Sydney Girls High School

MATHEMATICS EXTENSION 1 - DEC 2003

Question 1. (20 marks)

Marks

1. Solve $x^2 - 2x - 8 = 0$ — 2

2. Sketch $y = x^2 - 6$ showing the intercepts on both axes. — 3

3. For the equation $2x^2 + 7x - 3 = 0$ whose roots are α and β — 3

   Find (i) $\alpha + \beta$

   (ii) $\alpha\beta$

   (iii) $\alpha^2 + \beta^2$

4. Find the value of k for which the expression $kx^2 - 6x + 2$ is positive definite. — 2

5. Solve the inequality $-2x^2 + 3x + 14 > 0$ and graph the solution on a number line. — 3

6. For what values of k does $3x^2 + 2x + k = 0$ have no real roots. — 2

7. Find C if the roots of $4x^2 - 20x + C = 0$ differ by 2 — 2

8. Solve $x^4 = 4(x^2 + 8)$ — 3

Question 2. (20 marks)

Marks

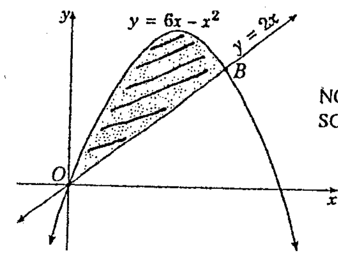
1. Integrate the following

   (i) $\int 6x^2 + \frac{4}{x^3}\,dx$ — 2

   (ii) $\int \frac{dx}{\sqrt{6x+1}}$ — 2

2. Evaluate: (i) $\int_{-2}^{-1} (x - \frac{1}{x})^2\,dx$ — 2

   (ii) $\int_0^4 \sqrt{t}(4 - t)\,dt$ — 2

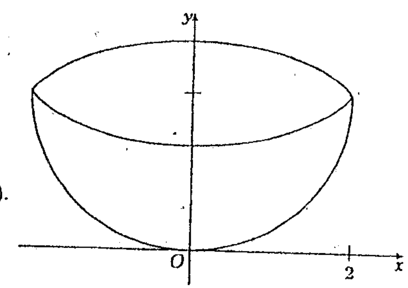
3. 

   NOT TO SCALE

   (i) Find the points of intersection of the line $y = 2x$ and the curve $y = 6x - x^2$ — 2

   (ii) Find the shaded area bounded by $y = 6x - x^2$ and $y = 2x$ — 2

4. A bowl is formed by rotating the part of the curve $y = \frac{x^2}{4}$ between $x = 0$ and $x = 2$ about the y axis. Find the volume of the bowl (exact answer). — 4



5. — 4

x	1	1.5	2	2.5	3
$f(x)$	5	1	-2	3	7

Use Simpson's Rule with five function values to evaluate $\int_1^3 f(x)\,dx$

Question 3 (20 marks)

Marks

1. Find the centre and radius of the circle

$$x^2 + y^2 + 4x + 8y + 11 = 0$$ 2

2. The focus of a parabola is $(0, 2)$ and its directrix is the line $y = -2$ 6

   (i) Sketch the parabola, indicating the coordinates of the vertex.
   (ii) Write down the focal length.
   (iii) Find the equation of the parabola.

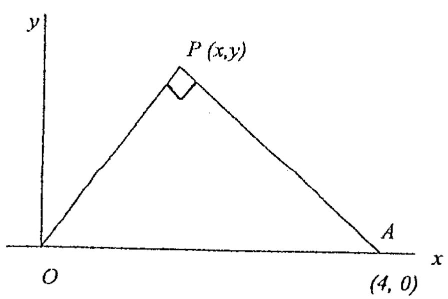
3. A point P moves so that its distance from the point $A\ (1, 5)$ is always twice its distance from the point $B(4, -1)$. 4
   Find the locus of P.

4. For the parabola $y = x^2 - 8x + 4$ Find:- 4

   (i) The coordinates of the vertex.
   (ii) The coordinates of the focus.

5. The point P moves so that always $PA \perp PO$ 4

   (i) Show that the locus of P is a circle.
   (ii) Find its centre and radius.



