Sydney Girls High School

2009 HSC Assessment Task 1

November 2008

MATHEMATICS

Extension 2

Year 12

Time allowed - 90 minutes

Topics: Curve Sketching and Circular Motion

Instructions

- Attempt all questions.
- Questions are NOT of equal value.
- All necessary working should be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Write on one side of the paper only.
- Questions do not necessarily appear in order of difficulty.
- Diagrams are not to scale
- Use $g = 10ms^{-2}$

Part A - Curve Sketching (42 marks)

Calculus is not required in this section

Question One (10 marks) **Marks**

Sketch each of the following showing important features including asymptotes where necessary

a) $y = \sqrt{4 - x^2}$ 2

b) $\frac{x^2}{9} + \frac{y^2}{4} = 1$ 2

c) $(x+1)(y+1) = 1$ 3

d) $y = x^2(x+2)(x-3)^3$ 3

Question Two (12 marks)

Sketch each of the following showing important features

a) $x^2 - y^2 = 16$ 3

b) $y = \sqrt{x^2 - 9}$ 3

c) $x^2 + y^2 - 2x + 4y - 4 = 0$ 3

d) $y = \frac{2}{(x-1)^2}$ 3

Question Three (7 marks)

a) Sketch the curve $y = 1 - x^2$. 1

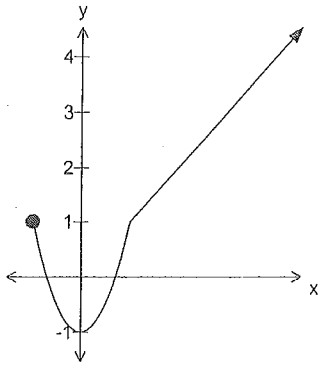
Hence or otherwise sketch:

b) $y = |1 - x^2|$ 1

c) $y = (1 - x^2)^2$ 2

d) $y = \frac{1}{1 - x^2}$ 3

Question Four (5 marks)
Given the graph of $y = f(x)$ below



Note that you may trace the original function on to your paper if it helps you

On separate axes sketch the graphs of:

a) $y = f(|x|)$ 1

b) $y^2 = f(x)$ 2

c) $y = 2^{f(x)}$ 2

Question Five (6 marks)

Given $f(x) = \dfrac{2}{x+1}$

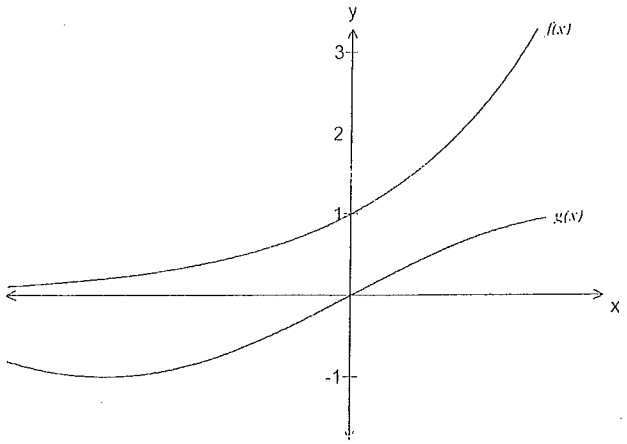
a) Sketch the function showing all relevant features 2

b) Write down the domain and range of $f(x)$ 2

c) Find an expression for $f^{-1}(x)$ 1

d) Sketch $f^{-1}(x)$ on the same set of axes as part a) 1

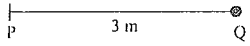
Question Six (2 marks)
Copy or trace the graphs of $f(x)$ and $g(x)$ below onto your exam paper. Hence sketch the graph of $y = f(x) \times g(x)$



Part B – Circular Motion (38 marks)

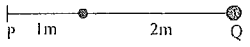
Use $g = 10ms^{-2}$ in all questions

Question Seven (5 marks)
A three metre piece of string PQ has a mass of 2kg attached at Q
The string rotates in a horizontal circle on a flat smooth surface about P and will break if the angular speed exceeds 8 rad.s^{-1}.



