23-May-05

SYDNEY GIRLS — SLICING No 6

1. the circle $(x - 2)^2 + y^2 = 1$ is rotated about the y axis. find the volume formed

2. Find the volume formed when the circle $(x - 3)^2 + (y-1)^2=1$ is rotated about a) the x axis.
   b) the Y axis

3. Find the volume when the circle $x^2 + (y - 4)^2 = 4$ is rotated about the axis.

ANSWERS: 1. $4\pi^2$ 2a) $2\pi^2$ b) $6\pi^2$ 3) $32\pi^2$

When finding volumes found by rotating an area around an axis then the slice must be:

- perpendicular to the axis of rotation for slicing method.
- parallel to the axis of rotation for shells method.

Slicing # 6.

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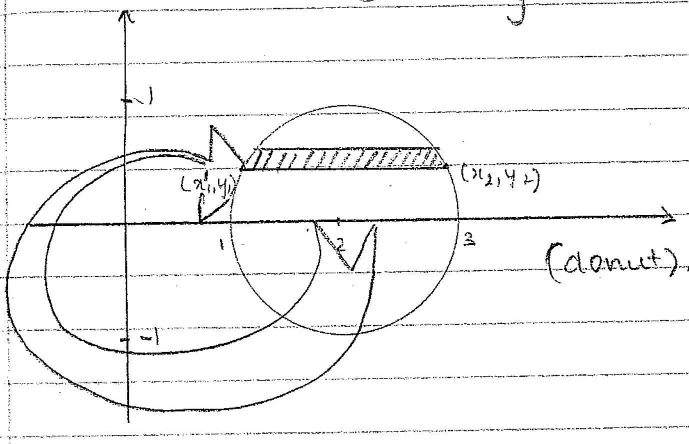
1.



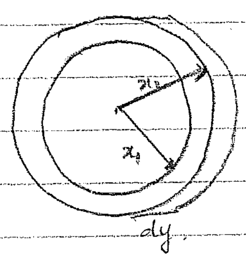
$(x-2)^2+y^2=1$

$x-2=\pm\sqrt{1-y^2}$

$x=2\pm\sqrt{1-y^2}$

$\therefore x_2=2+\sqrt{1-y^2}$

$x_1=2-\sqrt{1-y^2}$



$A=\pi x_2^2-\pi x_1^2$

$=\pi(x_2^2-x_1^2)$

$V_{slice}=\pi(x_2^2-x_1^2)\,dy$

$V_{solid}=\pi\int_{-1}^{1}(x_2^2-x_1^2)\,dy$

$=\pi\int_{-1}^{1}(x_2+x_1)(x_2-x_1)\,dy$

$=\pi\int_{-1}^{1}(2+a+2-a)(2+a-2+a)\,dy$ [where $a=\sqrt{1-y^2}$]