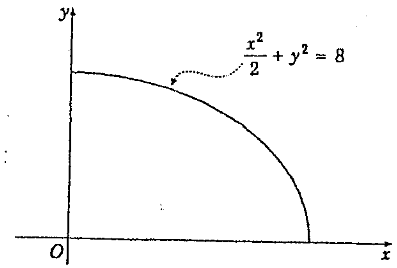
Question 4 (20 marks)

Marks

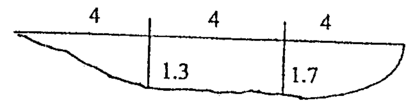
1. Find the values of k for which the equation
   $x^2 + (k + 2)\,x + 4 = 0$ has

   (i) equal roots. 2
   (ii) real and distinct roots. 2

2. Find the volume of the solid of revolution formed by rotation of the curve $\frac{x^2}{2} + y^2 = 8$ about the x axis. 4



3.

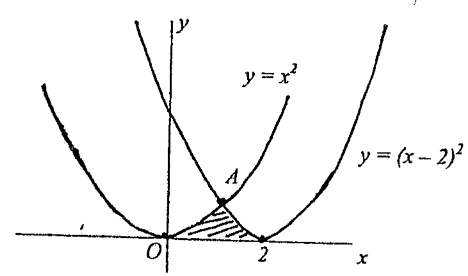


This diagram shows the cross section of a creek with the depths shown in metres at 4 metre intervals. The total width of the creek is 12 metres.

   (i) Use the trapezoidal rate to find an approximate value for the area of the cross section. 4

   (ii) Water flows through this section at a speed of 0.5 metres per second. Calculate the approximate volume of water that flows past this section in an hour. 2

4. (i) Find the coordinates of point A. 6
   (ii) Find the shaded area (to 1 decimal place).
   (iii) If this shaded area is rotated around the y axis, find the volume of revolution correct to 1 decimal place.



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Maths Solutions Y2003 Ext1 Task1

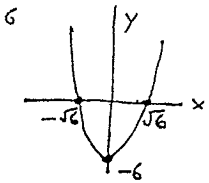
Question 1.

1 $x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$\therefore x = 4$ and $x = -2$ (2)

2 $y = x^2 - 6$



(3)

3 $2x^2 + 7x - 3 = 0$

i $\alpha + \beta = -b/a = -7/2 = -3\frac{1}{2}$

ii $\alpha\beta = c/a = -3/2 = -1\frac{1}{2}$

iii $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2(\alpha\beta)$

$= 12\frac{1}{4} + 3$

$= 15\frac{1}{4}$ (3)

4 For positive definite

$k > 0$ and $\Delta < 0$

$\Delta = b^2 - 4ac$

$= 36 - 4 \times k \times 2$

$= 36 - 8k$

$36 - 8k < 0$

$36 < 8k$

$\therefore k > 4\frac{1}{2}$ for pos. definite (2)

5 $-2x^2 + 3x + 14 > 0$

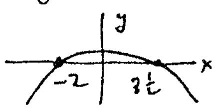
Put $-2x^2 + 3x + 14 = 0$

$\therefore 2x^2 - 3x - 14 = 0$

$(2x - 7)(x + 2) = 0$

$x = 3\frac{1}{2}$ and -2

Sketch $y = -2x^2 + 3x + 14$



(3)

$\therefore$ Solution to $-2x^2 + 3x + 14 > 0$

is $-2 < x < 3\frac{1}{2}$ = ans

6 $3x^2 + 2x + k = 0$

For no real roots $\Delta < 0$

$b^2 - 4ac < 0$

$4 - 4 \times 3 \times k < 0$

$4 < 12k$

$\therefore k > 1/3$ for no real roots (2)

7 $4x^2 - 20x + C = 0$

Let roots be α, $(\alpha - 2)$

$\alpha + (\alpha - 2) = -b/a = 20/4 = 5$

$2\alpha = 7 \quad \therefore \alpha = 3\,1/2$

$\alpha(\alpha - 2) = c/a = C/4$

$\therefore C = 4\alpha(\alpha - 2)$

$= 4 \times 7/2 \times 3/2$ (2)

8 $x^4 = 4(x^2 + 8)$

Let $a = x^2$

$a^2 = 4(a + 8)$

$a^2 - 4a - 32 = 0$

$(a - 8)(a + 4) = 0$

$a = 8$ and $a = -4$

$x^2 = 8$ $\qquad$ $x^2 = -4$

$\therefore$ No solution

$\therefore x = \pm\sqrt{8}$ or $\pm 2\sqrt{2}$ (3)

