24-may-05

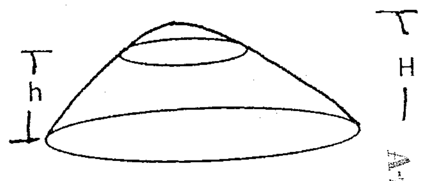
COMPLETE

SYDNEY GIRLS - SLICING NO 5

1. The base of a solid is an equilateral triangle of side "a" units, with one vertex at the origin and an altitude along the X axis. Each plane section perpendicular to the X axis is a square, one side of which lies on the base of the solid.

   Show that the area of the square is $4x^2/3$ units^2.

   Hence find the volume of the solid.



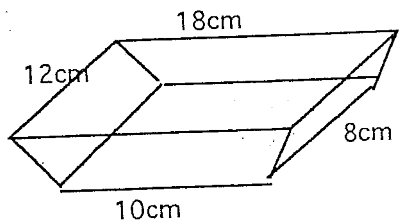
2. The diagram shows a mound of height H. At a distance h above the base, the horizontal cross section of the mound is elliptical in shape with equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\lambda^2$ where $\lambda = 1-\frac{h^2}{H^2}$, and x,y are appropriate co-ordinates in the plane of the cross section. Show that the volume of the mound is $\frac{8\pi abH}{15}$

3. The base of a brooch is the area enclosed by the curve $y^2 = x^3 - 6x^2 + 9x$ and the line $x = 4$. All cross sections perpendicular to the base are semicircles, whose diameters are perpendicular to the X axis. Find the volume of the broche.

4. A solid with a square base has cross sections whose sides vary in length from 5m at the base to 2m at the top. The height of the solid is 12m. Consider a slice of thickness ∂h and h units from the base.

   a) Show that $\partial V \approx (5 - h/4)^2 \partial h$.

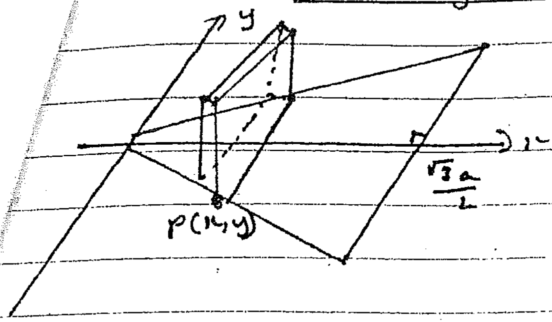
   b) Hence find the volume of the solid.



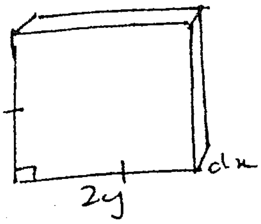
5. The diagram is of a cake tin, with rectangular base of sides 10 cm and 8 cm. Its top is also rectangular with dimensions 18cm and 12cm. The tin has a depth of 4 cm and each of its four side faces is a trapezium. Show that a section parallel to the base and h cm from the base is $2(h + 8)(h + 5)\partial h$ cm^3. Hence find the volume.

Answers: 1. $\frac{\sqrt{3}}{6}a^3$ 3. 4π 4b) 156m2 5b) $570\,2/3$ cm3

Slicing



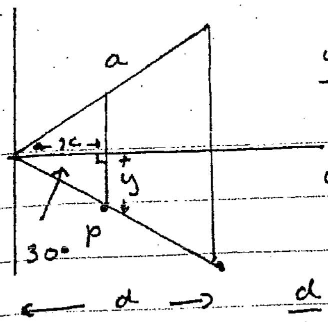
Slice



$V_{slice} = 4y^2 dx$

We need an expression for y (terms of x)

