SYDNEY GIRLS – SLICING No 8

1. Find the area enclosed by the parabola $x^2 = 4ay$ and its latus rectum. Hence find the volume of a solid base:

a) $x^2 + y^2 = 4$ and cross sectional area perpendicular to the X axis being that section of a parabola enclosed by its latus rectum, the latus rectum being on the base of the solid.

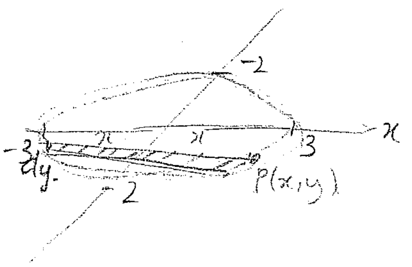
b) $4x^2 + 9y^2 = 36$ and cross sectional area perpendicular to the Y axis being that section of a parabola enclosed by its latus rectum, the latus rectum being on the base of the solid.

c) A square of side I and cross sectional area perpendicular to a diagonal being that section of a parabola enclosed by its latus rectum, the latus rectum being on the base of the solid.

ANSWERS: 1a) $\frac{64}{9}$, 1b) 16, 1c) $\frac{\sqrt{2}}{9}l^3$

b) $4x^2 + 9y^2 = 36$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$



$\perp y$ Area Parabola $= \frac{8a^2}{3}$

slice has base: $2x$

latus rectum $= 4a$

$2x = 4a$

$a = \frac{x}{2}$

Area solid $= \frac{8}{3}\left(\frac{x}{2}\right)^2$

$= \frac{2x^2}{3}$

$4x^2 = 36 - 9y^2$

$x^2 = \frac{36-9y^2}{4}$

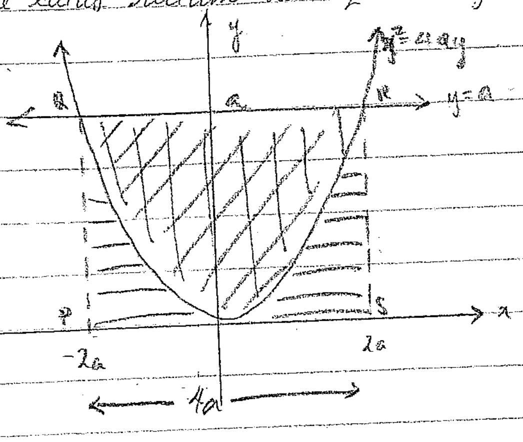
$x = \frac{\sqrt{36-9y^2}}{2}$

$V_{slice} = \frac{2}{3}x^2\,dy$

SLICING # 8

27-May-05.

1. The latus rectum has equation $y=a$



Points of intersection

$x^2 = 4a^2 \Rightarrow x = \pm 2a$

Area rectangle PQRS

$4a \times a$

$= 4a^2$

Area under parabola.

$A = 2\int_0^{2a} \frac{x^2}{4a}\, dx$

$= 2\left[\frac{x^3}{12a}\right]_0^{2a}$

$= \frac{1}{6a}\left[x^3\right]_0^{2a}$

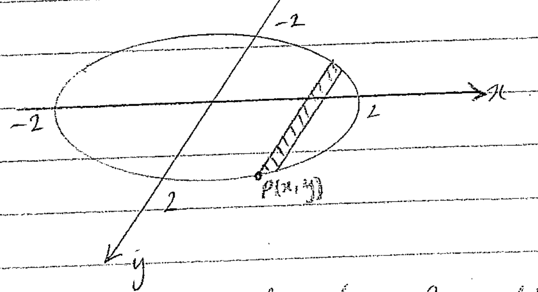
$= \frac{1}{6a} \cdot 8a^3$

$= \frac{4a^2}{3}$

$\therefore$ required area $= 4a^2 - \frac{4a^2}{3}$

$= \frac{8a^2}{3}$

a)



slice has base $2y$ which is the length of the latus rectum. Then from part (i), $2y = 4a \quad \therefore a = \frac{y}{2}$

Area of slice:

$\frac{8a^2}{3}$

$= \frac{8}{3}\left(\frac{y}{2}\right)^2$

$= \frac{2y^2}{3}$

Volume slice $= \frac{2y^2}{3}\, dx$

$x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4-x^2}$

V slice $= \frac{2}{3}(4-x^2)\, dx$

V solid $= 2\int_0^2 \frac{2}{3}(4-x^2)\, dx$

$= \frac{4}{3}\left[4x - \frac{x^3}{3}\right]_0^2$

$= \frac{4}{3}\left[8 - \frac{8}{3}\right]$

$= \frac{64}{9}$ units^3