Question 2

Tot = 20

1 i $\int 6x^2 + 4x^{-3}\,dx$

$= \frac{6x^3}{3} + \frac{4x^{-2}}{-2} + c$

$= 2x^3 - \frac{2}{x^2} + c$ (2)

ii $\int (6x+1)^{-\frac{1}{2}}\,dx$

$= \frac{(6x+1)^{1/2}}{6 \times 1/2} + c$

$= \frac{\sqrt{6x+1}}{3} + c$ (2)

2 i $\int_{-2}^{-1} x^2 - 2 + x^{-2}\,dx$

$= \left[\frac{x^3}{3} - 2x + \frac{x^{-1}}{-1}\right]_{-2}^{-1}$

$= \left[\frac{x^3}{3} - 2x - \frac{1}{x}\right]_{-2}^{-1}$

$= \left(-\frac{1}{3} + 2 + 1\right) - \left(-\frac{8}{3} + 4 + \frac{1}{2}\right)$

$= \frac{5}{6}$ (2)

ii $\int_0^4 4t^{\frac{1}{2}} - t^{3/2}\,dt$

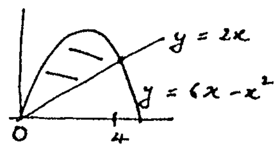
$= \left[\frac{2}{3} \cdot 4\,t^{3/2} - \frac{2}{5} t^{5/2}\right]_0^4$

$= \left[\frac{8}{3} t\sqrt{t} - \frac{2}{5} t^2\sqrt{t}\right]_0^4$

$= \left(\frac{8}{3} \times 4 \times 2 - \frac{2}{5} \times 16 \times 2\right) - (0)$

$= 8\frac{8}{15}$ (2)

3



i $2x = 6x - x^2$

$x^2 - 4x = 0$

$x(x - 4) = 0$

$\therefore x = 0$ and $x = 4$

$y = 0$ $\qquad$ $y = 8$

ans $(0,0)$ and $(4,8)$ (2)

ii Area $= \int_0^4 6x - x^2 - 2x\,dx$

$= \int_0^4 4x - x^2\,dx$

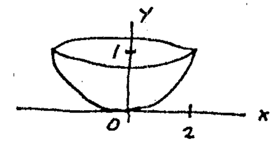
$= \frac{4x^2}{2} - \frac{x^3}{3}$

$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= \left(32 - \frac{64}{3}\right) - (0)$

$= 10\frac{2}{3}$ units^2 (2)

4



$y = \frac{x^2}{4}$

$\therefore x^2 = 4y$

Vol $= \pi\int_0^1 x^2\,dy$

$= \pi\int_0^1 4y\,dy$

$= \pi\left[2y^2\right]_0^1$

$= 2\pi$ units^3 (4)

5

x	$f(x)$	w	$w\,f(x)$
1	5	1	5
1.5	1	4	4
2	-2	2	-4
2.5	3	4	12
3	7	1	7

$\Sigma w f(x) = 24$

$\int_1^3 f(x)\,dx = \frac{h}{3}\Sigma w f(x)$

$= \frac{0.5}{3} \times 24$

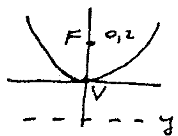
Question 3

1 $x^2 + 4x + 4 + y^2 + 8y + 16 = -11 + 20$

$(x+2)^2 + (y+4)^2 = 9$

Centre $= (-2, -4)$ Radius $= 3$ (2)

2 i



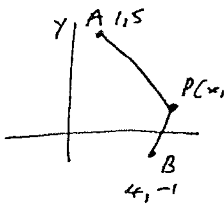
$V = (0,0)$ (6)

ii Focal length $= 2$ $(= a)$

iii Equation $x^2 = 4ay$

$\therefore x^2 = 8y$

3



(4)

Let $P = (x,y)$ = point on locus

Condition: $PA = 2 \times PB$

$\sqrt{(x-1)^2 + (y-5)^2} = 2 \times \sqrt{(x-4)^2 + (y+1)^2}$

$x^2 - 2x + 1 + y^2 - 10y + 25 = 4\{(x^2 - 8x + 16) + (y^2 + 2y + 1)\}$

$0 = 3x^2 + 3y^2 - 30x + 18y + 42$

$\therefore x^2 + y^2 - 10x + 6y + 14 = 0$ (Locus eqn)

