



2007

Year 10

# MATHEMATICS

## Half Yearly Examination

Time allowed – 75 minutes

Total Marks: 100

### Instructions:

- There are FIVE (5) questions of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each Part on a new page.
- Write on one side of paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your name and Maths class clearly at the top of each question and clearly number each question.

### Question 1. (20 marks)

a. Solve the following equations giving your answers in exact (surd) form if necessary.

(i)  $2x^2 - 10x = 0$

(ii)  $x^2 - 9x + 14 = 0$

(iii)  $8x^2 - 10x + 3 = 0$

(iv)  $(x - 3)(x - 4) = 20$

(v)  $3x^2 - 8x + 2 = 0$

b. The sum of the squares of two consecutive positive even numbers is 244

Let  $x$  = the smaller number. By forming and solving a quadratic equation, find both numbers.

c. Solve, giving your answer correct to 2 decimal places

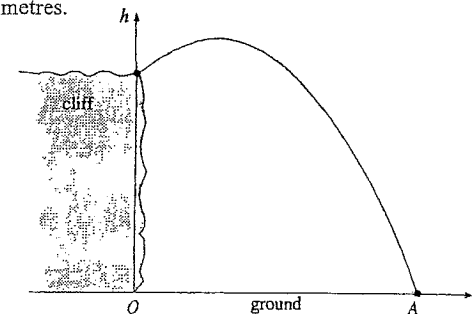
(i)  $2(x + 2) = \frac{1}{x}$

(ii)  $(2x + 3)^2 = (x + 4)(x + 3)$

d. A ball is thrown from the top of a cliff and hits the ground at A.

For the axes on the diagram, the flight path of the ball is the parabola  $h = 20 + 8x - x^2$

where  $x$  and  $h$  are in metres.



(i) Find how far from the base of the cliff, the ball hits the ground.

(ii) Find the maximum height of the ball above the ground.

**Question 2 (20 marks)**

a. From a standard pack of 52 playing cards, one card is selected at random. Find

- (i) Prob (either a King or a Club)
- (ii) Prob (a 4 and a Red Card)
- (iii) Prob (Queen given that the card is a Heart)
- (iv) Prob (Heart given that the card is a Jack)

b. A family has 3 children

(i) Draw a tree diagram showing all possible combinations of the children.

Find: (ii) Prob (exactly 2 girls)

(iii) Prob (at least 1 boy)

(iv) Prob (at most 1 girl)

(v) Prob (all children of the same sex)

c. Of 100 people who applied for a position with a company, 40 had work experience, 30 had a University Degree and 20 had both work experience and a University Degree.

- (i) Illustrate this information using a Venn Diagram.
- (ii) Find Prob (the applicant had either work experience or a University Degree or both)
- (iii) Find Prob (either work experience or a University Degree but not both)
- (iv) Find Prob (neither had work experience nor a University Degree)

**Question 2 (continued)**

d. 500 people were surveyed about a proposed under harbour tunnel. 250 of the 300 males surveyed were in favour of the tunnel, while a total of 150 people were against the proposal.

(i) Copy and complete the Contingency Table below:

	Male	Female	Totals
For	_____	_____	_____
Against	_____	_____	_____
Totals	_____	_____	_____

(ii) A person is selected at random. Find

1. Prob (female and against the proposal)
2. Prob (person is for the proposal)
3. Prob (for the proposal given that a female is chosen)

Question 3 (20 marks)

- a. Find the simple interest on \$2600 for 9 months at  $10\frac{1}{4}\%$  p.a.
- b. A block of land has increased in value by 20% of the previous year's value in each of the last 4 years. If its value 4 years ago was \$200,000, what is its value now?
- c. A company will sell its photocopy machine, which it purchased for \$11,000 once its value drops below \$7,000. How many years will this be if the rate of depreciation is 14% p.a.
- d. A customer takes out a loan of \$9000. The loan, together with simple interest, is repaid by making monthly payments of \$300 over 4 years. Find the rate of simple interest per annum as a percentage.
- e. Liliانا wants to borrow money to buy a house. The bank sent her an email with the following table attached.

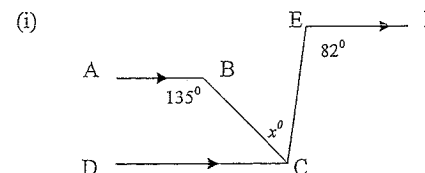
Monthly Repayments

Amount	Terms of loan				
	10 years	15 years	20 years	25 years	30 years
Borrowed	120 months	180 months	240 months	300 months	360 months
\$160 000	\$1941.24	\$1529.04	\$1338.30	\$1234.91	\$1174.02

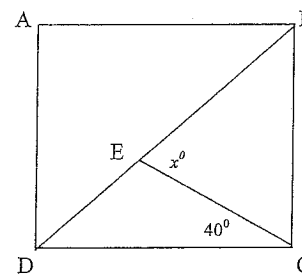
Liliana intends to borrow \$160,000 over 15 years. However if she chooses instead to borrow the same amount over 20 years instead, how much more interest will she pay?

