SYD. GIRLS H.S. — SLICING No 2

COMPLETE 18-May-05

1. A sphere of radius 5 cm has an 8 cm diameter hole bored through the centre. Find the volume remaining.

2. A sphere of radius 4 cm has a hole of diameter 4 cm bored through its centre. Find the volume remaining.

3. A solid, base an ellipse $16x^2 + 25y^2 = 400$ has each cross section perpendicular to the major axis in the shape of an isosceles triangle with altitude 6. Find the volume of the figure.

4. If, in the above question, the cross section was perpendicular to the minor axis, find the volume of the figure.

5. Two cuts are made on a circular log of radius 5 cm. The first is perpendicular to the axis and the second is inclined at 60° to the axis. Find the volume formed if the two cuts meet at the centre of the log

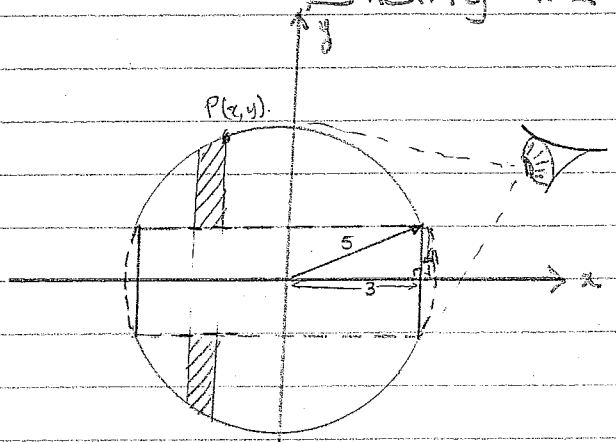
ANSWERS

1. 36π, 2. $32\sqrt{3}\pi$ 3. 60π 4. 60π 5. $\frac{250\sqrt{3}}{9}$

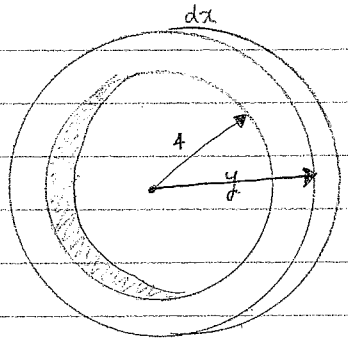
Slicing #2

18-May-05.

1.



Slice (ring)



$V_{slice} = (\pi y^2 - \pi 4^2)\,dx$.

$= \pi(y^2 - 16)\,dx$.

$V_{solid} = 2\pi \int_0^3 (y^2 - 16)\,dx$

but: $x^2 + y^2 = 25$.

$y^2 = 25 - x^2$.

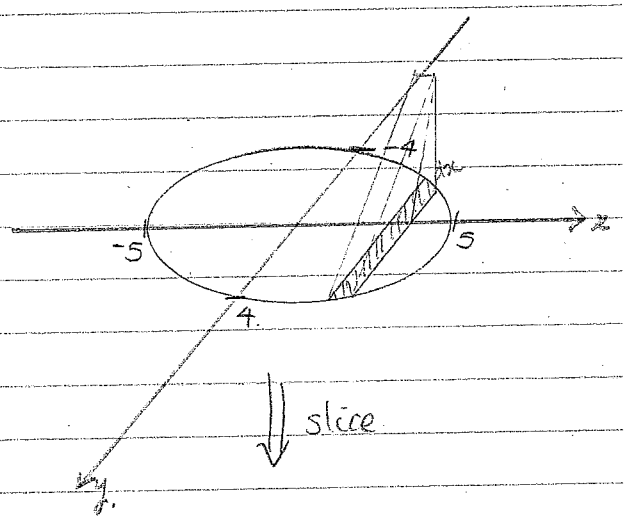
$= 2\pi \int_0^3 (25 - x^2 - 16)\,dx$

$= 2\pi \left[9x - \frac{x^3}{3}\right)_0^3$

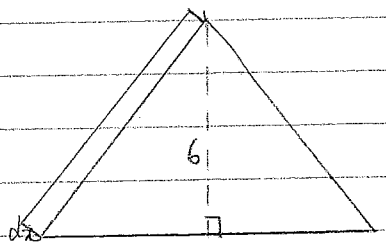
$= 36\pi$ units^3

3. $16x^2 + 25y^2 = 400$

$\frac{x^2}{25} + \frac{y^2}{16} = 1$



slice



2y

$V_{slice} = \frac{1}{2} bh \cdot dx$

$= 6y\,dx$.

$V_{solid} = \int_{-5}^{5} 6y\,dx$.

$= \int_{-5}^{5} 6\left(\frac{4}{5}\sqrt{25 - x^2}\right)dx$

$= \frac{24}{5} \int_{-5}^{5} \sqrt{25 - x^2}\,dx$

area semicircle

$= \frac{24}{5} \cdot \frac{1}{2} \cdot \pi r^2$

$= \frac{24}{5} \cdot \frac{1}{2} \pi 25$.

