SYDNEY GIRLS HIGH SCHOOL

LABOR OMNIA VINCIT

MATHEMATICS 2 Unit

Assessment Task 2 for 2008 HSC

March 4th, 2008

Reading Time 5 minutes
Time allowed: 90 minutes

Topics: Co-ordinate Geometry, Probability, Sequences and Series

DIRECTIONS TO CANDIDATES:

- Attempt all questions
- Questions are of equal value
- There are 5 questions with part marks shown
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work
- Board approved calculators may be used
- Answers may be left in either exact form or correct to three significant figures
- Write on one side of the paper only

Question 1

Consider the points A(4,-1), B(2,-3) and C(-2, 5)

a) Find (in simplest form), the distance AC [1]

b) Find D & E the midpoints of BA and BC [2]

c) Show DE is half the length of AC [2]

d) Find the gradient of AC [1]

e) Show AC is parallel to DE [1]

f) Find the angle AC makes with the positive X axis [2]

g) Find the equation of AC [2]

h) Find the equation of the line through B parallel to AC [1]

i) Find the equation of the line through B perpendicular to AC [2]

j) Find the perpendicular distance from B to AC [2]

k) Find the area of triangle ABC [2]

l) Find point H if ABCH is a parallelogram [1]

Question 2

a) If A(-6,1), B(-2,1) and C(2,3) are the vertices of a triangle,

i) Write down the equation of the perpendicular bisector of AB [1]
ii) Find the equation of the perpendicular bisector of BC [3]
iii) Show that the bisectors meet at the point D (-4,10) [2]
iv) Find the equation of the circle with centre D and radius DC [2]
v) Show that this circle also passes through B [2]

b) Prove the points A(1, -2), B(6,1), C(3,-4) and D(-2,-7) are the vertices of a rhombus. [6]

c) Find the distance between the lines $3x - 2y + 9 = 0$ and $3x - 2y + 3 = 0$ [4]

Question 3

a) Of the 45 students on a 611 bus, just 9 want to get out at Central. If two students are selected at random, find the probability that
   i) Both want to get out at Central
   ii) Neither get out at Central

[4] 0

b) Scarlet has 9 coloured pencils in her pencil case; 4 red, 3 crimson and 2 burgundy. If she draws out two pencils at random, find the probability that she draws:
   i) two red pencils
   ii) two pencils of the same colour

[4] 6

c) In a group of 24 Year 12 students, 13 study Physics, 11 study Chemistry and 9 study neither subject. If a student is chosen at random, find the probability she studies both Physics and Chemistry.

[4] 0

d) The probability of throwing a 6 with a die is $\frac{1}{6}$. If three dice are thrown, find the probability of
   i) 3 sixes
   ii) 0 sixes
   iii) 1 six.

[4] 0

e) A box of tennis balls has 6 new and 4 used balls.
   i) If two players randomly choose two balls to play a match, find the probability that they choose two new balls
   ii) After the match the balls are returned to the box and two other players randomly choose two balls to play their match. What is the probability that these players play with new balls?

2½

Question 4

a) Find the value of $\sum_{r=0}^{4} 3\left(2^r - 1\right)$ [3] 0

b) Consider the terms 101, 83, 65
   i) Show these terms form an arithmetic progression and hence find
   ii) The 12^{th} term
   iii) The sum of 12 terms
   iv) An expression for the sum of n terms
   v) The least number of terms necessary for the sum to become negative

[6] 5

c) If 2x, x-3 and x-7 are in geometric progression,
   i) Find a positive value of x
   ii) Find the 5^{th} term that corresponds to the positive value of x

[4] 0

d) For what values of x will the sequence $1+2x, (1+2x)^2, (1+2x)^3 \ldots$ have a limiting sum? [3] 6

e) In an AP, the 4^{th} term is 17 and the 7^{th} term is 26. Find the sum of 10 terms [4] 0

