17-May-05

SYD. GIRLS H.S. – SLICING No 1

COMPLETE

1. Find the volume of a solid, base a circle $x^2 + y^2 = 1$ and cross section perpendicular to the x axis being:
   a) a semi-circle
   b) a square
   c) a right angled isosceles triangle, hypotenuse on base
   d) an equilateral triangle

2. Find the volume of a solid, base an ellipse $x^2 + 4y^2 = 4$ and cross section perendicular to the Y axis being:
   a) a semi-circle
   b) a right angled isosceles triangle, hypotenuse on the base
   c) a square

3. Find the volume of a solid, base the area enclosed by the parabola $x^2 = 4y$ and the line $y = 4$, with cross section perpendicular to the Y axis being:
   a) a semi-circle
   b) an equilateral triangle.

Answers:

$1a)\frac{2\pi}{3},\ b)\frac{16}{3},\ c)\frac{4}{3},\ d)\frac{4\sqrt{3}}{3},\ 2a)\frac{8\pi}{3},\ b)\frac{16}{3},\ c)\frac{64}{3}$

$3a)16\pi,\ b)32\sqrt{3}$

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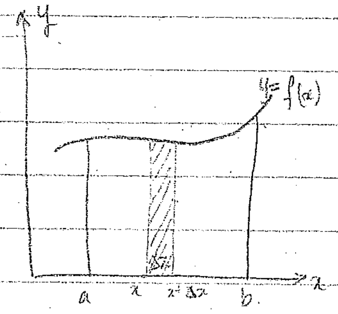
VOLUMES: SLICING #1 17-May-05

AREA UNDER A CURVE:

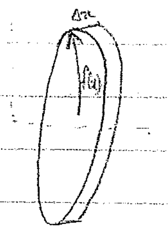


Area of strip = $f(x)\,\Delta x$

sum of strips = $\Sigma_a^b f(x)\,\Delta x$

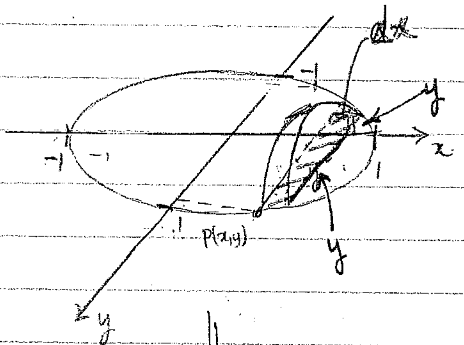
area under curve = $\int_a^b f(x)\,dx$

VOLUMES:

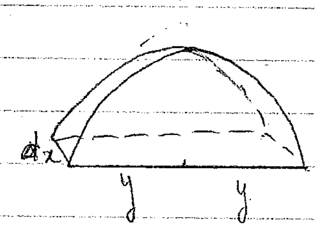


$V_{disk} = \pi f(x)^2 \Delta x$

①. a).



⇓ slice



$V_{slice} = \frac{1}{2}\pi r^2 h.$ ($\frac{1}{2}$ cylinder)

$= \frac{\pi}{2} y^2 dx$

$V_{solid} = \frac{\pi}{2}\int_{-1}^{1} y^2 dx$

$= \pi \int_0^1 y^2 dx$ ②

Now, $x^2+y^2=1$

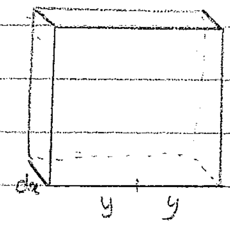
$y^2 = 1-x^2$ ① into ②

$V_{solid} = \pi \int_0^1 (1-x^2)\,dx$

$= \pi \left(x - \frac{x^3}{3}\right)_0^1$

$= \frac{2\pi}{3}$ units^3

b) $V_{slice} = 4y^2 dx$



but $x^2+y^2=1$

$\therefore y^2 = 1-x^2$

$V_{solid} = 4\int_{-1}^{1} y^2 dx$

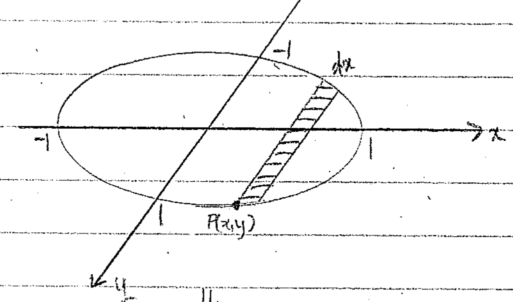
$= 4\int_{-1}^{1} (1-x^2)\,dx$

$= 8\int_0^1 (1-x^2)\,dx$

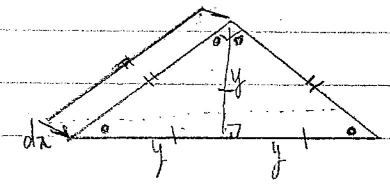
$= 8\left(x - \frac{x^3}{3}\right)_0^1$

$= \frac{16}{3}$ units^3

c).



