Sydney Girls High School

2005
TRIAL HIGHER SCHOOL
CERTIFICATE EXAMINATION

Mathematics

Extension 1

This is a trial paper ONLY. It does not necessarily reflect the format or the contents of the 2005 HSC Examination Paper in this subject.

General Instructions

- Reading Time – 5 mins
- Working time – 2 hours
- Attempt ALL questions
- ALL questions are of equal value
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied
- Board-approved calculators may be used
- Diagrams are not to scale
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

Question 1 (12 marks) Marks

(a) Find the point P which divides the interval AB externally in the ratio 1:2 where A = (-2, 0) and B = (3, -7). (3)

(b) Evaluate

$$\lim_{x\to 0}\left\{\frac{\sin\frac{x}{2}}{\frac{x}{4}}\right\}$$ (2)

(c) Solve $\frac{3x-2}{x} > 5$ (3)

(d) Find $\int \frac{x\,dx}{\sqrt{2x-5}}$ using a the substitution $u = 2x-5$ (4)

Question 2 (12 marks)

(a) Differentiate $y = 5\cos^{-1}(2x)$ (3)

(b) Sketch the graph of $y = 5\cos^{-1}(2x)$ showing the domain and range on your graph (3)

(c) Taking $x = 0.5$ radians as a first approximation to the root of $\cos x - x = 0$, Find a better approximation correct to 1 decimal place using one application of Newton's method. (3)

(d) Solve for $-2\pi \le \theta \le 2\pi$ (3)

$$1+\sqrt{3}\tan\theta = 0$$

Then write down the general solution to this equation.

Question 3 (12 marks) marks

(a) Points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on parabola $x^2 = 4ay$

i) Derive the equation of chord PQ (1)

ii) If chord PQ subtends a right angle at the origin, show that $pq = -4$ (1)

iii) Find the equation of the locus of the midpoint of chord PQ. (2)

(b) If α, β, γ are the roots of $2x^3 + 8x^2 - x + 6 = 0$ (3)

Find i) $\alpha + \beta + \gamma$

ii) $\alpha\beta + \alpha\gamma + \beta\gamma$

iii) $\alpha^2 + \beta^2 + \gamma^2$

(c) i) Find the zeros of the polynomial function $P(x) = x^4 + 3x^3 + 2x^2$ (2)

ii) Without using calculus sketch the function.

(d) Find the equation of the curve which passes through the point $\left(3, \frac{\pi}{2}\right)$ and has

$\frac{dy}{dx} = \frac{1}{\sqrt{9 - x^2}}$ (3)

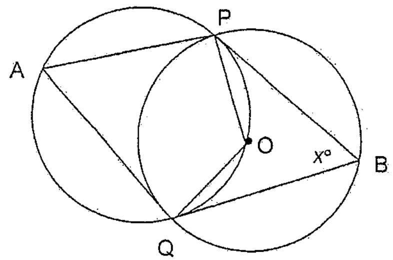
Question 4 (12 Marks)

(a) i) Find the domain and range for which the function $y = x^2 + 6x$ is increasing. (2)

ii) Hence find the inverse function over this above domain making y the subject. State the domain and range for this inverse function. (3)

Question 4 (12 marks) marks

(b)



The centre, O of the circle PBQ lies on the circumference of circle APQ (3)

$APBQ$ is a parallelogram

i) Copy this diagram

ii) Find angle PBQ (x^o) Give reasons.

(c) Prove by the Principle of Mathematical Induction that $n^3 + 2n$ is divisible by 3 for all integers $n \geq 1$ (4)

Question 5 (12 Marks)

(a) Find the acute angle between the lines $4x - 3y - 2 = 0$ and $3x - y - 2 = 0$ (3)

Answer in radians correct to 2 decimal places.

(b) By using the substitution $u^2 = 1 + x^3$ (3)

Evaluate as an exact value. $\int_0^1 \frac{3x^2}{2\sqrt{1+x^3}} dx$

(c) i Express $4\cos\theta - 3\sin\theta$ in the form $A\cos(\theta + \alpha)$ where A>0 and α is a subsidiary angle in the range $0 \leq \alpha \leq 90^\circ$ (3)

ii Hence or otherwise solve for $0 \leq \theta \leq 360^\circ$ $4\cos\theta - 3\sin\theta = -1$ (3)

Answer to the nearest minute.