a) Find the breaking strain (maximum tension of the string) 2

b) If the mass at Q is replaced by a 3kg mass and an additional 1kg mass is attached 1 m from P, find the maximum number of revolutions per minute the new system can be rotated at before the string breaks 3



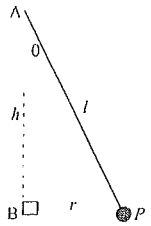
Question Eight (4 marks)
A car of mass 800 kg passes over a circular bridge of radius 40 metres

a) Find the force exerted by the bridge (at its peak) if the car is travelling at $10ms^{-1}$ 2
b) At what speed will the car leave the top of the bridge? 2

Question Nine (4 marks)
At a height of $4.22\times10^{7}m$ above the earths centre the acceleration due to gravity is $0.22ms^{-2}$. Calculate the period in hours of a satellite in a circular orbit at this height

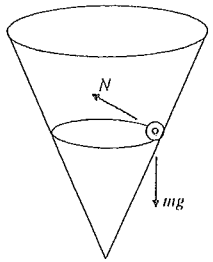
Question Ten (7 marks)

A particle of mass m, attached by a light rod to a point A moves with constant angular velocity ω in a horizontal circle of radius r metres, h metres below A. The length of the rod is l metres



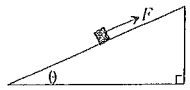
a) Draw a diagram to show the forces acting on the mass at the point P 1
b) Find an expression for the time(T) taken for the mass to complete one revolution in terms of h and g 4
c) What effect will the doubling of the mass at P have on the motion? Give a brief reason for your answer 2

Question Eleven (6 marks)
A cone with semi vertical angle 30° is placed with its vertex at the bottom. A ball moves around the inner side of the cone at $\frac{3}{\pi}$ rev.s^{-1}



How high is the ball from the vertex of the cone?

Question Twelve (12 marks)
A railway line is banked at an angle θ to the horizontal as shown in the diagram below. The frictional force is directed up the plane



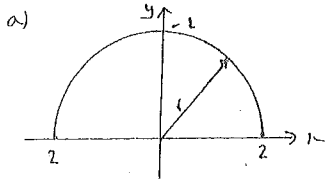
The resultant force of circular motion is composed of the force of gravity, the normal force and the sideways frictional force F.

a) Copy the diagram and on it show $\frac{mv^2}{r}$, mg and N 1
b) Resolve F and N in the vertical and horizontal directions 2
c) Hence determine an expression for F independent of N. 2
d) The railway line is banked due to the radius of curvature of a corner being 500 metres. What should the angle of elevation θ be for a train travelling at 72 km/hr to exert no lateral force on the rails? 3
e) What should the angle of elevation be if a 20 tonne train travelling at 90 km/hr is to exert as much force up the track as a 40 tonne train travelling at 54 km/hr exerts down the track 4

S.C.H.S Extn 2 Task 1 Dec 2008 [HSC 2009]

Question One

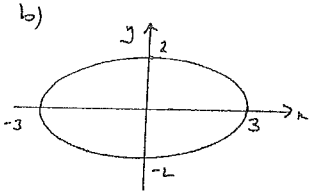
a)



(2)

semi circle
$-2 \le x \le 2$
$0 \le y \le 2$
$r = 2$

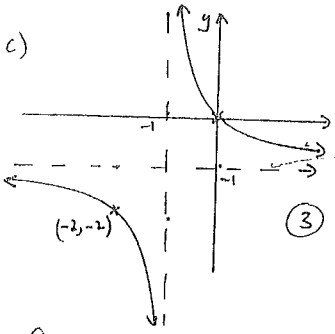
b)



intercepts $x = \pm 3$
$y = \pm 2$
ellipse

(2)

c)