4 $y = x^2 - 8x + 4$

$y - 4 = x^2 - 8x$

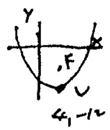
$y - 4 + 16 = x^2 - 8x + 16$

$y + 12 = (x-4)^2$

$(x-4)^2 = 4 \times \frac{1}{4}(y+12)$

i Vertex $= (4, -12)$

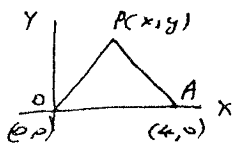
ii



Focus $= (4, -11\frac{3}{4})$

(4)

5



i Let $P = (x,y)$ = point on locus

Condition: $PA \perp PO$ $(m_1 \cdot m_2 = -1)$

$\frac{y}{x} \cdot \frac{y}{(x-4)} = -1$

$y^2 = -x^2 + 4x$

$\therefore x^2 + y^2 - 4x = 0$ is locus which is equation of circle.

ii $x^2 - 4x + y^2 = 0$

$x^2 - 4x + 4 + y^2 = 4$

$(x-2)^2 + y^2 = 4$

$\therefore$ Centre $= (2, 0)$ Radius $= 2$

(4)

Question 4

1 $x^2 + (k+2)x + 4 < 0$

i $\Delta = (k+2)^2 - 16$

$= k^2 + 4k - 12$

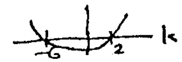
$\Delta = 0$ for equal roots

$(k+6)(k-2) = 0$

$\therefore k = -6$ or 2 ans (2)

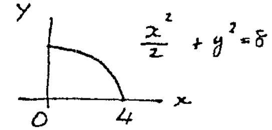
ii $\Delta > 0$ for real & distinct roots

$(k+6)(k-2) > 0$



$\therefore k < -6,\ k > 2$ ans (2)

2



$Vol = \pi \int_0^4 y^2 \, dx$

$= \pi \int_0^4 8 - \frac{x^2}{2} \, dx$

$= \pi \left[8x - \frac{x^3}{6} \right]_0^4$

$= \pi \left[\left(32 - \frac{64}{6}\right) - (0) \right]$

$= \frac{64\pi}{3}$ units^3 (4)

3 i

x	y	ω	$\omega \times y$
0	0	1	0
4	1.3	2	2.6
8	1.7	2	3.4
12	0	1	0
		$\Sigma \omega x y$	6

Area $\doteqdot \frac{h}{2} \times \Sigma \omega \times y$

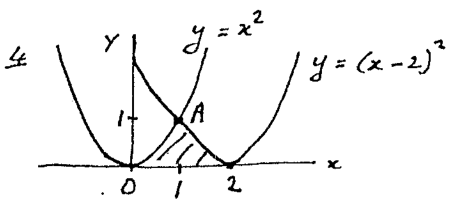
$= \frac{4}{2} \times 6$

$= 12 \ m^2$ (4)

ii Volume $= 0.5 \times 12 \times 60 \times 60$

$= 21600 \ m^3$ (2)

4



i $x^2 = (x-2)^2$

$x^2 = x^2 - 4x + 4$

$4x = 4$

$x = 1, \ y = 1$

$\therefore$ Point $A = (1,1)$

ii Area $= \int_0^1 x^2 \, dx + \int_1^2 (x-2)^2 \, dx$

$= \left[\frac{x^3}{3} \right]_0^1 + \left[\frac{(x-2)^3}{3} \right]_1^2$

$= \left(\frac{1}{3}\right) - (0) + (0) - \left(-\frac{1}{3}\right)$

$= \frac{2}{3}$ units^2

(6)

iii) $V = \pi \int x_1^2 \, dy - \pi \int x_2^2 \, dy$

$\pm\sqrt{y} = x - 2$

From diagram, $x - 2 = -\sqrt{y}$

$x = 2 - \sqrt{y}$

$x^2 = 4 - 4\sqrt{y} + y$

$\therefore V = \pi \int_0^1 (4 - 4\sqrt{y} + y - y) \, dy$

$= \pi \left[4y - \frac{8}{3} y^{3/2} \right]_0^1$

$= 4\pi/_3$ units^3