Question 6 (12 marks)

(a) Evaluate as an exact answer (2)

$$\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right)$$

(b) A particle moves such that when its position is x metres to the right of the origin its velocity $v = \sqrt{2x+4}$ m/s

(i) show that the acceleration is constant throughout the motion. (1)

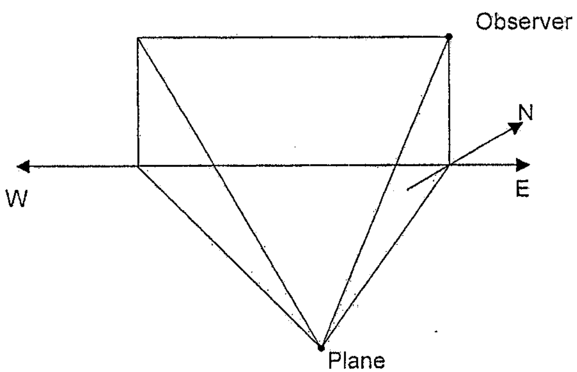
(ii) show that $t = \int (2x+4)^{-\frac{1}{2}}.dx$ (1)

(iii) if initially $x = 0$, show that $x = \dfrac{t^2+4t}{2}$ (2)

(iv) Hence find the velocity when $t = 5$ seconds (1)

(c) From the top of a mountain 1000 metres high a plane is sighted on an airstrip at a bearing of 160° from the base of the mountain. The angle of elevation of the mountain top from the plane is 30°. The plane takes off and climbs at a constant speed on a constant bearing. After 1 minute it is observed 2km due West of the observer at the same height as the observer. (Altitude 1000 metres).

Find
i) the course of the plane as a true bearing from the airstrip (to nearest degree) (2)
ii) the angle of the climb, (to nearest degree) (1)
iii) the speed of the plane in km/h (to nearest whole number) (2)

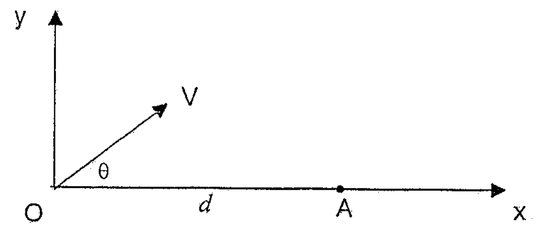


Question 7 (12 marks)

marks

(a) A dump funnel drops a steady stream of sand on the ground at the rate of $8m^3$ per minute. The sand falls to form a cone shape so that the height (h) metres of the cone is twice the radius (r) metres.
Find the rate at which the height (h) is changing when the height is 2 metres (answer correct to two decimal places). (3)

(b)



A projectile is fired from O, with initial speed of V m/s at an angle of elevation Θ, at a target at point A which is d metres distant from O.

i. Show that the position (x,y) of the projectile at time t seconds after the start is given by
$$x = Vt\cos\theta,\quad y = Vt\sin\theta - \tfrac{1}{2}gt^2$$ (2)

ii. Show that the projectile is above the x – axis for a total of $\dfrac{2V\sin\theta}{g}$ seconds (1)

iii. Show that the horizontal range is $\dfrac{V^2\sin 2\theta}{g}$ metres. (1)

iv. At the exact instant of firing, the target moves away from A in a positive direction at a constant speed of W metres/s.

If the projectile hits the moving target show that $W = V\cos\theta - \dfrac{gd}{2V\sin\theta}$ (1)

Ext 1 H.S.C. Trial 2005. S.G.H.S

Question 1

(a)

Externally in ratio $1 : 2$ $\quad A = (-2, 0) = x_1\ y_1$

$\therefore\ k_1 : k_2 = 1 : -2$ $\quad B = (3, -7) = x_2\ y_2$

$x = \dfrac{k_1 x_2 + k_2 x_1}{k_1 + k_2} = \dfrac{1 \times 3 - 2 \times -2}{1 - 2} = -7$

$y = \dfrac{k_1 y_2 + k_2 y_1}{k_1 + k_2} = \dfrac{1 \times -7 - 2 \times 0}{1 - 2} = 7$