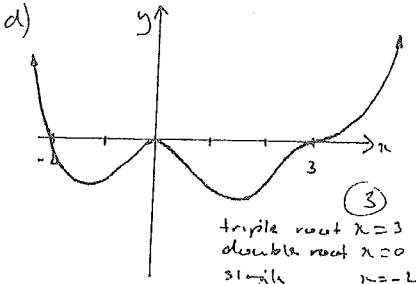


asymptotes
$x = -1$, $y = -1$

(3)

Points on each branch of curve

d)



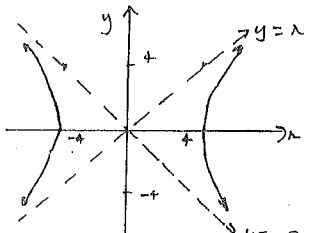
(3)

triple root $x = 3$
double root $x = 0$
single $x = -2$
when $x > 3, y > 0$ } curve
$x < -2, y > 0$ } in

Question Two

a) $x^2 - y^2 = 16$

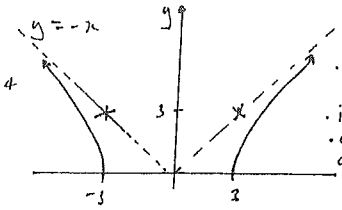
$(x+y)(x-y) = 16$



asympt $y = \pm x$
intercepts $x = \pm 4$
symmetric

(3)

b) $y = \sqrt{x^2 - 9}$



- asymptotes $y = \pm x$ for $y \ge 0$
- intercepts $x = \pm 3$
- curve must approach asymptote not curve away

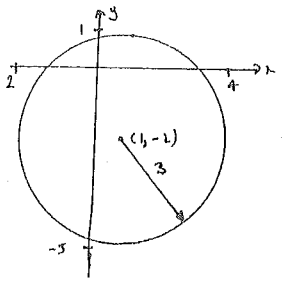
(3)

c) $x^2 + y^2 - 2x + 4y - 4 = 0$

$x^2 - 2x + 1 + y^2 + 4y + 4 = 4 + 1 + 4$

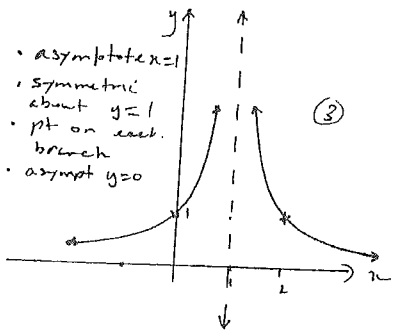
$(x-1)^2 + (y+2)^2 = 9$

circle centre $(1, -2)$ $r = 3$



(3)

d) $y = \dfrac{2}{(x-1)^2}$

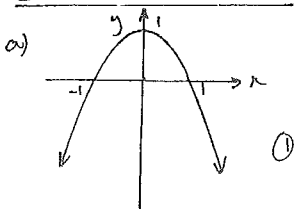


- asymptotes $x = 1$
- symmetric about $y = 1$
- pt on each branch
- asympt $y = 0$

(3)

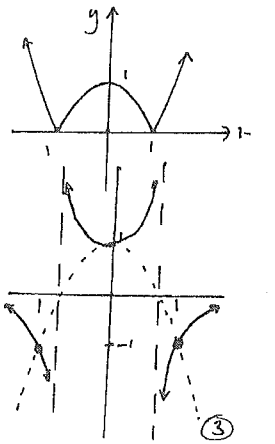
Question Three

a)



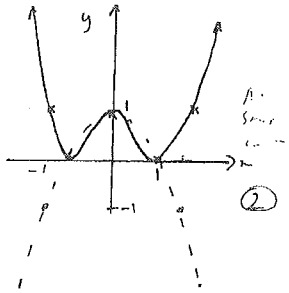
(1)

b)



(1)

c)



