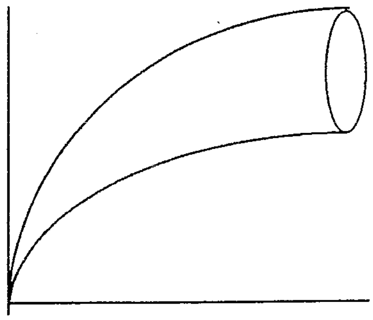
COMPLETE

SYD. GIRLS H.S. – SLICING No 3

1. Two cuts are made on a circular log of radius 5 cm. The first is perpendicular to the axis and the second is inclined at 30° to the axis. the volume formed if the two cuts meet at the centre of the log

2. The cross sections perpendicular to the X axis of the horn shaped figure shown are circles with diameters joining the two curves $y = x^{1/3}$ and $y = {}^{4}/_{3}\, x^{1/3}$. The solid extends from the origin to the position where $x = 1$. Find the volume of the figure.



3. The area of an ellipse with semi major and semi minor axes a and b is πab.
   Find the volume of the above figure if the cross sectional shape is an ellipse where the major axis is twice the length of the minor axis, the major axis being parallel to the y axis.

4. A solid has its base the area enclosed by the curves $y=x$ and $x^2 = 4y$. Find the volume formed if the cross sectional area parallel to
   a) the x axis.
   b) the y axis.
   is a semi circle.

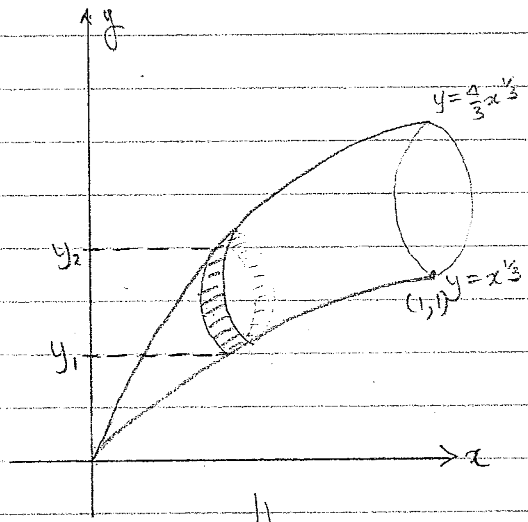
5. The base of a solid is the area enclosed by the curves $y = x^2$ and $x = y^2$. Find the volume of the solid if the cross sectional area parallel to
   a) the X axis is a square
   b) the Y axis is an equilateral triangle.

ANSWERS

1 $250\sqrt{3}/_3$ 2. $\pi/_{60}$ 3. $\pi/_{120}$ 4a) $4\pi/_{15}$ 4b) $4\pi/_{15}$ 5a) $9/_{70}$
5b) $9\sqrt{3}/_{280}$

SLICING #3. 20-may-05

2.



$A=\pi ab$ ① & $b=\frac{1}{2}a$ ②

Now, from Eq ② & diagram

$y_2-y_1 = 2a$

ie, $\frac{4}{3}x^{1/3} - x^{1/3} = 2a.$

$\frac{1}{3}x^{1/3} = 2a.$

$a=\frac{1}{6}x^{1/3}$

from ②, $b=\frac{1}{12}x^{1/3}.$

from ①, $A=\pi\cdot\frac{1}{6}x^{1/3}\cdot\frac{1}{12}x^{1/3}$

$=\frac{\pi}{72}x^{2/3}$

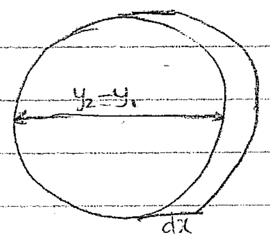
$V_{solid}=\frac{\pi}{72}\int_0^1 x^{2/3}\,dx$