$\therefore$ Point $P = (-7, 7)$

(3)

b $\displaystyle \lim_{x \to 0} \frac{\sin \frac{x}{2}}{\frac{x}{4}} = \left(\lim_{x \to 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} \right) \times \frac{1}{\frac{1}{2}} = 2$

(2)

c $\dfrac{3x - 2}{x} > 5$

1st C.V $x = 0$

Let $\dfrac{3x-2}{x} = 5$

$3x - 2 = 5x$

$-2 = 2x$

$\therefore\ x = -1$ (2nd C.V)

$-1 \qquad 0$

Test $x = -2$, $\frac{-8}{-2} = 4 < 5$

Test $x = -\frac{1}{3}$, $\frac{-3}{-1/3} = 9 > 5$

Test $x = 1$, $\frac{1}{1} < 5$

Ans: $-1 < x < 0$

(3)

d $\displaystyle \int \frac{x\,dx}{\sqrt{2x-5}}$

Let $u = 2x - 5$ $\quad \therefore\ x = \dfrac{u+5}{2}$

$\dfrac{du}{dx} = 2$

$\therefore\ dx = \frac{1}{2} du$

$\displaystyle \int \frac{x\,dx}{\sqrt{2x-5}} = \int \frac{u+5}{2} \cdot \frac{1}{u^{1/2}} \cdot \frac{1}{2} du$

$\displaystyle = \frac{1}{4} \int u^{\frac{1}{2}} + 5u^{-\frac{1}{2}}\,du$

$= \dfrac{1}{4} \left[\dfrac{2u^{3/2}}{3} + 5u^{1/2} \times 2 \right] + c$

$= \dfrac{1}{6} u^{3/2} + \dfrac{5}{2} u^{1/2} + c$

$= \dfrac{1}{6} (2x-5)^{3/2} + \dfrac{5}{2} (2x-5)^{\frac{1}{2}} + c$

(4)

Question 2

(a) $y = 5 \cos^{-1}(2x)$ $\quad$ Let $u = 2x$

$\therefore\ \dfrac{du}{dx} = 2$

$y = 5 \cos^{-1} u$

$\therefore\ \dfrac{dy}{du} = \dfrac{-5}{\sqrt{1-u^2}}$

$\therefore\ \dfrac{dy}{dx} = \dfrac{-5}{\sqrt{1-4x^2}} \cdot 2 = \dfrac{-10}{\sqrt{1-4x^2}}$

(3)

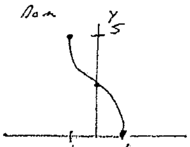
(b) $y = 5 \cos^{-1}(2x)$

$-1 \le 2x \le 1$

$\therefore\ -\frac{1}{2} \le x \le \frac{1}{2}$ Dom

$0 \le \cos^{-1}(2x) \le \pi$

$0 \le 5\cos^{-1}(2x) \le 5\pi$



(3)

(c) $f(x) = \cos x - x$

$f'(x) = -\sin x - 1$ $\quad$ let $a = 0.5$

$a_1 = a - \dfrac{f(a)}{f'(a)} = 0.5 - \dfrac{(\cos 0.5 - 0.5)}{(-\sin 0.5 - 1)}$

$= 0.5 + \dfrac{(\cos 0.5 - 0.5)}{(\sin 0.5 + 1)}$

$= 0.755$

$\doteqdot 0.8$

(3)

(d) $1 + \sqrt{3} \tan\theta = 0$ $\quad$ for $-2\pi \le \theta \le 2\pi$

$\tan\theta = -\frac{1}{\sqrt{3}}$

$\therefore\ \theta = \dfrac{5\pi}{6}, \dfrac{11\pi}{6}, -\dfrac{7\pi}{6}, -\dfrac{\pi}{6}$

(3)

General Soln $\quad \theta = n\pi + \left(-\dfrac{\pi}{6}\right)$

$\therefore\ \theta = n\pi - \dfrac{\pi}{6}$

(c) The tide rises and falls in simple harmonic motion with the time between successive high tides being 12 hours. A ship is due to sail from a wharf. On the morning it is to sail, high tide at the wharf occurs at 6am. The water depths at the wharf at high tide and low tide are 12 metres and 4 metres respectively.

i.) Show that the water depth, y metres, at the wharf is given by $y = 8 + 4\cos\left(\dfrac{\pi}{6}\right)$ when t is the number of hours after high tide. (1)

ii) A nearby bridge obstructs the ships exit from the wharf. The ship can only leave if the water depth at the wharf in 10 metres or less. Find the earliest possible time that the ship can leave the wharf. (1)

iii) Under the bridge is a sandbar. In order for the ship to sail through, the water level must be at least 3 metres above low tide level. Find the latest possible time that the ship can leave the wharf to the nearest minute assuming the wharf must be cleared before midday, (2) Question 4.

