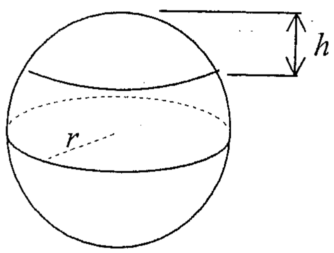
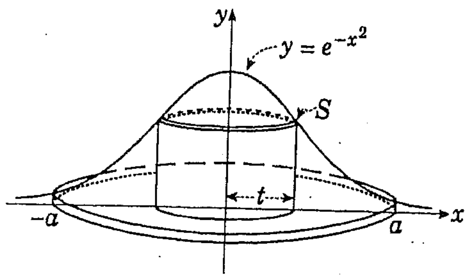
Volumes No. 9
Revision and Consolidation

1. Find the volume of the solid formed by rotating $(x-2)^2 + y^2 = 1$ around the Y axis using the method of cylindrical shells.
2. The region bounded by the curve $y = (x-1)^2$ and the x- and y- axes is rotated about the line $y = -\frac{1}{2}$. Find the volume of the solid of revolution
3. The region bounded by the curve $y = x(4-x)$ and the x-axis is rotated about the y-axis. Find the volume of the solid of revolution formed.
4. Find the volume of the solid that results when the region enclosed by the semicircle $y = \sqrt{25-x^2}$ and the line $y = 3$ is revolved about the x-axis.
5. The base of a certain solid is a circle with diameter AB of length 2a units. Find the volume of the solid if each cross section perpendicular to AB is:
   a. a. a square b. an equilateral triangle
6. The region bounded by the curve $y = \log_e x$, the line $y = 1$ and the coordinate axes, is rotated about the x-axis:
   a. By slicing perpendicular to the axis of revolution show:
   $$V = \pi + \pi \int_1^e \left\{1 - (\log_e x)^2\right\} dx$$
   b. By forming cylindrical shells show: $V = 2\pi \int_0^1 ye^y dy$.
   c. Hence find the volume of the solid formed.
7. The region bounded by the curve $y = \sin^{-1} x$, the x-axis and the ordinate $x = 1$ is rotated about the line $y = -1$. Find the volume of the solid formed.
8. The base of a solid is the region enclosed by $y = \sin x$, $y = 0$, $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$ and every cross-section perpendicular to the x-axis is a square with one side on the base. Find the volume of the solid.
9. The figure below shows a spherical segment of height h cut of from a sphere of radius r by a horizontal plane. Show that its volume is $V = \frac{1}{3}\pi h^2 (3r - h)$



10. The region under the curve $y = e^{-x^2}$ and above the x-axis for $-a \le x \le a$ is rotated about the y-axis to form a solid.(1990 HSC)



Divide the resulting solid into cylindrical shells S of radius t as in the diagram. Show that each shell S has approximate volume $\delta V = 2\pi t e^{-t^2}\delta t$ where δt is the thickness of the shell.

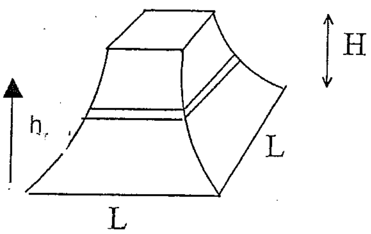
a) Hence calculate the volume of the solid.

b) What is the limiting value of the solid as a approaches infinity?

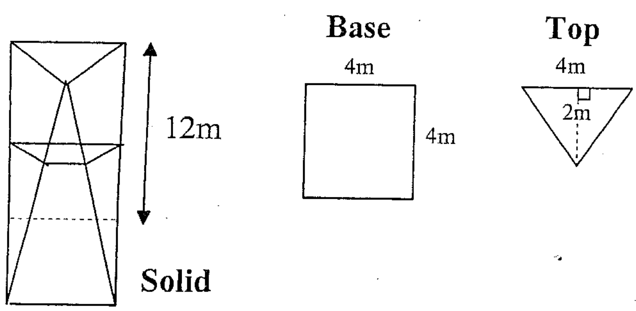
11. a) Sketch the curve $y^2 = x^2(2-x)$

b) Find the volume of the solid formed by rotating the loop of the curve about the x-axis.

12. A stone building of height H metres has the shape of a flat-topped square pyramid as shown in the figure. The cross section at height h metres is a square with sides parallel to the base of length $l(h)\frac{L}{\sqrt{h+1}}$ where L is the side length of the square base in metres. Find the volume of the building given H = L = 30



13. The base of a solid is a square of side 4m. The solid rises to a vertical height of 12m. The top of the solid is an isosceles triangle with base 4m and altitude 2m.



Find the volume of the solid. (The rear face is perpendicular to the base)

Answers

1. $125\sqrt{3}\pi$

2. $\frac{8\pi}{15}$ 3. $\frac{128\pi}{3}$ 4. $\frac{256\pi}{3}$ 5. a) $\frac{16a^3}{3}$, b) $\frac{4\sqrt{3}a^3}{3}$

6. b) $\pi + \pi\int_1^e \left\{1-\left(\log_e x\right)^2\right\}dx$, 2π 7. $\pi\left(\frac{\pi^2}{4}+\pi-4\right)$

8. $40,000\pi$ 9. $\frac{1}{4}(\pi+2)$ 10. $\pi\left(1-e^{-a^2}\right)$, π 11. $\frac{4\pi}{3}$ 12. $900\log_e 31$

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