Question 5

a) Find the sum of the multiplies of 7 that lie between 2000 and 3000 [5] 4

b) What is the least number of terms of the GP 2000, 400, 80 ...that must be taken so that the term is less than 1. 0

c) In a GP, $U_2 = -27$ and $U_5 = 8$, find U_6 and the infinite sum [5] 0

d) Find A , B and the infinite sum of the expression:

$$T_n = A\left(\frac{1}{2}\right)^n + B\left(\frac{1}{3}\right)^n \text{ if } T_1 = 7 \text{ and } T_2 = 3$$

[5] 2

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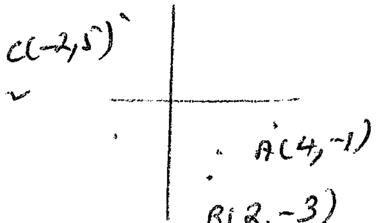


a) $d_{AC} = \sqrt{(4+2)^2+(5+1)^2}$

$= 6\sqrt{2}$

b) $D = \left(\frac{2+4}{2}, \frac{-1-3}{2}\right)$

$= (3,-2)$

$E = \left(\frac{2-2}{2}, \frac{5-3}{2}\right)$

$= (0,1)$

c) $DE = \sqrt{3^2+3^2}$

$= 3\sqrt{2} = \frac{1}{2}$ of BC

d) $m_{AC} = \frac{5+1}{-2-4} = -1$

e) $m_{DE} = \frac{-2-1}{3-0} = -1 = m_{AC}$

f) $\tan\theta = -1$

$\therefore \theta = 135^0$

g) $y - y_1 = m(x - x_1)$

$\therefore y+1 = -(x-4)$

$y+1 = -x+4$

$x+y-3=0$

h) $y+3 = -(x-2)$

$y+3 = -x+2$

$x+y+1=0$

i) $y-y_1 = m(x-x_1)$

$y+3 = +1(x-2)$

$0 = x-y-5$

j) $d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

$= \left|\frac{(1)(2)+(1)(-3)-3}{\sqrt{1+1}}\right|$

$= \left|\frac{2-3-3}{\sqrt{2}}\right|$

$= \frac{4}{\sqrt{2}}$

k) $A = \frac{1}{2}\cdot bh$

$= \frac{1}{2}\cdot 6\sqrt{2}\cdot\frac{4}{\sqrt{2}}$

$= 12$

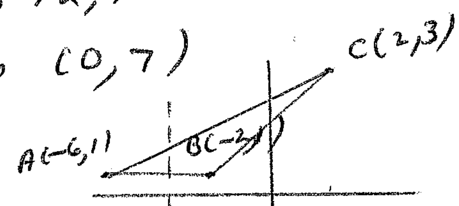
$\therefore$ area $= 12u^2$

l) $B \to A$ is $+2, +2$

$\therefore C \to H$ is $+2, +2$

& H is $(0,7)$

Q2.a)



i) $x = -4$

ii) Midpoint BC is $\left(\frac{-2+2}{2}, \frac{1+3}{2}\right) = (0,2)$

$m_{BC} = \frac{3-1}{2+2} = \frac{1}{2}$

$\therefore$ Bisector is

$y-2 = -2(x-0)$

$y-2 = -2x$

$2x+y-2=0$

iii) Sub $x=-4$

$\therefore -8+y-2=0$

$\therefore y = 10$

$\therefore D$ is $(-4,10)$

iv) $DC = \sqrt{(-4-2)^2+(10-3)^2}$

$= \sqrt{36+49} = \sqrt{85}$

& Eqn is $(x+4)^2+(y-10)^2 = 85$

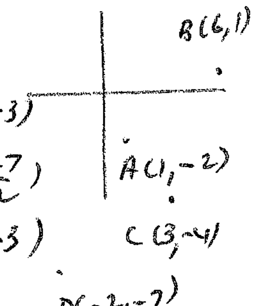
v) at $B(-2,1)$

$LHS = (-2+4)^2+(1-10)^2$

$= 2^2+9^2$

$= 85 = RHS$

$\therefore$ circle passes through B



b) Midpoint AC is $\left(\frac{1+3}{2}, \frac{-2-4}{2}\right) = (2,-3)$

Midpoint BD is $\left(\frac{6-2}{2}, \frac{1-7}{2}\right)$

$= (2,-3)$

$m_{AC} = \frac{-2+4}{1-3} = -1$

$m_{BD} = \frac{1+7}{6+2} = +1$

$\therefore AC \perp BD$ & $\therefore$ diagonals bisect at 90^0, the figure is a rhombus