$=\frac{\pi}{72}\left[\frac{3x^{5/3}}{5}\right]_0^1$

$=\frac{3\pi}{360}$

$=\frac{\pi}{120}$ units^3

Slice



radius $=\frac{y_2-y_1}{2}$

$=\frac{\frac{4}{3}x^{1/3}-x^{1/3}}{2}$

$=\frac{1}{6}x^{1/3}$

V slice $\pi r^2 dx$

$=\pi\left(\frac{x^{1/3}}{6}\right)^2 . dx.$

$=\pi . \frac{x^{2/3}}{36}dx.$

V solid $=\frac{\pi}{36}\int_0^1 x^{2/3}\,dx$

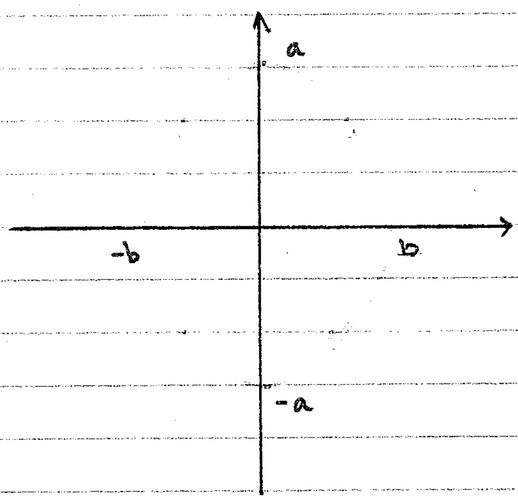
$=\frac{\pi}{36}\left(\frac{3x^{5/3}}{5}\right)_0^1$

4a.

$=\frac{3\pi}{180}$

$=\frac{\pi}{60}$ units^3

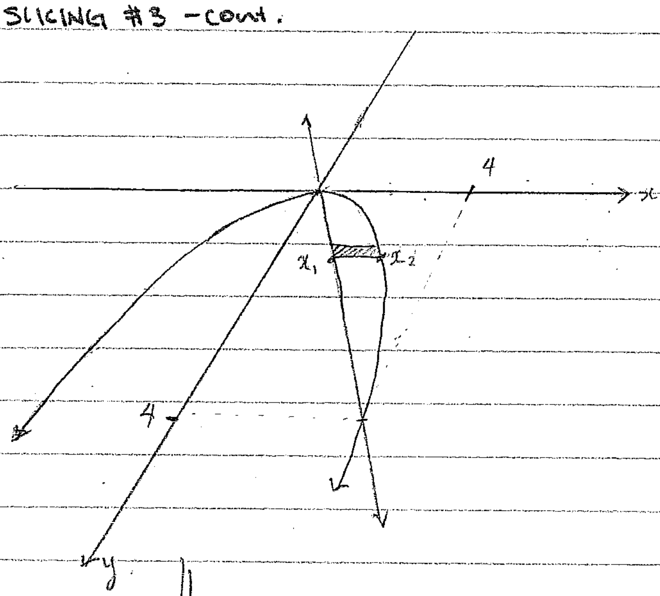
3.



SLICING #3 - cont.

20-May-05

4a



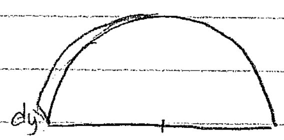
$\Downarrow$

Slice.

$x^2=4y$

∴ $x=2\sqrt{y}$

∴ $y=x$



radius $= \dfrac{x_2-x_1}{2}$

$= \dfrac{2\sqrt{y}-y}{2}$

$V_{slice} = \frac{1}{2}\pi r^2 \, dy$

$= \frac{1}{2}\left[\frac{1}{2}(2\sqrt{y}-y)\right]^2\pi \, dy$

$= \frac{\pi}{8}\left(4y-4y^{3/2}+y^2\right)dy$

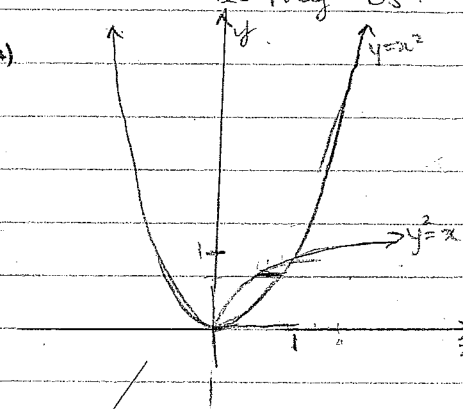
$V_{solid} = \frac{\pi}{8}\int_0^4 (4y-4y^{3/2}+y^2)\,dy$