$=\pi\int_{-1}^{1}(4)(2a)\,dy$

$=8\pi\int_{-1}^{1}(\sqrt{1-y^2})\,dy$

$\sqrt{1-y^2}$ is a semicircle, $r=1$

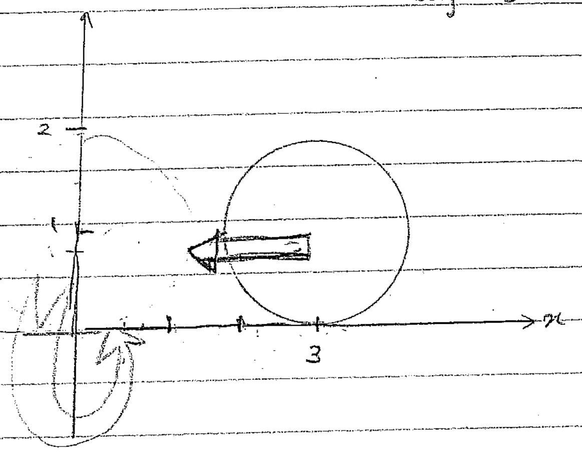
$\frac{1}{2}\pi r^2$

$=\frac{\pi}{2}$

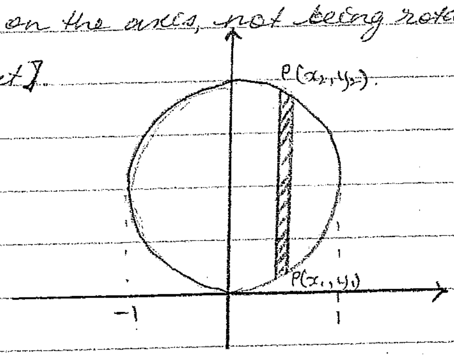
$V_{solid}=8\pi\cdot\frac{\pi}{2}$

$=4\pi^2$ units^3

2a)



Now, $(x-3)^2+(y-1)^2=1$ ~~has the~~ rotated about the x-axis has the same volume as $x^2+(y-1)^2=1$, rotated about the y-axis [adjust centre so it lies on the axis, not being rotated about].



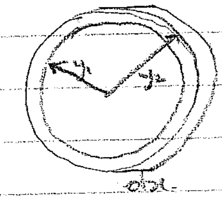
Now, $x^2+(y-1)^2=1$

$y-1=\pm\sqrt{1-x^2}$

$y=1\pm\sqrt{1-x^2}$

$y_2=1+\sqrt{1-x^2}$

$y_1=1-\sqrt{1-x^2}$



$V_{slice}=\pi(y_2^2-y_1^2)\,dx$

$V_{solid}=\pi\int_{-1}^{1}(y_2^2-y_1^2)\,dx$

$=\pi\int_{-1}^{1}(y_2+y_1)(y_2-y_1)\,dx$

$(a=\sqrt{1-x^2}$

$=\pi\int_{-1}^{1}(1+a+1-a)(1+a-1+a)\,dx$

$=\pi\int_{-1}^{1}(2)(2a)\,dx$

$=4\pi\int_{-1}^{1}(\sqrt{1-x^2})\,dx$

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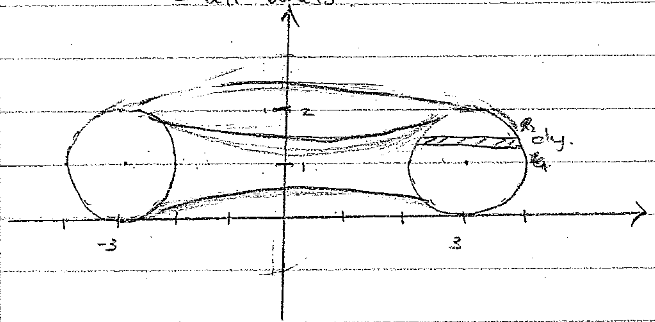
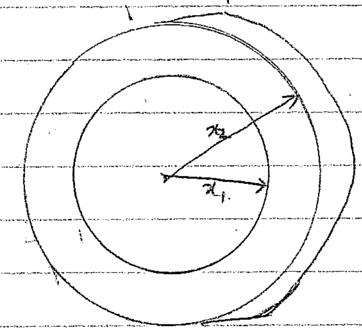
$\sqrt{1-x^2}$ is a semicircle, $r=1$