⇓ SLICE



($\theta = 45°$)

$V_{slice} = \frac{1}{2} bh \times dx$

$= y^2 dx$ $\quad x^2+y^2=1$

$V_{solid} = \int_{-1}^{1} y^2 dx$ $\quad y^2 = 1-x^2$

$= 2\int_0^1 (1-x^2)\,dx$

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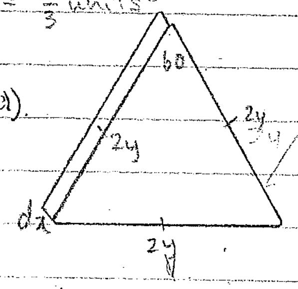
$= 2\left(x - \frac{x^3}{3}\right)_0^1$

$= \frac{4}{3}$ units^3

d)



$V_{slice} = \frac{1}{2} ab \sin C \, dx$

$= \frac{1}{2} \cdot 4y^2 \frac{\sqrt{3}}{2} \, dx$

$= \sqrt{3} y^2 \, dx$

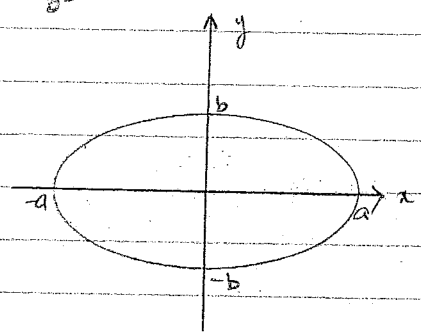
$V_{solid} = 2\int_0^1 \sqrt{3} y^2 \, dx \qquad y^2 = 1 - x^2$

$= 2\sqrt{3}\left(x - \frac{x^3}{3}\right)_0^1$

$= \frac{4}{3}\sqrt{3}$ units^3

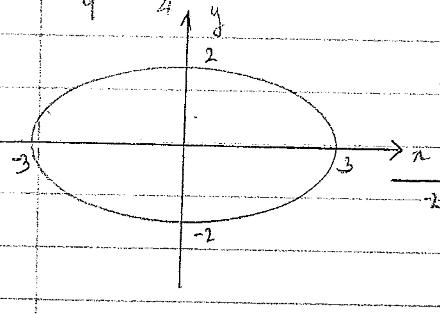
2. Ellipses

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ represents an ellipse.

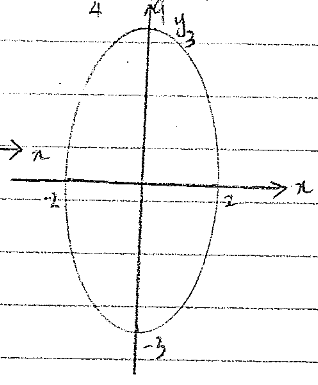


example.

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

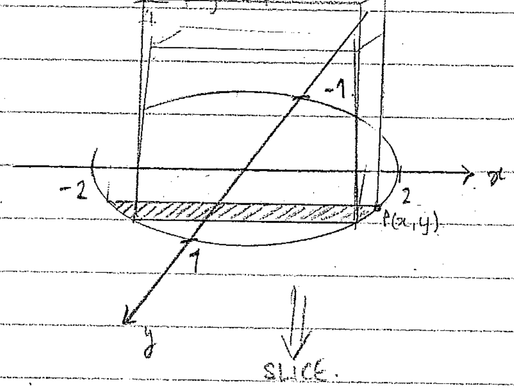


$\frac{x^2}{4} + \frac{y^2}{9} = 1$

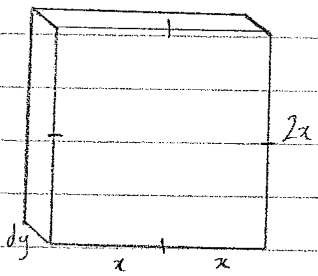


2c) $x^2 + 4y^2 = 4$

$= \frac{x^2}{4} + y^2 = 1$



SLICE.