Base



$\frac{y}{x} = \tan 30°$

$y = x \tan 30°$

$= \frac{x}{\sqrt{3}}$

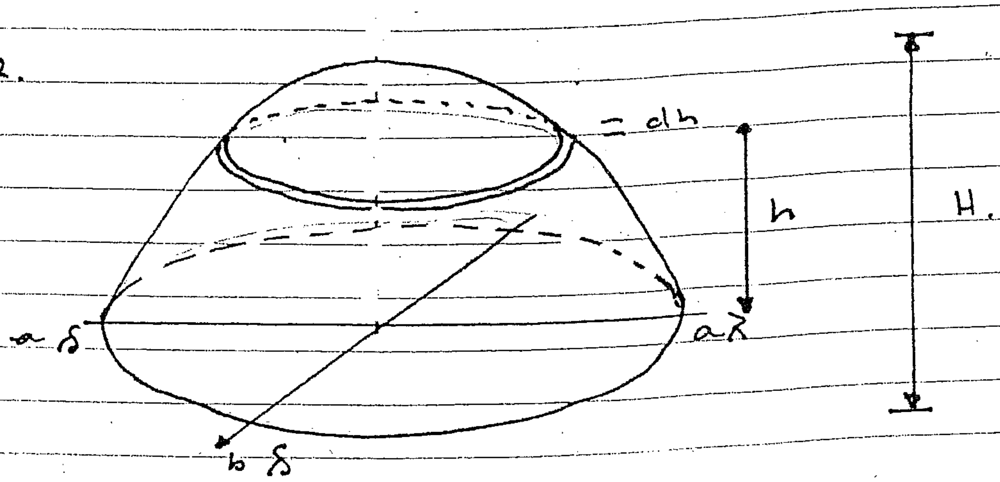
$\frac{d}{a} = \cos 30°$

$d = a \cos 30°$

$d = \frac{\sqrt{3}a}{2}$

$V_{solid} = \int_0^{\frac{\sqrt{3}a}{2}} 4y^2 \, dy$

2.



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \lambda^2$

$\frac{x^2}{a^2\lambda^2} + \frac{y^2}{b^2\lambda^2} = 1$ (standard form

$\underset{(slice)}{Area} = \pi \times (a\lambda) \times (b\lambda)$

$= \pi ab\lambda^2$

$V_{solid} = \int_0^H \pi ab \lambda^2 \, dh$

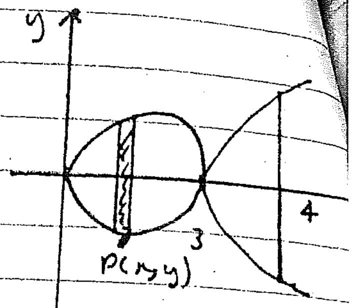
3. $y^2 = x^3 - 6x^2 + 9x$

$y = \pm\sqrt{x^3 - 6x^2 + 9x}$

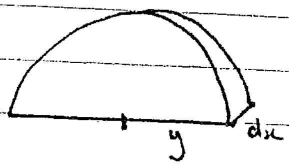
$= \pm\sqrt{x(x^2 - 6x + 9)}$

$= \pm\sqrt{(x-3)^2 x}$

$= \pm(x-3)\sqrt{x}$

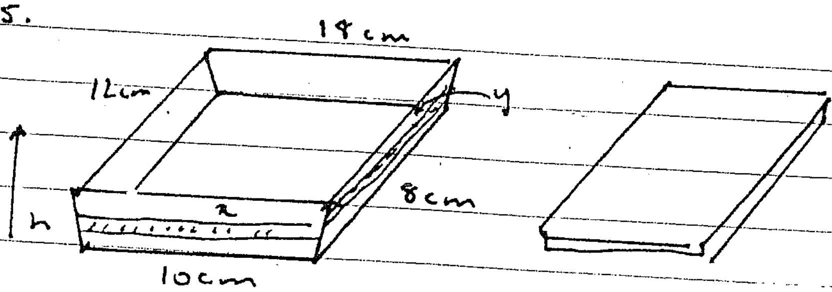


Slice



$V_{slice} = \frac{\pi}{2} y^2 \, dx$

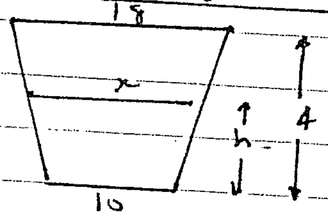
5.

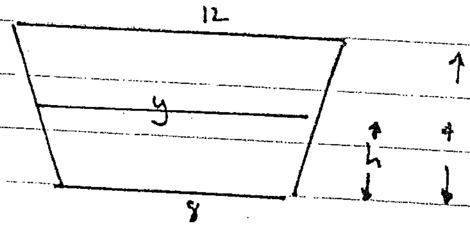


Area Slice =

$V_{slice} = xy \, dh$

Cross Sections





Now x, y are linear functions of h

$x = mh + b$

when $h = 0$, $x = 10$, $b = 10$

$x = mh + 10$

when $h = 4$, $x = 18$

$18 = 4m + 10$

$8 = 4m \quad m = 2$

$x = 2h + 10$

$y = mh + b$

when $h = 0$, $x = 8$, $b =$

$y = mh + 8$

when $h = 4$, $y = 12$

$12 = 4m + 8$, $m = 1$

$y = h + 8$

$V_{slice} = xy \, dh$

$= 2(h+5)(h+8) \, dh$

Slicing #5

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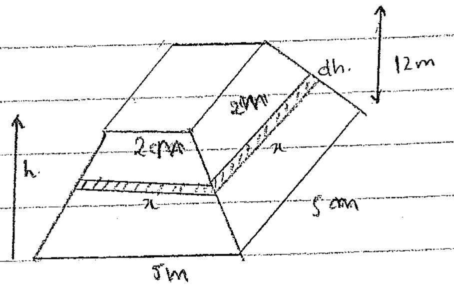
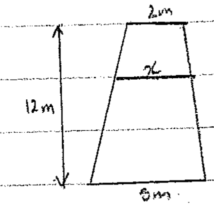
4.