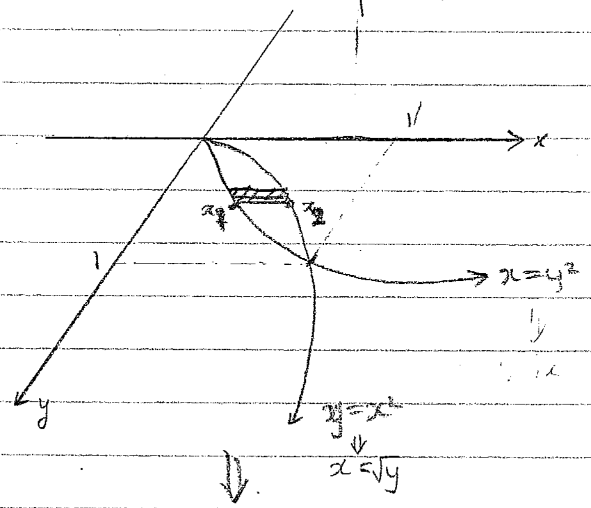
$= \frac{\pi}{8}\left[2y^2-\frac{8y^{5/2}}{5}+\frac{y^3}{3}\right]_0^4$

$= \frac{\pi}{8}\left(32-51\tfrac{1}{5}+21\tfrac{1}{3}\right)$

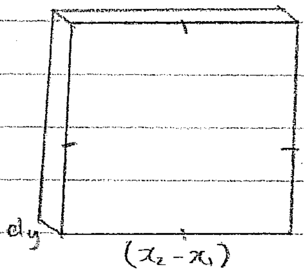
$= \frac{4\pi}{15}$ units^3

5a)





$\Downarrow$



$V_{slice} = (x_2-x_1)^2\,dy$

$= (\sqrt{y}-y^2)^2\,dy$

$= (y^4-2y^{5/2}+y)\,dy$

$V_{solid} = \int_0^1 (y^4-2y^{5/2}+y)\,dy$

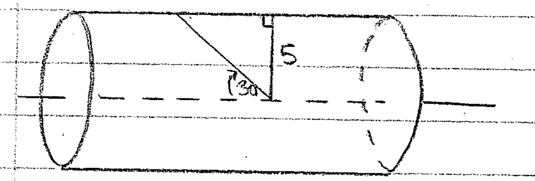
$= \left(\frac{y^5}{5}-\frac{4y^{7/2}}{7}+\frac{y^2}{2}\right)_0^1$

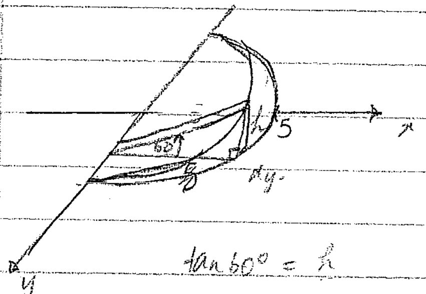
$= \frac{1}{5}-\frac{4}{7}+\frac{1}{2}$

$= \frac{9}{70}$ units^3

SLICNG #3.

1.