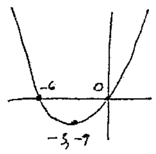
a i $f : y = x^2 + 6x$

i Put $y = 0$, $0 = x(x+6)$

$\therefore x = 0$ and -6

Concave up.

$\therefore$ Turning point $= (-3, -9)$



(2)

Increasing curve for Dom: $x \geq -3$

Range: $y \geq -9$

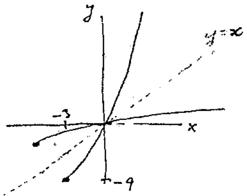
ii f^{-1} : $x = y^2 + 6y$

$y^2 + 6y + 9 = x + 9$

$(y+3)^2 = x + 9$

$y + 3 = \pm\sqrt{x+9}$

$y = -3 \pm \sqrt{x+9}$



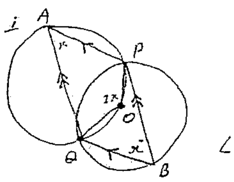
$\therefore y = -3 + \sqrt{x+9}$ is inverse function

with Domain $x \geq -9$

Range $y \geq -3$

(3)

(b) i



ii Aim: Find $\angle PBQ$ (x)

Solution

$\angle PAQ = \angle PBQ = x$ (opp $\angle$s parm.)

$\angle POQ = 2 \times \angle PBQ = 2x$ ($\angle$ at Centre $= 2 \times \angle$ at circum)

$\angle PAQ + \angle POQ = 3x = 180°$ (opp $\angle$s cyc quad)

$\therefore x = 60°$

(3)

Question 4

(c) Prove $n^3 + 2n$ is divisible by 3 for all $n \geq 1$

(4)

Let $n = 1$, $1^3 + 2 \times 1 = 3$ which is divisible by 3

$\therefore$ True for $n = 1$

Assume True for $n = k$

$\therefore k^3 + 2k = 3M$ where M = integer

Prove True for $n = k+1$

Let $n = k+1$

$(k+1)^3 + 2(k+1) = k^3 + 3k^2 + 3k + 1 + 2k + 2$

$= (k^3 + 2k) + 3k^2 + 3k + 3$

$= 3M + 3(k^2 + k + 1)$

$= 3(M + k^2 + k + 1)$

which is divisible by 3 since $(M + k^2 + k + 1)$ = integer.

$\therefore$ True for $n = k+1$

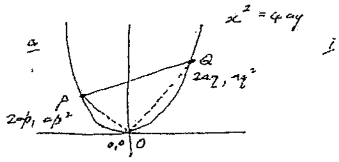
Conclusion. If true for $n = k$, then it is true for $n = k+1$

Since true for $n = 1$, it is true for $n = 2$, then $n = 3$ and so on for all $n \geq 1$

$\therefore$ By Math Induction $n^3 + 2n$ is divisible by 3 for all $n \geq 1$

Question 3

a



i Grad $PQ = \dfrac{aq^2 - ap^2}{2aq - 2ap}$

$= \dfrac{a(q-p)(q+p)}{2a(q-p)}$

$= \dfrac{p+q}{2}$

Equation of PQ is

$y - ap^2 = \dfrac{p+q}{2}(x - 2ap)$

$2y - 2ap^2 = (p+q)x - 2ap^2 - 2apq$

$\therefore (p+q)x - 2y - 2apq = 0$

(1)

ii Grad $OP = \dfrac{ap^2 - 0}{2ap - 0} = \dfrac{p}{2} = m_1$

Grad $OQ = \dfrac{aq^2 - 0}{2aq} = \dfrac{q}{2} = m_2$

Since $OP \perp OQ$

$m_1 \times m_2 = -1$

$\dfrac{p}{2} \times \dfrac{q}{2} = -1$

$\therefore pq = -4$

(1)

iii Mid-pt of PQ (x, y) $= \left(\dfrac{2ap + 2aq}{2}, \dfrac{ap^2 + aq^2}{2}\right)$