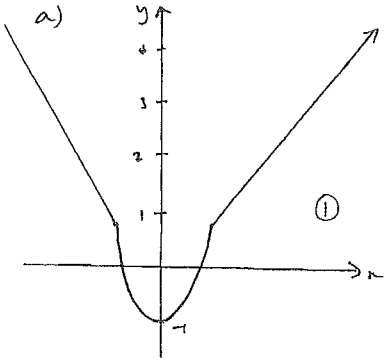
(2)

d)

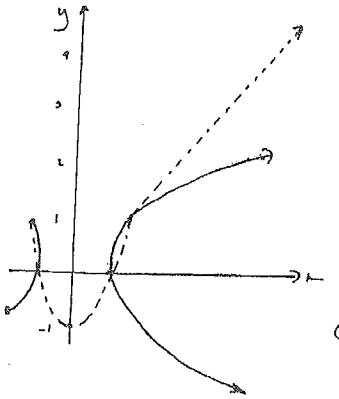
(3)

Question Four

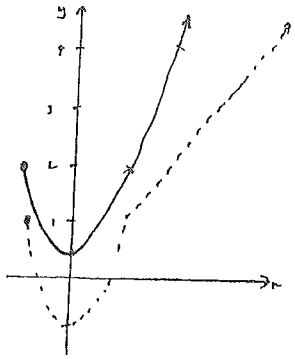
a)



(1)



(2)



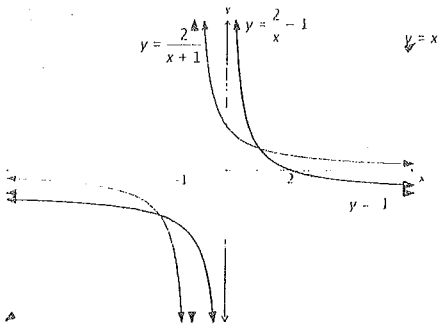
(2)

1, 4, 5, 2,

PART A:

Question 5:

a) $f(x) = \dfrac{2}{x+1}$



b) $D: x \neq -1,\ R: y \neq 0$

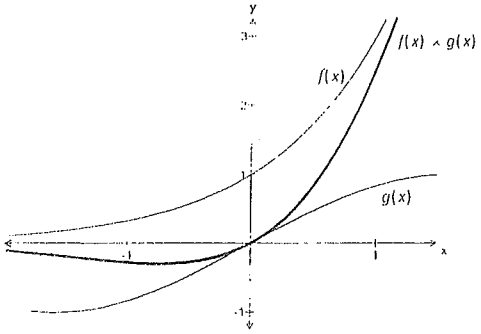
c) $D: x \neq 0,\ R: y \neq -1$

$x(y+1) = 2$

$y = \dfrac{2}{x} - 1$

d) See (a).

Question 6:



PART B:

Question 7:

a)

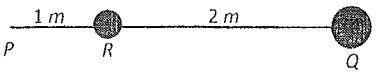
$T = mr\omega^2$

$= 2 \times 3 \times 8^2$

$= 384$

Breaking strain of string is 384 Newtons

b)



$T_{RQ} = mr\omega^2$

$= 3 \times 3 \times \omega^2$

$= 9\omega^2$

$T_{PR} = mr\omega^2 + T_{RQ}$

$= 1 \times 1 \times \omega^2 + 9\omega^2$

$= 10\omega^2$

$10\omega^2 = 384$

$\omega^2 = 38.4$

$\omega \cong 6.2 \text{ rads}^{-1}$

$f = \dfrac{\omega}{2\pi}$

$\cong \dfrac{6.2}{2\pi}$

$\cong 0.99 s^{-1}$

Approx. 59 revolutions per minute.

Question 8:

a)

$mg - N = \dfrac{mv^2}{r}$

$800 \times 10 - N = \dfrac{800 \times 10^2}{40}$

$8000 - N = 2000$

$N = 6000$

Force exerted by bridge is 6000 Newtons

b) When car leaves bridge, $N = 0$:

$mg = \dfrac{mv^2}{r}$

$v^2 = rg$

$v^2 = 40 \times 10$

$= 400$

$v = \pm 20$

Reqd. speed is 20 ms^{-1}.