a) area slice $= x^2$

$\delta V =$ Volume slice

$= x^2 dh.$



x is a linear fn of h

$\therefore x = mh + b.$

when $h = 0$, $x = 5.$

$5 = b.$

$x = mh + 5$

$\boxed{x = 5 - \frac{h}{4}.}$

when $h = 12$, $x = 2.$

$2 = 12m + b.$

$2 = 12m + 5.$

$-3 = 12m.$

$m = \frac{1}{-4}.$

hence, Vslice $= x^2 dh.$

$= (5 - \frac{h}{4})^2 dh.$

$\delta V = (5 - \frac{h}{4})^2 \delta h.$

b) Volume solid $= \int_0^{12} (5 - \frac{h}{4})^2 \delta h.$

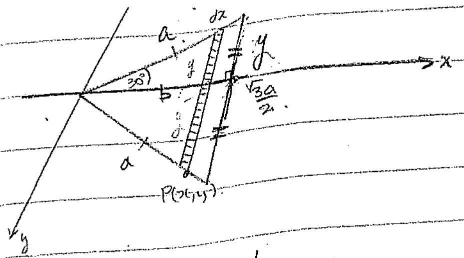
$= \int_0^{12} (25 - \frac{\cancel{10}^{5} h}{\cancel{4}_2} + \frac{h^2}{16}) dh.$

$= \left[25h - \frac{5h^2}{4} + \frac{h^3}{48}\right]_0^{12}$

$= 300 - 180 + 36$

$= 156$ ~~units~~ m^3

1.



$\cos 30° = \frac{b}{a}$

$b = a \cos 30°$

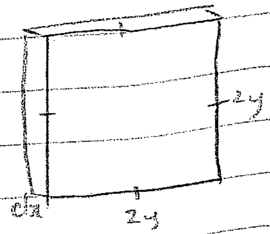
$b = \frac{\sqrt{3}a}{2}.$

Area square $= a^2$

$x = \frac{\sqrt{3}a}{2}.$

$\frac{2x}{\sqrt{3}} = a.$

$a^2 = \frac{4x^2}{3}$ units²



V slice $= 4y^2 dx$

$\tan 30° = \frac{y}{x}$

$\frac{1}{\sqrt{3}} = \frac{y}{x}$

$y = x/\sqrt{3}.$

V solid $= 4 \int_0^{\frac{\sqrt{3}a}{2}} \left(\frac{x}{\sqrt{3}}\right)^2 dx$

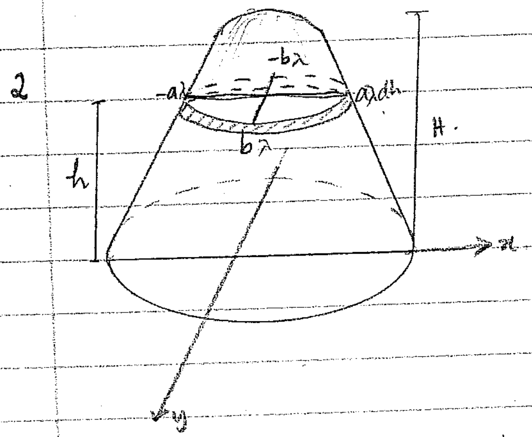
$= 4 \int_0^{\frac{\sqrt{3}a}{2}} \frac{x^2}{3} dx.$

$= 4 \left[\frac{x^3}{9}\right]_0^{\frac{\sqrt{3}a}{2}}$

$= 4 \cdot \frac{3\sqrt{3}a^3}{72}$

$= \frac{\sqrt{3}a^3}{6}$ units³

2



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\lambda^2 \quad [\text{divide by } \lambda^2]$$

$$\frac{x^2}{a^2\lambda^2}+\frac{y^2}{b^2\lambda^2}=1.$$

Area slice $= \pi \cdot a\lambda \cdot b\lambda$

$= \pi ab\lambda^2$.