$= \left[a(p+q), \dfrac{a}{2}(p^2 + q^2)\right]$

Now $p + q = \dfrac{x}{a}$ and $\dfrac{2y}{a} = p^2 + q^2$

$(p+q)^2 = \dfrac{x^2}{a^2}$

$= (p+q)^2 - 2pq$

$= (p+q)^2 + 8$

$\therefore (p+q)^2 = \dfrac{2y}{a} - 8$

$\therefore \dfrac{x^2}{a^2} = \dfrac{2y}{a} - 8$

$\therefore x^2 = 2ay - 8a^2$ is locus of mid-pt of chord PQ

(2)

Question 3

(b) $2x^3 + 8x^2 - x + 6 = 0$ i $\alpha + \beta + \gamma = \dfrac{-b}{a} = \dfrac{-8}{2} = -4$

ii $\alpha\beta + \alpha\gamma + \beta\gamma = \dfrac{c}{a} = \dfrac{-1}{2}$

(3)

iii $(\alpha + \beta + \gamma)(\alpha + \beta + \gamma)$

$= \alpha^2 + \alpha\beta + \alpha\gamma + \beta\alpha + \beta^2 + \beta\gamma + \gamma\alpha + \gamma\beta + \gamma^2$

$= (\alpha^2 + \beta^2 + \gamma^2) + 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$\therefore \alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= (-4)^2 - 2(-\tfrac{1}{2})$

$= 16 + 1$

$= 17$

(c) i $P(x) = x^4 + 3x^3 + 2x^2$

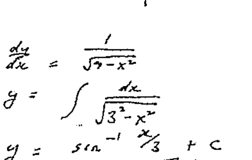
$= x^2(x^2 + 3x + 2)$

$= x^2(x+2)(x+1)$

$\therefore$ Zeros are $0, -2, -1$

(2)

ii



Test $x = 1$

$P(1) = 1 + 3 + 2 = 6$

(d) $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{9 - x^2}}$

$y = \displaystyle\int \frac{dx}{\sqrt{3^2 - x^2}}$

$y = \sin^{-1} \dfrac{x}{3} + c$

Substitute $(3, \tfrac{\pi}{2})$

$\tfrac{\pi}{2} = \sin^{-1} 1 + c$

$\dfrac{\pi}{2} = \dfrac{\pi}{2} + c$

$\therefore c = 0$

Hence Curve equation is

$y = \sin^{-1} \dfrac{x}{3}$

(3)

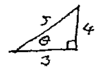
Question 6

(a)

$\sin\left(2\cos^{-1}\frac{3}{5}\right)$

Let $\theta = \cos^{-1}\frac{3}{5}$

$\cos\theta = 3/5$



$= \sin(2\theta)$

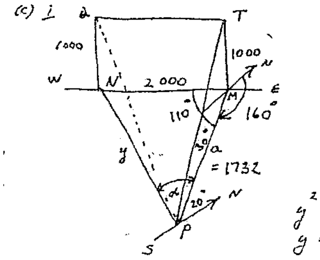
$= 2\sin\theta\cos\theta$

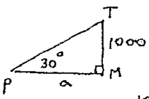
$= 2 \times \frac{4}{5} \times \frac{3}{5}$

$= \frac{24}{25}$

(2)

(c) i





$\tan 30^\circ = \frac{1000}{a}$

$\therefore a = 1732\ m$

$y^2 = 2000^2 + 1732^2 - 2 \times 2000 \times 1732 \times \cos 110$

$y = 3060.9\ m$

In ΔPMN $\quad \frac{\sin\alpha}{2000} = \frac{\sin 110^\circ}{3060.9}$

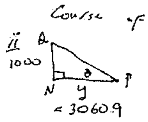
$\therefore \sin\alpha = \frac{2000 \times \sin 110^\circ}{3060.9}$

$\therefore \alpha = 37^\circ 53' \doteqdot 38^\circ$

(2)

Course of plane $= 360^\circ - 20^\circ - 38^\circ = 302^\circ\ T$

ii



$\tan\theta = \frac{1000}{3060.9}$

$\therefore \theta = 18^\circ$

$\therefore$ Angle of climb $= 18^\circ$

(1)

iii $QP^2 = 1000^2 + 3060.9^2$

$\therefore QP = 3220\ m$

Speed $= 3220 / \frac{1}{60}\ m/h$

$= 193\ km/h$

(2)