$mr\omega^2 = ma$

$4.22 \times 10^7 \times \omega^2 = 0.22$ ✓

$\omega = 7.220277876 \times 10^{-5}$ ✓

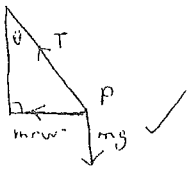
$T = \dfrac{2\pi}{7.2... \times 10^{-5}}$ ✓

$= 87021.13781 s$

$= 24.17253826 h$

$= 24 h$ ✓

c) a)



✓

b) $T\sin\theta = mr\omega^2$

$T\cos\theta = mg$ ✓

$\tan\theta = \dfrac{r\omega^2}{g}$

$\dfrac{r}{h} = \dfrac{r\omega^2}{g}$ ✓

$\omega^2 = \dfrac{g}{h}$

$\omega = \sqrt{\dfrac{g}{h}}$ ✓

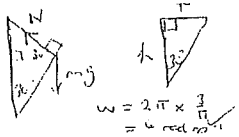
$T = \dfrac{2\pi}{\sqrt{\frac{g}{h}}}$ ✓

$= 2\pi\sqrt{\dfrac{h}{g}}$

c) No effect because ✓

T is not dependent ✓

on the mass



$\omega = 2\pi \times \frac{3}{\pi}$ ✓

$= 6 \text{ rad s}^{-1}$

$N\sin 30° = mg$

$N\cos 30° = mr\omega^2$

$\tan 30° = \dfrac{g}{3\omega^2}$ ✓

$\dfrac{1}{\sqrt{3}} = \dfrac{10}{36r}$

$r = \dfrac{5\sqrt{3}}{18}$ ✓

$\tan 30 = \dfrac{r}{h}$ ✓

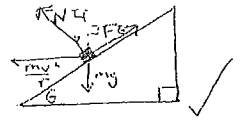
$\dfrac{1}{\sqrt{3}} = \dfrac{r}{h}$ ✓

$h = \sqrt{3}\, r$

$= \dfrac{\sqrt{3} \times 5\sqrt{3}}{18}$

$= \dfrac{5}{2} m$ ✓

12 a)



✓

b) $N\cos\theta + F\sin\theta - mg = 0$ ✓

$N\sin\theta - F\cos\theta = \dfrac{mv^2}{r}$ ✓

c) $N\sin\theta\cos\theta + F\sin^2\theta = mg\sin\theta$

$N\sin\theta\cos\theta - F\cos^2\theta = \dfrac{mv^2}{r}\cos\theta$

$F = mg\sin\theta - \dfrac{mv^2}{r}\cos\theta$ ✓

d) $F = 0$ $\quad v = \dfrac{72000}{3600} = 20 ms^{-1}$

$0 = m \times 10 \times \sin\theta - \dfrac{m \times 20^2}{500}\cos\theta$ ✓

$\dfrac{4\cos\theta}{5} = 10\sin\theta$

$\tan\theta = \dfrac{2}{25}$ ✓

$\theta = 4° 34' 20.11''$

$\cong 4° 34'$ ✓

e) $F_{up} = F_{down}$

$v_{2t} = \dfrac{40\cos\theta}{3600}$

$= 25 ms^{-1}$

$v_{4t} = \dfrac{54000}{3600}$

$= 15 ms^{-1}$ ✓

$4000 \times 10 \times \sin\theta -$

$4000 \times \dfrac{25^2\cos\theta}{500} =$

$2000 \times \dfrac{25^2\cos\theta}{500} -$

$20000 \times 10 \times \sin\theta$

$60000\cos\theta = 43\cos\theta$

$\tan\theta = \dfrac{43}{600}$

$\theta = 4° 5' 57.4''$

$\cong 4° 6'$ ✓