$A = \frac{\pi \cancel{r}}{2} \cdot 1^2$

$V_{solid} = 4\pi \cdot \frac{\pi \cancel{r}}{2}$

$= 2\pi^2$ units^3





$(x-3)^2 + (y-1)^2 = 1$

$(x-3)^2 = 1-(y-1)^2$

$x-3 = \pm\sqrt{1-(y-1)^2}$

$x = 3 \pm \sqrt{1-(y-1)^2}$

$V_{slice} = \pi x_2^2 - \pi x_1^2 \cdot dy$

$= \pi(x_2^2 - x_1^2)\,dy$

$V_{solid} = \pi \int_0^2 (x_2^2 - x_1^2)\,dy$

$= \pi \int_0^2 (x_2 + x_1)(x_2 - x_1)\,dy$

$= \pi \int_0^2 (3+a+3-a)(3+a-3+a)\,dy \quad [a = \sqrt{1-(y-1)^2}]$

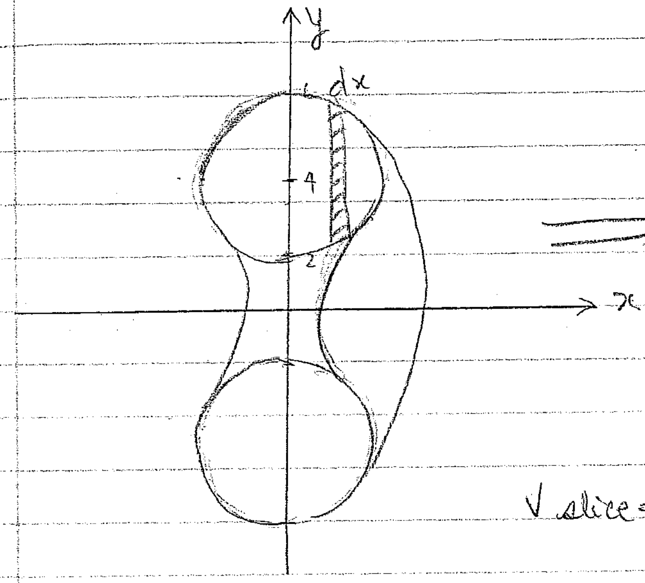
$= \pi \int_0^2 (6)(2a)\,dy$

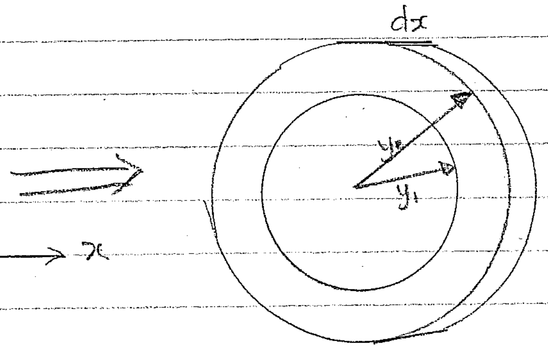
$= 12\pi \left[\frac{\sqrt{1-(y-1)^2}}{2}\right]^2 dy$

$= 6\pi \left[1-(y-1)^2\right]_0^2$

$= 6\pi \left[(1-1) - (1-\right.$

3. Find the volume when the circle $x^2+(y-4)^2=4$ is rotated about the x-axis.





$$V\ \text{slice} = \pi y_2^2 - \pi y_1^2 \cdot dx$$

$$= \pi\left(y_2^2 - y_1^2\right) dx$$

$$x^2+(y-4)^2=4$$
$$(y-4)^2 = 4-x^2$$
$$y-4 = \pm\sqrt{4-x^2}$$
$$y = 4 \pm \sqrt{4-x^2}$$
$$\left[\begin{array}{l} y_2 = 4+\sqrt{4-x^2} \\ y_1 = 4-\sqrt{4-x^2} \end{array}\right]$$

$$V\ \text{solid} = \pi \int_2^6 \left(y_2^2 - y_1^2\right) dx$$

$$= \pi \int_2^6 (y_2+y_1)(y_2-y_1)\, dx$$

$$= \pi \int_2^6 (8)\left(2\sqrt{4-x^2}\right) dx$$

$$= 16\pi \int_2^6 \sqrt{4-x^2}\, dx$$

$y=\sqrt{4-x^2}$ is semicircle $\rightarrow r=2$.

$$\frac{1}{2}\pi\, 2^2$$

$$= 2\pi$$

$$V\ \text{solid} = 16\pi\, 2\pi$$
$$= 32\pi^2 \text{ units}^3.$$