- Use separate paper
- Show all necessary working

S.G.H.S. – CALCULUS TEST – YR11 – EXT 1

Question One (12 marks)
Differentiate with respect to x

a) $y = 5x^3 - 2x^{-2}$

b) $y = (4x^3 + 7)^5$

c) $y = \dfrac{8x-2}{2+8x}$

d) $y = x\sqrt{x}$

e) $y = \dfrac{1}{\sqrt{8x-1}}$

f) $y = x^{\frac{2}{3}}(1 - x^3)$

Question Two (4 marks)
Evaluate

a) $\lim\limits_{x \to 4} 4x - 1$

b) $\lim\limits_{x \to 3} \dfrac{x-3}{x^2-9}$

c) $\lim\limits_{x \to \infty} \dfrac{3x^2+3x}{6x-x^2}$

Question Three (7 marks)

a) Sketch the graph of $y = \frac{2}{3}x^3 - 8x$ locating all relevant features.

b) For what values of x is $y = \frac{2}{3}x^3 - 8x$ increasing ?

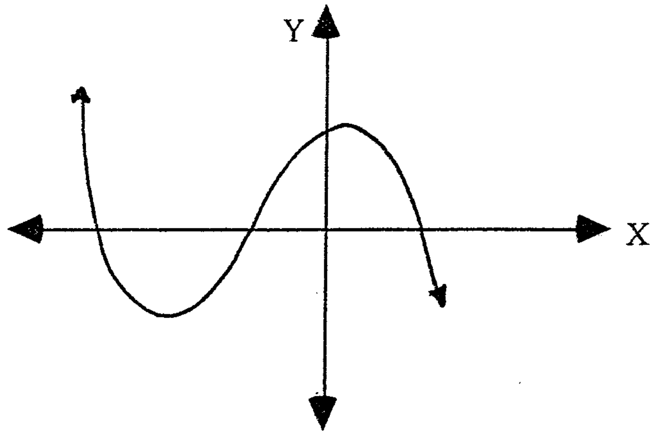
Question Four (3 marks)
Derive from first principles $y = 2x^2 - x$

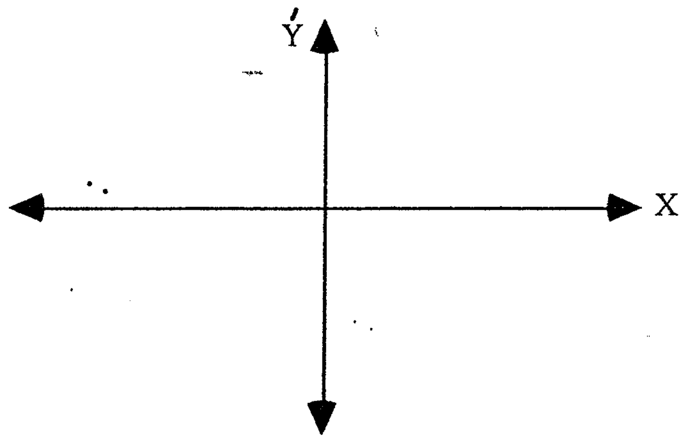
Question Five (6 marks)
A fence 300 metres long is to be erected so that it encloses three sides of a rectangular paddock. The fourth side of the paddock is formed by a creek. Find the dimensions of the paddock such that the enclosed area is a maximum.

PTO

Question Six (2 marks)

The graph of $y = f(x)$ is drawn below





a) Trace the graph of $y = f(x)$ and the second set of axes onto your answer sheet.

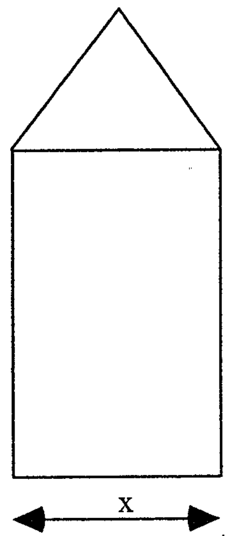
b) On the second set of axes draw a sketch of $y' = f(x)$

Question Seven (4 marks)

Find the equation of the normal to $y = 5x^3 - 3x$ at the point where $x = -1$

Question Eight (6 marks)

The figure below represents a large window made up of a rectangle and an equilateral triangle.



a) The beading around the outside of the window is 12m long. Show that the area, of the window is given by $A = 6x - \dfrac{\left(6-\sqrt{3}\right)x^2}{4}$

b) Hence calculate the dimensions of the window so that it allows the maximum light through answer correct to 2dp).

