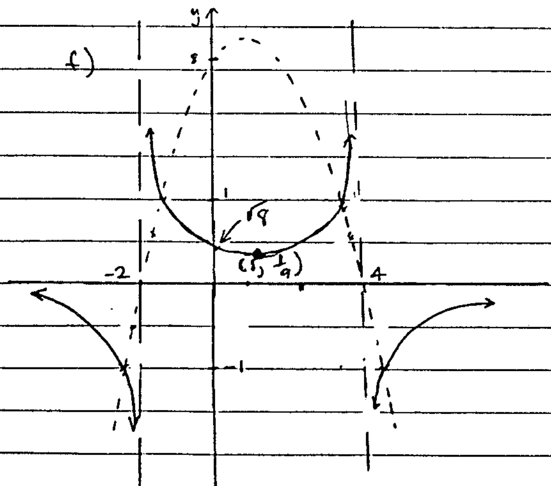
d) $y = -2f(x)$



e) $y = |f(x)|$



f)



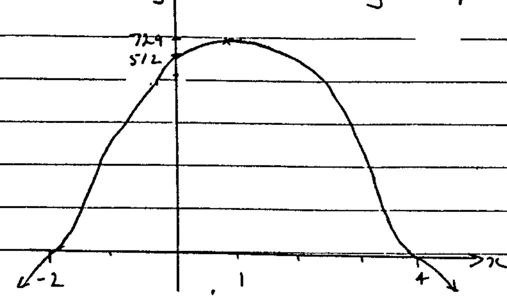
g) $y = [f(x)]^3$

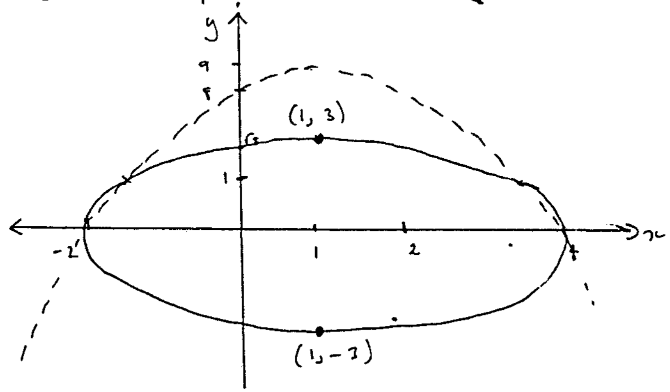
$= [(x+2)(4-x)]^3$

$= (x+2)^3(4-x)^3$ ie triple roots at $x=-2$, $x=4$

y incpt $2^3 \times 4^3 = 512$

Stny pt $x=1$, $y=729$



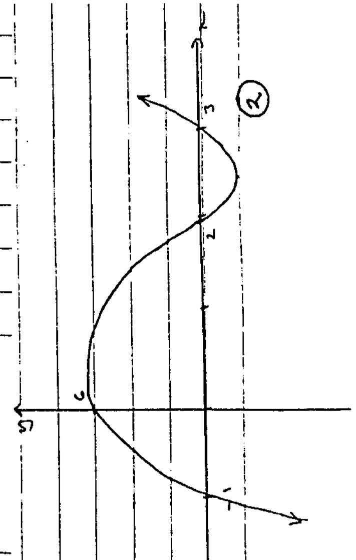


Question Two

a)

$$\begin{array}{r} x^2 - x - 2 \\ x-3\,\overline{)\,x^3 - 4x^2 + x + 6} \\ \underline{x^3 - 3x^2} \\ -x^2 + x \\ \underline{-x^2 + 3x} \\ -2x + 6 \\ -2x + 6 \end{array}$$

2



ie $P(x) = (x-3)(x^2 - x - 2)$

$= (x-3)(x-2)(x+1)$ 1

ie single roots at $x = -1, 2, 3$

b) Show $\dfrac{x^2+x-2}{x-2} = x+3 + \dfrac{4}{x-2}$

LHS $= \dfrac{x+3}{1} + \dfrac{4}{x-2}$

$= \dfrac{(x+3)(x-2)+4}{x-2}$

$= \dfrac{x^2+x-6+4}{x-2}$

$= \dfrac{x^2+x-2}{x-2}$ $=$ RHS (1)