Question 6

(a) i $\quad a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left(\frac{1}{2}(2x+4)\right) = \frac{d}{dx}(x+2) = 1$

or $a = \frac{dv}{dt} = 1$

$\therefore$ acc is constant $= 1\ m/s^2$ independent of t

(1)

(b) $\quad v = (2x+4)^{\frac{1}{2}}\ m/s$

ii $\quad \frac{dx}{dt} = (2x+4)^{\frac{1}{2}}$

$\frac{dx}{(2x+4)^{1/2}} = dt$

(1)

$\therefore \int dt = \int (2x+4)^{-\frac{1}{2}}\,dx$

$\therefore t = \int (2x+4)^{-\frac{1}{2}}\,dx$

iii $\quad t = 0, \quad x = 0$

$t = \frac{(2x+4)^{\frac{1}{2}}}{(2 \times \frac{1}{2})} + c$

$t = (2x+4)^{1/2} + c$

$0 = (4)^{1/2} + c$

$0 = 2 + c$

$c = -2$

(2)

$t = \sqrt{2x+4} - 2$

$2 + t = \sqrt{2x+4}$

$(2+t)^2 = 2x+4$

$4 + 4t + t^2 = 2x + 4$

$\therefore x = \frac{t^2 + 4t}{2}$

iv $\quad v = \frac{dx}{dt} = \frac{2t+4}{2} = t+2$

Let $t = 5, \quad \therefore vel = 7\ m/s$

(1)

Question 5

(a)

$4x - 3y - 2 = 0$

$3y = 4x - 2$

$y = \frac{4}{3}x - \frac{2}{3}$

$m_1 = \frac{4}{3}$

$3x - y - 2 = 0$

$y = 3x - 2$

$\therefore m_2 = 3$

(3)

$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{(3 - \frac{4}{3})}{(1 + 3 \times \frac{4}{3})} = \frac{1}{3}$

$\theta = 0.32$ radians

Question 5

(b)

$\int_0^1 \frac{3x^2}{2\sqrt{1+x^3}}\,dx$

$= \int_1^{\sqrt{2}} du$

$= [u]_1^{\sqrt{2}}$

$= \sqrt{2} - 1$

Let $u^2 = 1 + x^3$

$u = (1+x^3)^{1/2}$

$\frac{du}{dx} = \frac{1}{2} \cdot 3x^2 (1+x^3)^{-\frac{1}{2}}$

$= \frac{3x^2}{2\sqrt{1+x^3}}$

$\therefore du = \frac{3x^2}{2\sqrt{1+x^3}}\,dx$

(3)

Change limits

$x = 0, \quad u = 1$

$x = 1, \quad u = \sqrt{2}$

Q5 (c) i $\quad 4\cos\theta - 3\sin\theta$

$A\cos(\theta - \alpha) = A\cos\theta\cos\alpha - A\sin\theta\sin\alpha$

$= (A\cos\alpha)\cos\theta - (A\sin\alpha)\sin\theta$

$= 4\cos\theta - 3\sin\theta$

(3)

$A^2\cos^2\alpha + A^2\sin^2\alpha = 4^2 + 3^2$

$A^2 = 25$

$\therefore A = 5$

$\cos\alpha = 4/5 \quad \sin\alpha = 3/5$

$\tan\alpha = 3/4$

$\therefore \alpha = 36^\circ 52'$

Hence $4\cos\theta - 3\sin\theta = 5\cos(\theta + 36^\circ 52')$

ii $\quad 4\cos\theta - 3\sin\theta = -1$

$5\cos(\theta + 36^\circ 52') = -1$

$\cos(\theta + 36^\circ 52') = -1/5$

$\theta + 36^\circ 52' = 101^\circ 32'$ or $258^\circ 28'$

(3)

$\therefore \theta = 64^\circ 40'$ or $221^\circ 36'$

Question 7

(c) i Amplitude $= \frac{1}{2}(12-4) = 4 = a$

$\therefore$ Centre of S.H.M $= 8$

Period $= 12 = \frac{2\pi}{n}$ $\quad \therefore n = \pi/6$

$y = 8 + a\cos(nt + \alpha)$

$= 8 + 4\cos(\frac{\pi}{6}t + \alpha)$

Let $t = 0$ at high tide, $y = 12$

$12 = 8 + 4\cos(\frac{\pi}{6}\times 0 + \alpha)$

$1 = \cos\alpha$

$\alpha = 0$ $\quad$ (1)