c) $3x-2y+9=0$, $3x-2y+3=0$

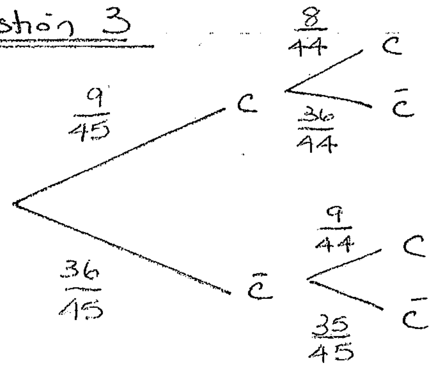
$(-1,0)$ lies on line 2

$\therefore d = \left|\frac{(3)(-1)+(-2)(0)+9}{\quad}\right| = \frac{6}{\sqrt{13}}$

Question 3

/20

a)



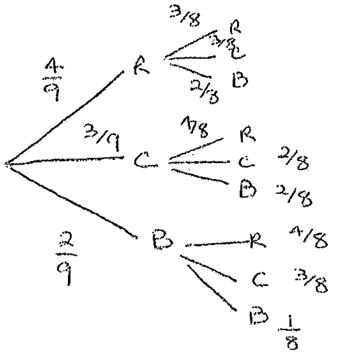
(i) $P(C,C) = \frac{9}{45} \times \frac{8}{44}$

$= \frac{2}{55}$ (2)

(ii) $P(\bar{C},\bar{C}) = \frac{36}{45} \times \frac{35}{44}$

$= \frac{7}{11}$ (2)

b)



(i) $P(R,R) = 4/9 \times 3/8$

$= 1/6$. (2)

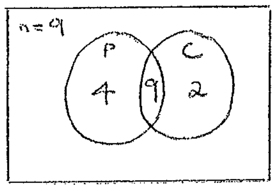
(ii) $P(R,R) + P(C,C)\ P(B,B)$

$= \left(\frac{4}{9} \times \frac{3}{8}\right) + \left(\frac{3}{9} \times \frac{2}{8}\right) + \left(\frac{2}{9} \times \frac{1}{8}\right)$

$= \frac{1}{6} + \frac{1}{12} + \frac{1}{36}$

$= \frac{5}{18}$ (2)

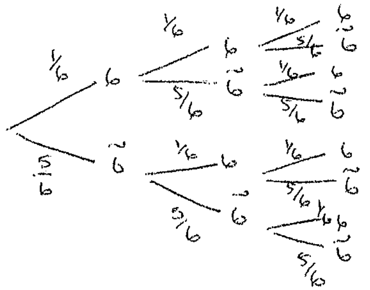
c)



$P(\text{Both}) = \frac{9}{24}$

$= \frac{3}{8}$ (4)

d)



(i) $P(6,6,6) = 1/6 \times \frac{1}{6} \times \frac{1}{6}$

$= \frac{1}{216}$ (1)

(ii) $P(\text{no } 6) = 5/6 \times 5/6 \times 5/6$

$= \frac{125}{216}$ (1)

(iii) $P(\text{one six}) = \left(\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}\right) 3$ (2)

$= 25/$

1 ✓ 5 ✓
2 ✓ 6 ✓
3 ✓ 7 ✓
4 ✓ 8 ✓

New - 6

Used - 4

i) $P(N,N) = \frac{6}{10} \times \frac{5}{9}$

$= \frac{1}{3}$. (2)

(ii) First lot

$P(1N) = P(N,U) + P(U,N)$

$= \frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{6}{9}$

$= \frac{8}{15}$

$P(\text{no new}) = P(U,U)$

$= \frac{4}{10} \times \frac{3}{9}$

$= \frac{2}{15}$

P (2^{nd} lot of getting 2 new balls)

If first lot (i) got 2 new → 4N 6U

(ii) got 1 new → 5N 5U

(iii) got no new → 6N 4U

(2)

∴ P(2^{nd} lot of getting 2 new balls)

$= \frac{1}{3}\left(\frac{4}{10} \times \frac{3}{9}\right) + \frac{8}{15}\left(\frac{5}{10} \times \frac{4}{9}\right) + \frac{2}{15}\left(\frac{6}{10} \times \frac{5}{9}\right)$

$= \frac{2}{45} + \frac{16}{135} + \frac{2}{45} = \frac{28}{135}$.