$\tan 60° = \frac{h}{x}$

$x\sqrt{3} = h$

$V_{slice} = \frac{1}{2} b h . dy$

$= \frac{1}{2} x \, x\sqrt{3} . dy$

$= \frac{\sqrt{3}}{2} x^2 \, dy$

$V_{solid} = \sqrt{3} \int_0^5 x^2 \, dy$

base is semi-circle: $\sqrt{25-x^2}$

$y' = \sqrt{25-x^2}$

$x = \sqrt{25-y^2}$

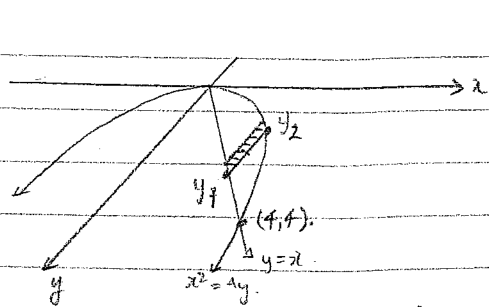
$= \sqrt{3} \int_0^5 (25-y^2) dy$

$= \sqrt{3} \left[25y - \frac{y^3}{3}\right]_0^5$

$= \sqrt{3} \left(125 - 41\frac{2}{3}\right]$

$= \frac{250\sqrt{3}}{3}$

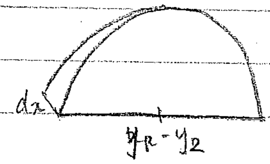
4b)



$y = x$

$x^2 = 4y$

$y = \frac{x^2}{4}$



$radius = \frac{y_1 - y_2}{2}$

$= \frac{x - \frac{x^2}{4}}{2}$

$V_{slice} = \frac{1}{2} \pi \left[\frac{1}{2}\left(x - \frac{x^2}{4}\right)^2\right] dx$

$= \frac{1}{8}\pi \left(x - \frac{x^2}{4}\right)^2 dx$

$= \frac{\pi}{8}\left(x^2 - \frac{x^3}{2} + \frac{x^4}{16}\right)$

$V_{solid} = \frac{\pi}{8} \int_0^4 \left(x^2 - \frac{x^3}{2} + \frac{x^4}{16}\right) dx$

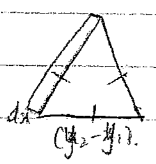
$= \frac{\pi}{8}\left(\frac{x^3}{3} - \frac{x^4}{8} + \frac{x^5}{80}\right]_0^4$

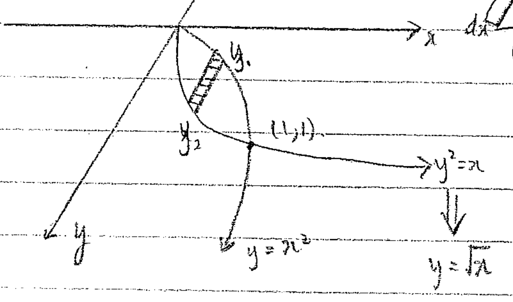
$= \frac{\pi}{8}\left(21\frac{1}{3} - 32 + 12\frac{4}{5}\right)$

$= 2\frac{2}{15} \cdot \frac{\pi}{8}$ units^3

$= \frac{4\pi}{15}$ units^3

5b)





$V_{slice} = \frac{1}{2}(y_2 - y_1)^2 \sin 60° . dx$

$= \frac{1}{2}\left(\sqrt{x} - x^2\right)^2 \frac{\sqrt{3}}{2} . dx$

$= \frac{\sqrt{3}}{4}\left(x - 2x^2\sqrt{x} + x^4\right) dx$

$$= \frac{\sqrt{3}}{4}\left(x - 2x^{5/2} + x^4\right) dx$$

$$V_{solid} = \frac{\sqrt{3}}{4}\int_0^1 \left(x - 2x^{5/2} + x^4\right) dx$$

$$= \frac{\sqrt{3}}{4}\left(\frac{x^2}{2} - \frac{4x^{7/2}}{7} + \frac{x^5}{5}\right)_0^1$$

$$= \frac{\sqrt{3}}{4}\left(\frac{1}{2} - \frac{4}{7} + \frac{1}{5}\right)$$

$$= \frac{9\sqrt{3}}{280} \text{ units}^3$$