$V_{slice} = 4x^2 \, dy$

$V_{solid} = \int_{-1}^{1} 4x^2 \, dy \qquad x^2 + 4y^2 = 4$

$x^2 = 4 - 4y^2$

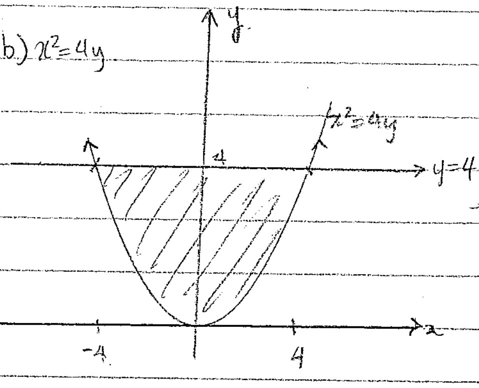
$= 2\int_0^1 4(4 - 4y^2) \, dy$

$= 2\int_0^1 (16 - 16y^2) \, dy$

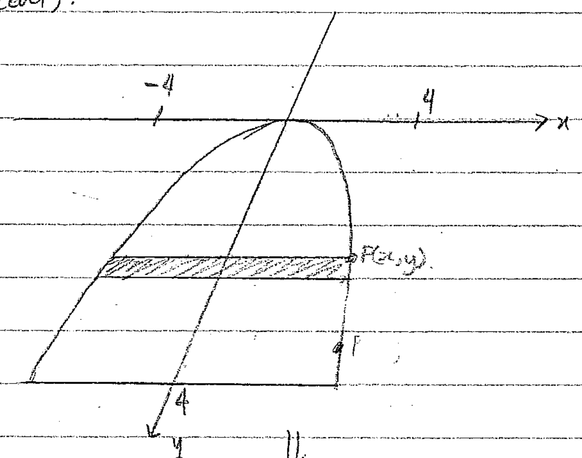
$= 2\left(16y - \frac{16y^3}{3}\right)_0^1$

$= \frac{64}{3}$ units^3

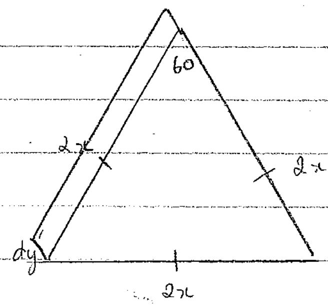
3.b) $x^2 = 4y$



3 b (cont).



⇓ SLICE



$V_{slice} = \frac{1}{2} ab \sin C \, dy$

$= \frac{1}{2} \cdot 2x \cdot 2x \cdot \frac{\sqrt{3}}{2} \cdot dy$

$= \sqrt{3} x^2 \, dy$

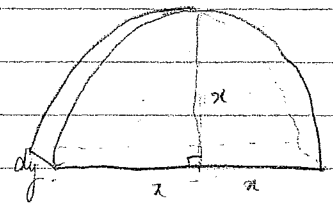
$V_{solid} = \int_0^4 \sqrt{3} x^2 \, dy \quad \Rightarrow x^2 = 4y$

$= \int_0^4 4\sqrt{3} y \, dy$

$= 4\sqrt{3} \left(\frac{y^2}{2} \right)_0^4$

$= 32\sqrt{3}$ units^3

a).



$V_{slice} = \frac{1}{2} \pi r^2 \, dy$

$= \frac{1}{2} \cdot \pi x^2 \, dy$

$= \frac{\pi}{2} x^2 \, dy$

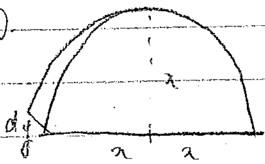
$V_{solid} = \frac{\pi}{2} \int_0^4 x^2 \, dy \quad \Rightarrow x^2 = 4y$

$= 2\pi \int_0^4 y \, dy$

$= 2\pi \left(\frac{y^2}{2} \right)_0^4$

$= 16\pi$ units^3

2 a)



$V_{slice} = \frac{1}{2}\pi r^2 h$

$= \frac{\pi}{2} x^2 \cdot dy$

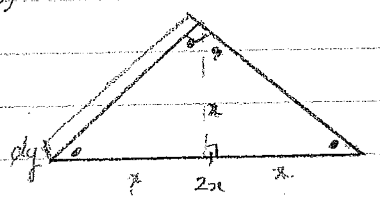
$V_{solid} = \int_{-1}^{1} \frac{\pi}{2} x^2 \, dy$

$= \pi \int_{0}^{1} 4(1-y^2)\, dy$

$= 4\pi \left(y - \frac{y^3}{3}\right)_0^1$

$= \frac{8\pi}{3}$ units^3

b)



$V.slice = \frac{1}{2} b.h.\, dy$

$= x^2 \, dy$

$V_{solid} = \int_{-1}^{1} x^2 \, dy$

$= 2\int_{0}^{1} x^2 \, dy$

$= 2\int_{0}^{1} 4(1-y^2)\, dy$

$= 8\left(y - \frac{y^3}{3}\right)_0^1$

$= \frac{16}{3}$ units^3