$= 60\pi$ units^3

$25y^2 = 400 - 16x^2$

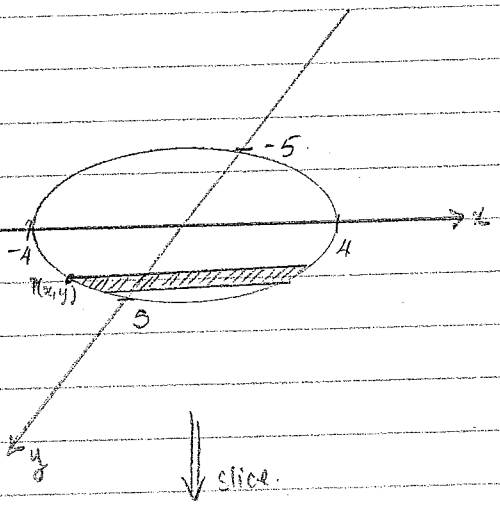
$25y^2 = 16(25 - x^2)$

$y^2 = \frac{16}{25}(25 - x^2)$

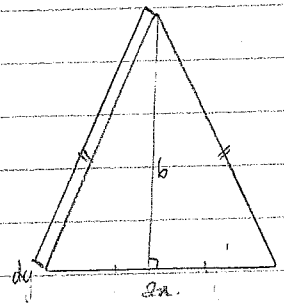
$y = \frac{4}{5}\sqrt{25 - x^2}$

18-May-05

4.



slice.



$V_{slice} = \frac{1}{2} bh \cdot dy$

$= 6x\,dy$

$V_{solid} = \int_{-5}^{5} 6x\,dy$

$= 6\int_{-5}^{5} x\,dy$

$16x^2 + 25y^2 = 400$

$16x^2 = 25(16 - y^2)$

$x^2 = \frac{25}{16}(16 - y^2)$

$x = \frac{5}{4}\sqrt{16 - y^2}$

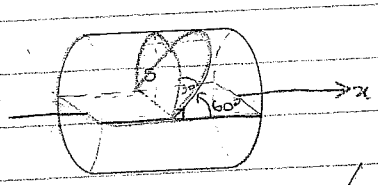
$= \frac{30}{4}\int_{-5}^{5} \sqrt{16 - y^2}\,dy$

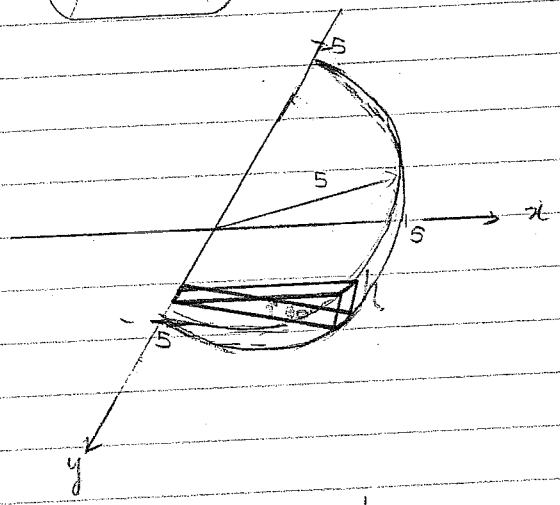
$= \frac{30}{4} \cdot \frac{1}{2}\pi 4^2$

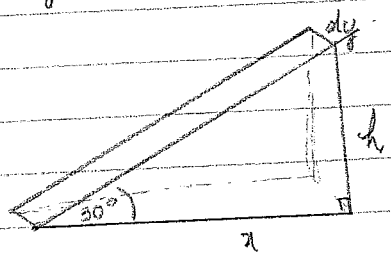
$= 60\pi$ units^3

SLICING #2.

5.







$\frac{h}{x} = \tan 30°$

$h = \frac{x}{\sqrt{3}}$.

$V_{slice} = \frac{1}{2} bh \cdot dy$

$= \frac{x^2}{2\sqrt{3}} dy$

$V_{solid} = 2\int_0^5 \frac{x^2}{2\sqrt{3}} dy$

but base $= \sqrt{25-x^2}$ (semi circle).

$y^2 = 25 - x^2$

$x^2 = 25 - y^2 \Rightarrow x = \sqrt{25-y^2}$

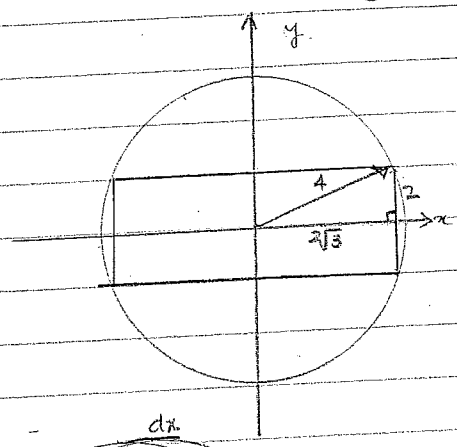
(x is positive)

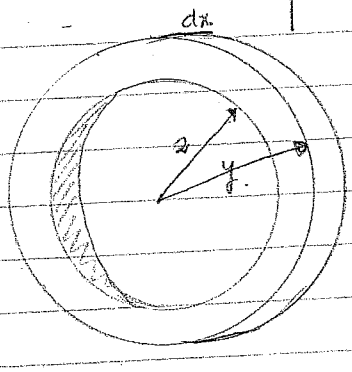
$= \frac{1}{\sqrt{3}} \int_0^5 (25 - y^2) dy$

$= \frac{1}{\sqrt{3}} \left(25y - \frac{y^3}{3}\right)_0^5$

$= \frac{250}{3\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{250\sqrt{3}}{9}$ units^3

2.





$V_{slice} = (\pi y^2 - \pi x^2) dx$

$= \pi (y^2 - x^2) dx$

$V_{solid} = 2\pi \int_0^{2\sqrt{3}} (y^2 - x^2) dx$

$x^2 + y^2 = 16$.

$y^2 = 16 - x^2$.

$= 2\pi \int_0^{2\sqrt{3}} (16 - x^2 - x^2) dx$

$= 2\pi \left(12x - \frac{x^3}{3}\right)_0^{2\sqrt{3}}$

$= 2\pi (24\sqrt{3} - 8\sqrt{3})$

$= 32\sqrt{3}\pi$ units^3