Vertical Asymptote at $x = 2$ (1)

Other: As $x \to +\infty$, $\frac{4}{x-2} \to 0$, $y \to x+3$ [from above]

$x \to -\infty$, $\frac{4}{x-2} \to 0$, $y \to (x+3)$ [from below]

$\therefore$ $y = x+3$ is an asymptote (1)

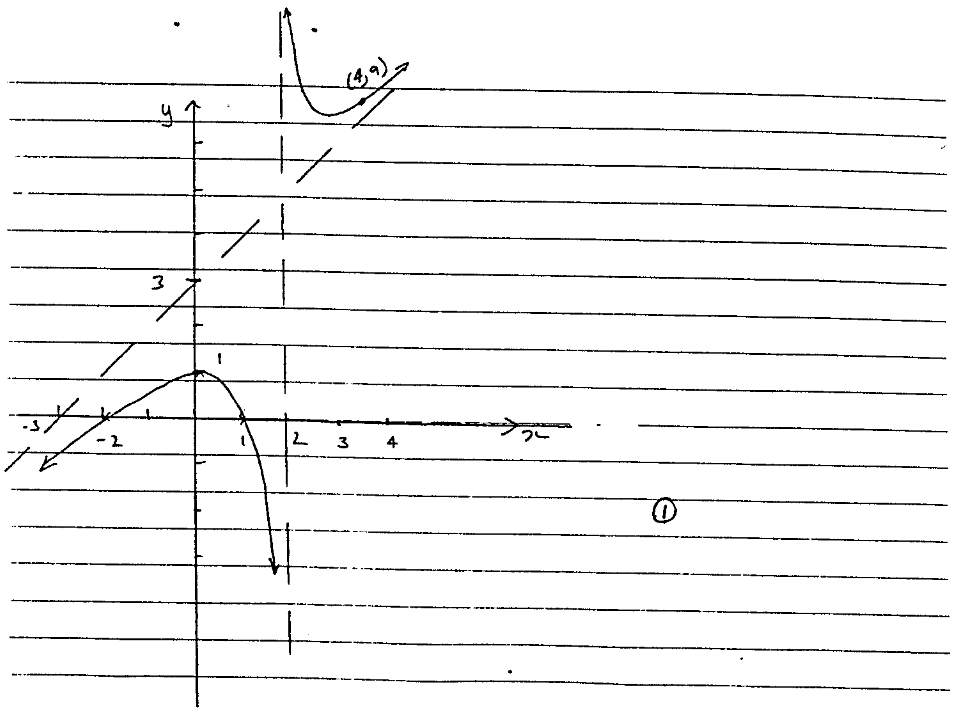
Intercepts

X: If $y=0$, $x^2+x-2=0$

$(x+2)(x-1)=0$

$x=-2$ or $x=1$ (1)

Y: If $x=0$, $y=1$ (1)



(1)

c) $y = \dfrac{|x+1|}{x+1}$ Now $|x+1| = 0$ for $x=-1$

$= (x+1)$ for $x > -1$

$= -(x+1)$ for $x < -1$

then the above can be written

y undefined for $x=-1$

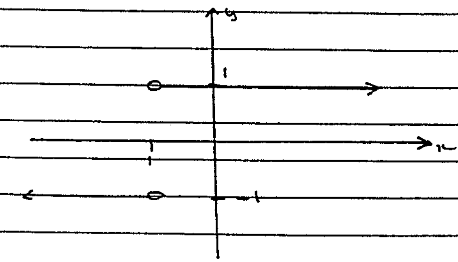
$y = \dfrac{x+1}{x+1}$

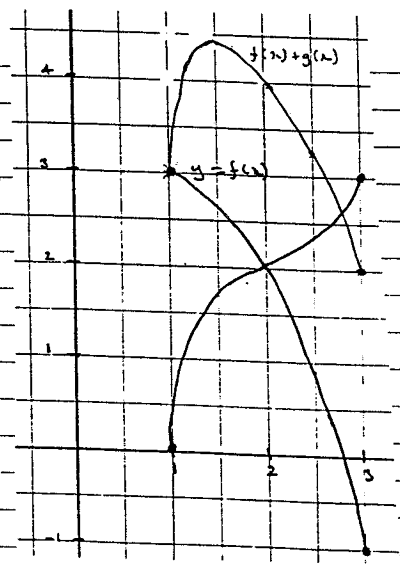
$= 1$ for $x > -1$

$y = \dfrac{-(x+1)}{(x+1)}$ for $x < -1$

$= -1$

(3)