$\therefore y = 8 + 4\cos(\frac{\pi t}{6})$

ii $10 \geq 8 + 4\cos(\frac{\pi t}{6})$

$2 \geq 4\cos(\pi t/6)$

$\frac{1}{2} \geq \cos(\pi t/6)$

$\therefore \frac{\pi t}{6} \geq \pi/3$ $\quad$ (1)

$t \geq 2$

$\therefore$ Earliest time to leave $= 6$ am $+ 2$h

$= 8$ am.

iii $8 + 4\cos(\frac{\pi t}{6}) \geq 7$

$4\cos(\frac{\pi t}{6}) \geq -1$

$\cos(\frac{\pi t}{6}) \geq -1/4$

$\frac{\pi t}{6} \leq 1.8235$

$t \leq 3.48$ hours $\quad$ (2)

$t \leq 3$h 29 min

$\therefore$ Latest possible time to leave $= 6$am $+ 3$h 29min

$= 9:29$ am

Question 7

(a)

$h = 2r$

$\frac{dV}{dt} = 8$ m^3/min

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2(2r)$

$h = 2$ m $= 2r$

$\therefore r = 1$ m

$V = \frac{2}{3}\pi r^3$

$\frac{dV}{dr} = \frac{2}{3}\times 3\pi r^2 = 2\pi r^2$

$\frac{dV}{dt} = \frac{dV}{dr}\cdot\frac{dr}{dt}$

$8 = 2\times\pi\times r^2 \times \frac{dr}{dt}$ $\quad$ (3)

$\frac{dr}{dt} = \frac{8}{(2\times\pi\times 1^2)} = \frac{8}{2\pi}$

$h = 2r \quad \therefore dh/dr = 2$

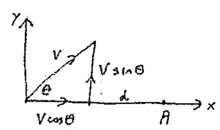
$\frac{dh}{dt} = \frac{dh}{dr}\cdot\frac{dr}{dt}$

$= 2\times\frac{8}{(2\pi)} = \frac{8}{\pi} = 2.55$ m/min

$\therefore$ Rate height is changing $= 2.55$ m/min.

Question 7

(b)



i Horizontal motion

$\ddot{x} = 0$

$\dot{x} = 0 + c$

$V\cos\theta = c$

$\therefore \dot{x} = V\cos\theta$

$x = Vt\cos\theta$

Vertical motion

$\ddot{y} = -g$

$\dot{y} = -gt + c$

$V\sin\theta = 0 + c$

$\therefore \dot{y} = -gt + V\sin\theta$

$y = -g\frac{t^2}{2} + Vt\sin\theta$ $\quad$ (2)

$\therefore$ Position $(x, y) = (Vt\cos\theta,\ Vt\sin\theta - \frac{1}{2}gt^2)$

ii Let $y = 0$

$Vt\sin\theta - \frac{1}{2}gt^2 = 0$

$t(V\sin\theta - \frac{1}{2}gt) = 0$

$t = 0$ and $t = \frac{2V\sin\theta}{g}$

$\therefore$ Time above X-axis is $\frac{2V\sin\theta}{g}$ seconds $\quad$ (1)

iii Substitute flight time above in $X = Vt\cos\theta$

$x = V\cos\theta\left(\frac{2V\sin\theta}{g}\right)$

$= \frac{V^2}{g}(2\sin\theta\cos\theta) = \frac{V^2\sin 2\theta}{g}$ (m) = horizontal range $\quad$ (1)

iv In t secs, Target is $(d + Wt)$ metres from origin (O)

In t secs, Projectile has moved $Vt\cos\theta$ horizontally

At collision $d + Wt = Vt\cos\theta$

$Wt = Vt\cos\theta - d$

$W = V\cos\theta - d/t$

$= V\cos\theta - \frac{d}{\left(\frac{2V\sin\theta}{g}\right)}$

$\therefore W = V\cos\theta - \frac{gd}{2V\sin\theta}$ $\quad$ (1)