V solid $= \pi\int_0^H ab\,\lambda^2\,dh$

$= \pi ab\int_0^H \lambda^2\,dh$

$= \pi ab\int_0^H \left(1-\frac{2h^2}{H^2}+\frac{h^4}{H^4}\right) dh$

$= \pi ab\left\{h-\frac{2h^3}{3H^2}+\frac{h^5}{5H^4}\right\}_0^H$

$= \pi ab\left\{H-\frac{2H^3}{3H^2}+\frac{H^5}{5H^4}\right\}$

$= \pi ab\left\{\frac{15H-10H+3H}{15}\right\}$

$= \frac{8\pi abH}{15}$.

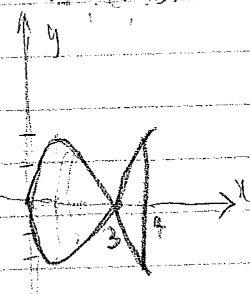
3. $y^2 = x^3-6x^2+9x$

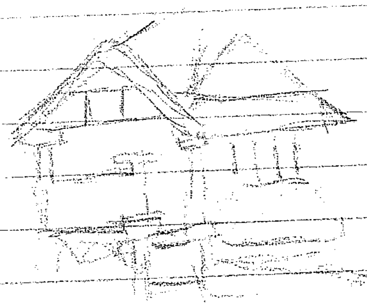
$= x(x^2-6x+9)$

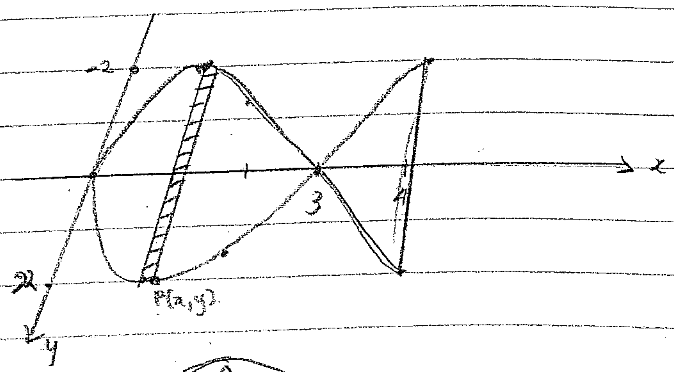
$= x(x-3)^2$

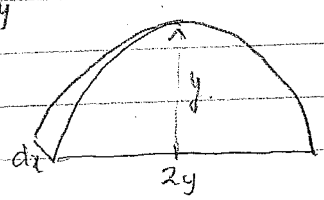
$y = \pm\sqrt{x(x-3)^2}$

$= \pm\sqrt{x}\,(x-3)$









V slice $= \frac{1}{2}\pi r^2\,dx$

$= \frac{\pi}{2}y^2\,dx$.

V solid $= \frac{\pi}{2}\int_0^4 y^2\,dx$. $\quad y^2 = x^3-6x^2+9x$

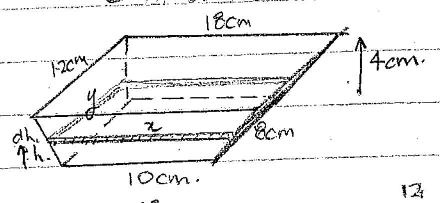
$= \frac{\pi}{2}\int_0^4 (x^3-6x^2+9x)\,dx$

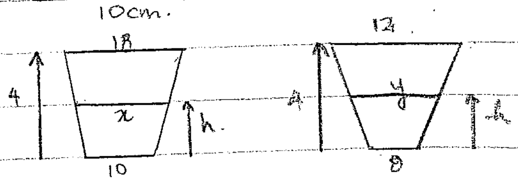
$= \frac{\pi}{2}\left[\frac{x^4}{4}-2x^3+\frac{9x^2}{2}\right]_0^4$

$= \frac{\pi}{2}\left[64-128+72\right]$

$= 4\pi$ units³

5.





$x = mh+b$.	$y = mh+c$.
when $h=0$, $x=10$.	when $h=0$, $y=8$.
$10=b$.	$8=c$.
when $h=4$, $x=18$.	when $h=4$, $y=12$
$18=4m+10$.	$12=4m+8$
$m=2$.	$m=1$.
$x=2h+10$.	$y=h+8$.

$V_{slice} = xy\,dh$

$= (2h+10)(h+8)\,dh$

$= 2(h+5)(h+8)\,dh$

$V_{solid} = 2\int_0^4 (h+5)(h+8)\,dh$

$= 2\int_0^4 (h^2+13h+40)\,dh$

$= 2\left[\frac{h^3}{3} + \frac{13h^2}{2} + 40h\right]_0^4$

$= 2\left(\frac{64}{3} + 104 + 160\right)$

$= 570\frac{2}{3}$ units^3