(3)

$T_2 = 0$

$\cos\theta_1 = mg$

$\sin\theta_1 = mr\omega^2$

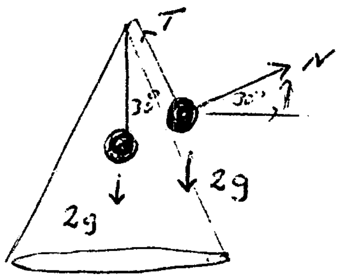
$\therefore \tan\theta_1 = \frac{r\omega^2}{g}$

$\frac{3}{4} \times \frac{10}{2.4} = \omega^2$

$\omega = \sqrt{\frac{300}{96}} = \sqrt{\frac{25}{8}} \doteq 1.77$

$\therefore \omega > 1.77$ rad/sec.

8.



$T\cos 30 + N\sin 30 = 2g$

$mr\omega^2 = T\sin\theta - N\cos\theta$

$T.\frac{\sqrt{3}}{2} + N.\frac{1}{2} = 20 \Rightarrow T\sqrt{3} + N = 40$

$2.r.4 = T.\frac{1}{2} - N.\sqrt{3}/2$

$\therefore T - N\sqrt{3} = 16r$

Now $T = 2g$

$= 2 \times 10$

$= 20\,N$

$20\sqrt{3} + N = 40$

$20 - N\sqrt{3} = 16r$

$N = 40 - 20\sqrt{3}$

$\therefore 20 - \sqrt{3}(40 - 20\sqrt{3}) = r.16$

$20 - 40\sqrt{3} + 60 = 16r$

$80 - 40\sqrt{3} = 16r$

$\frac{\cancel{40}^{5}}{\cancel{16}_{2}}(2 - \sqrt{3}) = r$

$r = \frac{5}{2}(2 - \sqrt{3})$

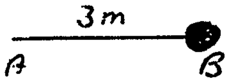
Now $\sin 30 = \frac{r}{l}$

$\therefore l = 5(2 - \sqrt{3})$

$\doteq 1.34m$

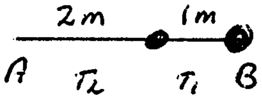
$\therefore$ 0.66m of the string is below the top of the cone.

Q4.



$\omega = 60\, rpm$
$= 60 \times \frac{2\pi}{60}$
$= 2\pi$ rad/sec

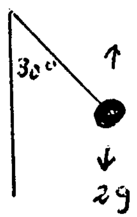
$T = mr\omega^2$
$= 3 \times 3 \times (2\pi)^2$
$\doteqdot 36\pi^2 N \doteqdot 355N$



$T_1 = mr\omega^2$
$= 2 \times 3 \times \omega^2 = 6\omega^2$
$T_2 = T_1 + 1 \times 2 \times \omega^2 = 2\omega^2$
$= 8\omega^2$

$\therefore 8\omega^2 = 36\pi^2$
$\omega^2 = \frac{9\pi^2}{2}$
$\omega = \frac{3\pi}{\sqrt{2}} \doteqdot 2.12\pi$ r/s
$\doteqdot 63.6$ rpm.

Q5.