$\therefore \sum_{r=0} 3(2^r-1)$

$= 3(0)+3(1)+3(3)+3(7)+3(15)$

$= 3\times 26$

$= 78$

b) 101, 83, 65

i) $\frac{101+65}{2} = \frac{166}{2} = 83$

$\therefore$ terms are in AP

ii) $T_{12} = a+11d$

$= 101+11(-18)$

$= -97$

iii) $S_{12} = 6(2a+11d)$

$= 6(202-198)$

$= 24$

iv) $S_n = \frac{n}{2}(2(101)+(n-1)(-18))$

$= n(101+(n-1)(-9))$

$= n(110-9n)$

v) If $S_n<0$, $110n-9n^2<0$

$-9n<-110$

$n>12\frac{2}{9}$

$\therefore n=13$

c) $2x, x-3, x-7$

$(x-3)^2 = 2x(x-7)$

$x^2-6x+9 = 2x^2-14x$

$0 = x^2-8x-9$

$0 = (x-9)(x+1)$

$\therefore x = 9, -1$

If $x>0$, then $x=9$

ii) $T_1 = 18$, $T_2 = 6$, $T_3 = 2$

$\therefore T_4 = 2/3$, $T_5 = 2/9$

d) $1+2x, (1+2x)^2 \ldots$

limiting sum if $|2x+1|<1$

$\therefore 2x+1<1$, $2x+1>-1$

$\therefore x<0$, $x>-1$

$\therefore -1<x<0$

e) $T_4 = a+3d = 17$

$T_7 = a+6d = 26$

$\therefore 3d = 9$, $d=3$

& $a+9=17$, $a=8$

$\therefore S_{10} = 5(16+9\times 3)$

$= 5(16+27)$

$= 215$

5a) $\frac{2000}{7} = 285.7$, $\frac{3000}{7} = 428.6$

$\therefore 286\times 7 = 2002$

$428\times 7 = 2996$

$286 \to 428 = 143$ terms

$S_{143} = \frac{143}{2}(2002+2996)$

$= 357357$

b) 2000, 400, 80 ...

$a = 2000$, $r = \frac{1}{5}$

$T_n < 1$

$\therefore 2000\left(\frac{1}{5}\right)^{n-1} < 1$

$\frac{1}{5^{n-1}} < \frac{1}{2000}$

$\therefore 5^{n-1} > 2000$

$\therefore n > \frac{\log 2000}{\log 5}+1$

$n > 5.7$ $\therefore n=6$

c) $U_2 = ar = -27$

$a_5 = ar^4 = 8$

$\therefore r^3 = \frac{8}{-27}$, $r = -2/3$

$\therefore a(-2/3) = -27$ $\therefore a = 40.5$

$U_6 = (\frac{81}{2})(-2/3)^5 = 5\frac{1}{3}$

$S_\infty = \frac{40.5}{1+2/3} = \frac{81}{2}\times\frac{3}{5} = 24.3$

d) $T_n = A\left(\frac{1}{2}\right)^n + B\left(\frac{1}{3}\right)^n$

$T_1 = 7$, $\therefore \frac{A}{2}+\frac{B}{3} = 7$ (1)

$T_2 = 3$, $\therefore \frac{A}{4}+B/9 = 3$ (2)

$\times$ (1) by $-\frac{1}{2}$: $-A/4 - B/6 = -7/2$

adding $B/9 - B/6 = -1/2$

$\therefore B = 9$

Sub in (1), $\frac{A}{2}+3=7$, $\therefore A = 8$

$\therefore S_\infty = \{4+2+1\ldots\}+\{3+1+\frac{1}{3}+\frac{1}{9}\ldots\}$

$= \frac{4}{1-1/2} + \frac{3}{1-1/3}$

$= 12\frac{1}{2}$.