Yr Eleven Calculus Test I Solutions

Q1 (2 marks each)

a) $y = 5x^3 - 2x^{-2}$

$\frac{dy}{dx} = 15x^2 + 4x^{-3}$

b) $y = (4x^3+7)^5$

$\frac{dy}{dx} = 5(4x^3+7)^4(12x^2)$

$= 60x^2(4x^3+7)^4$

c) $y = \frac{8x-2}{2+8x}$

$\frac{dy}{dx} = \frac{(2+8x)(8)-(8x-2)(8)}{(2+8x)^2}$

$= \frac{16+64x-64x+16}{(2+8x)^2}$

$= \frac{32}{(2+8x)^2}$

d) $y = x\sqrt{x}$

$= (x)(x^{\frac{1}{2}})$

$= x^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

$= \frac{3\sqrt{x}}{2}$ (2)

e) $y = (8x-1)^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}(8x-1)^{-\frac{3}{2}}$

$= \frac{-4}{\sqrt{(8x-1)^3}}$

f) $y = x^{\frac{2}{3}}(1-x^3)$

$= x^{\frac{2}{3}} - x^{\frac{11}{3}}$

$\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} - \frac{11}{3}x^{\frac{8}{3}}$

$= \frac{2}{3\sqrt[3]{x}} - \frac{11}{3\sqrt[3]{x^8}}$

Q2

a) $\lim_{x\to 4} 4x-1 = 4(4)-1$
$= 15$ (1)

b) $\lim_{x\to 3} \frac{x-3}{(x-3)(x+3)}$

$= \lim_{x\to 3} \frac{1}{x+3}$ (1)

$= \frac{1}{6}$

c) $\lim_{x\to\infty} \frac{3x^2+3x}{6x-x^2}$

$= \lim_{x\to\infty} \frac{3+\frac{3}{x}}{\frac{6}{x}-1}$

$= \frac{3}{-1}$ (2)

$= -3$

Q3

On the Y axis $x=0$
$y=0$ (1)

On the X axis $y=0$

$0 = \frac{2}{3}x^3 - 8x$

$0 = x(\frac{2}{3}x^2-8)$

$x=0$ or $\frac{2}{3}x^2 - 8 = 0$

$x^2 = 12$

$x = \pm\sqrt{12}$ (1)

$= \pm 2\sqrt{3}$

$\frac{dy}{dx} = 2x^2 - 8$

for a stny pt $\frac{dy}{dx}=0$

$0 = 2(x-2)(x+2)$

$x=-2$ or $x=2$

$y = \frac{32}{3}$ $\quad y = -\frac{32}{3}$

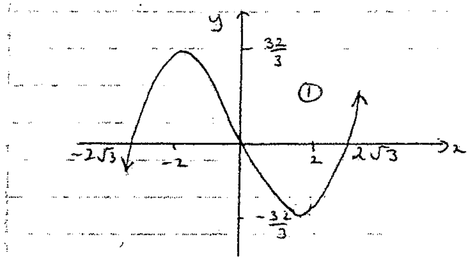
x	-2^-	-2	-2^+	x	2^-	2	2^+
$\frac{dy}{dx}$	+	0	−	$\frac{dy}{dx}$	−	0	+

∴ max at $(-2, \frac{32}{3})$ (1), min at $(2, -\frac{32}{3})$

Function is odd (1)

$F(-a) = -F(a)$

as $x\to\infty$, $y\to\infty$ (1)
$x\to-\infty$, $y\to-\infty$



(1)

b) increasing for $x<-2$ and $x>2$ (1)

Q4 (3 marks)

Let $P(x,y)$ be a point on the curve and $Q(x+\Delta x, y+\Delta y)$ a neighbouring point