$T\cos 30 = mg$
$T \sin 30 = mr\omega^2$
$\therefore T.\frac{\sqrt{3}}{2} = 20 \Rightarrow T = \frac{40}{\sqrt{3}} N$
& $T.\frac{1}{2} = 2.r.\left(\frac{2\pi}{3}\right)^2$

$20 rpm = 20 \times \frac{2\pi}{60_3}$

$\therefore \frac{40}{\sqrt{3}}.\frac{1}{2} = 2r.\frac{4\pi^2}{9}$
$\frac{20}{\sqrt{3}} \times \frac{9}{8\pi^2} = r$

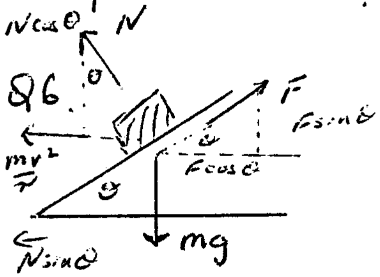
$\therefore r = \frac{45}{2\sqrt{3}\pi^2} \doteqdot 1.32 m$

$\sin 30 = r/\ell$, $\therefore \ell = 2r$
$= \frac{45}{\pi^2\sqrt{3}}$
$\doteqdot 2.63$

$h_1 = \frac{g}{\omega_1^2}$ $h_2 = \frac{g}{\omega_2^2}$
$= \frac{10 \times 4\pi^2}{9}$, $h_2 = \frac{10 \times 16\pi^2}{9} \frac{9}{16\pi^2}$
$= 0.57$

$\therefore \cos\theta = \frac{.57}{2.63}$
$\theta \doteqdot 77°32'$
$\sin 77°32' = \frac{r}{\ell}$
$\therefore r = \ell \sin 77°32'$
$\doteqdot 2.58 m$

Q6.



$N\cos\theta + F\sin\theta = mg$
$\frac{mv^2}{r} = N\sin\theta - F\cos\theta$

$N\cos^2\theta + F\sin\theta\cos\theta = mg\cos\theta$
$N\sin^2\theta - F\cos\theta\sin\theta = \frac{mv^2}{r}\sin\theta$
$\therefore N = \frac{mv^2}{r}\sin\theta + mg\cos\theta$

$N\cos\theta\sin\theta + F\sin^2\theta = mg\sin\theta$
$N\cos\theta\sin\theta - F\cos^2\theta = \frac{mv^2}{r}\cos\theta$
Subtracting
$F = mg\sin\theta - \frac{mv^2}{r}\cos\theta$

$F = 0$, $v = 25 km/hr$, $r = 400$
$\therefore 10\sin\theta = \frac{625}{400}\cos\theta$
$\tan\theta = \frac{625}{4000} = 0.16$
$\therefore \theta \doteqdot 8°53'$

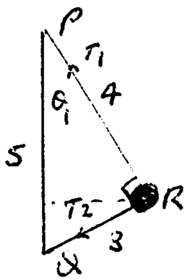
e) $F_1 = 20{,}000(10\sin\theta - \frac{900}{400}\cos\theta)$
$= 20{,}000(10\sin\theta - \frac{9}{4}\cos\theta)$

$F_2 = 50{,}000(10\sin\theta - \frac{400}{400}\cos\theta)$
$= 50{,}000(10\sin\theta - \cos\theta)$

Now $|F_1| = |F_2|$

$2(10\sin\theta - \frac{9}{4}\cos\theta) = |5(10\sin\theta - \cos\theta)|$
$\therefore 20\sin\theta - \frac{9}{2}\cos\theta = -50\sin\theta + 5\cos\theta$
$70\sin\theta = \frac{19}{2}\cos\theta$
$\tan\theta = \frac{19}{140}$
$\therefore \theta \doteqdot 7°44'$

Q7.



$T_1\cos\theta_1 = T_2\cos\theta_2 + m$
$T_1\sin\theta_1 + T_2\sin\theta_2 = m r$
$T_1\frac{4}{5} = T_2\frac{3}{5} + 20$
$4T_1 = 3T_2 + 100$ (1
$T_1\frac{3}{5} + T_2\frac{4}{5} = 2 \times \frac{12}{5} \times$
$3T_1 + 4T_2 = 600$ (2

$\times(1)$ by 4: $16T_1 - 12T_2 = 400$
$\times(2)$ by 3: $9T_1 + 12T_2 = 1800$
$\therefore 25T_1 = 2200$, $\therefore T_1 = 88 N$

Sub in (1) $352 = 3T_2 + 100$
$\therefore T_2 = 84 N$