$y = 2x^2 - x$

subst co-ords of Q

$y+\Delta y = 2(x+\Delta x)^2 - (x+\Delta x)$

$= 2x^2 + 4x\Delta x + (\Delta x)^2 - (x+\Delta x)$

$y+\Delta y = 2x^2 + 4x\Delta x + (\Delta x)^2 - x - \Delta x$

$\Delta y = 4x\Delta x + (\Delta x)^2 - \Delta x$

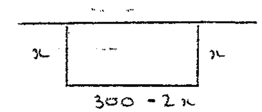
$\frac{\Delta y}{\Delta x} = \frac{4x\Delta x + (\Delta x)^2 - \Delta x}{\Delta x}$

$\frac{\Delta y}{\Delta x} = 4x + \Delta x - 1$

Now $\frac{dy}{dx} = \lim_{\Delta x\to 0} 4x+\Delta x - 1$

$= 4x-1$

Q5



$A = LB$

$= x(300-2x)$

$A = 300x - 2x^2$ (1)

$\frac{dA}{dx} = 300 - 4x$ (1)

for stny pt $\frac{dA}{dx} = 0$ (1)

$0 = 300 - 4x$

$x = 75$ (1)

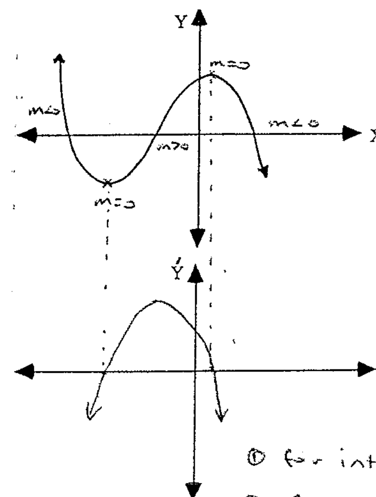
x	75^-	75	75^+
$\frac{dA}{dx}$	+	0	−

(1)

∴ max when $x = 75$

and dimensions 75 m × 150 m (1)

Q6



(1) for intercepts

(1) for concavity

(7) $y = 5x^3 - 3x \quad \therefore \frac{dy}{dx} = 15x^2 - 3$

at $x = -1$, $m_1 = 15 - 3 = 12 \quad \therefore$ required $m_2 = -\frac{1}{12}$

$y = -5 + 3 = -2$

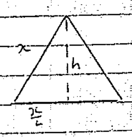
$y + 2 = -\frac{1}{12}(x+1)$

$12y + 24 = -x - 1$

$x + 12y + 25 = 0$

Q8

a)



Area $A = \frac{1}{2} \times b \times h$

$b = x$

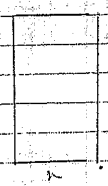
$x^2 = h^2 + \frac{x^2}{4}$

$\therefore \frac{3x^2}{4} = h^2$

$h = \frac{\sqrt{3}x}{2}$

$\therefore$ Area $\Delta = \frac{1}{2} \times x \times \frac{\sqrt{3}x}{2}$

$= \frac{\sqrt{3}x^2}{4}$ (1)



$A = L \times B$, $B = x$

$L = \frac{12 - 3x}{2}$

$= 6 - \frac{3x}{2}$ (1)

$A = x(6 - \frac{3x}{2})$

$= 6x - \frac{3x^2}{2}$

$\therefore A = \frac{\sqrt{3}x^2}{4} + 6x - \frac{3x^2}{2}$

$= 6x + \frac{\sqrt{3}x^2 - 6x^2}{4}$

$= 6x - \frac{(6-\sqrt{3})x^2}{4}$ (1)

$\frac{dA}{dx} = 6 - \frac{(6-\sqrt{3})x}{2}$ (1)

for max/min $\frac{dA}{dx} = 0$ (1)

x: $\frac{dA}{dx}$ + 0 − $\therefore$ max

$0 = 6 - \frac{6-\sqrt{3}}{2}x$

$\frac{6-\sqrt{3}}{2}x = 6$

$x = 6 \times \frac{2}{6-\sqrt{3}}$

$x = \frac{12}{6-\sqrt{3}}$ $(\doteqdot 2.81)$ (1)

$h \doteqdot 1.73$