Question 4 (20 marks)

- a. Find  $x^\circ$  giving reasons - Copy both diagrams



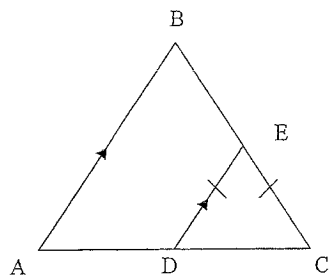
- (ii) ABCD is a square



- b. The sum of the interior angles of a regular polygon is  $1980^\circ$
- (i) How many sides has the polygon?
- (ii) Find the size of each interior angle to the nearest minute.
- c. Find the sum of the interior angles of a regular polygon whose exterior angles are each  $18^\circ$

Question 4 (continued)

d.

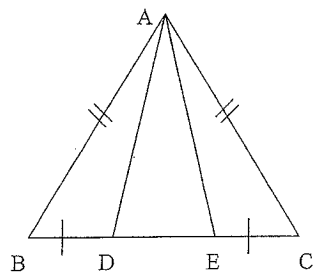


In the diagram  $AB \parallel DE$  and  $DE = CE$

Copy this diagram.

Prove (giving reasons) that  $AB = BC$

e.



Given  $AB = AC$  and  $BD = CE$

Copy this diagram

Prove  $AD = AE$

Question 5 (20 marks)

a. Sketch the curves and mark one critical point on each curve

(i)  $y = 2x^2 - 2$

(ii)  $y = \frac{1}{x}$

(iii)  $y = 2^x - 1$

(iv)  $x^2 + y^2 = 25$

b. Find the equation of a parabola whose X intercept is -2 and y intercept is 6

c. For the parabola  $y = 2x^2 + 4x + 2$

Find (i) The concavity

(ii) The axis of symmetry

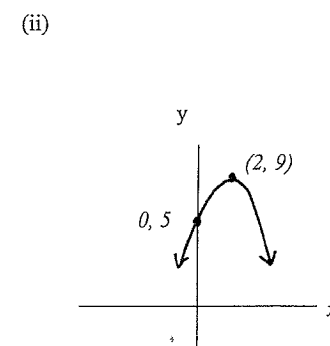
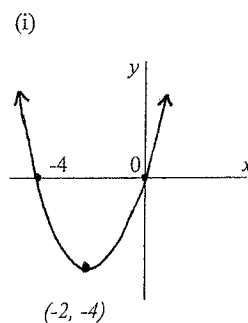
(iii) The vertex

(iv) The minimum value

(v) The y intercept

(vi) The x intercepts

d. Find the equations of the following parabolae



--- THE END ---

Question 1

i  $2x^2 - 10x = 0$   
 $2x(x-5) = 0$   
 $\therefore x = 0$  or  $5$  (2)

ii  $x^2 - 9x + 14 = 0$   
 $(x-7)(x-2) = 0$   
 $\therefore x = 7$  or  $2$  (2)

i  $8x^2 - 10x + 3 = 0$   
 $(4x-3)(2x-1) = 0$   
 $\therefore x = \frac{3}{4}$  or  $\frac{1}{2}$  (2)

i  $(x-3)(x-4) = 20$   
 $x^2 - 7x + 12 = 20$   
 $x^2 - 7x - 8 = 0$   
 $(x-8)(x+1) = 0$   
 $\therefore x = 8$  or  $-1$  (2)

$3x^2 - 8x + 2 = 0$   
 $x = \frac{8 \pm \sqrt{64 - 24}}{6}$   
 $= \frac{8 \pm \sqrt{40}}{6}$   
 $= \frac{8 \pm 2\sqrt{10}}{6}$   
 $= \frac{4 \pm \sqrt{10}}{3}$  (2)

Let  $x =$  smaller no.  
 $\therefore x+2 =$  larger no.  
 $x^2 + (x+2)^2 = 244$   
 $x^2 + x^2 + 4x + 4 = 244$   
 $2x^2 + 4x - 240 = 0$   
 $x^2 + 2x - 120 = 0$   
 $(x+12)(x-10) = 0$   
 $\therefore x = -12$  or  $10$   
 The two consecutive nos are  $-10$  and  $12$

i  $2(x+2) = \frac{1}{x}$   
 $2x^2 + 4x - 1 = 0$   
 $x = \frac{-4 \pm \sqrt{16+8}}{4}$  (2)  
 $x = 0.22$  or  $-2.22$

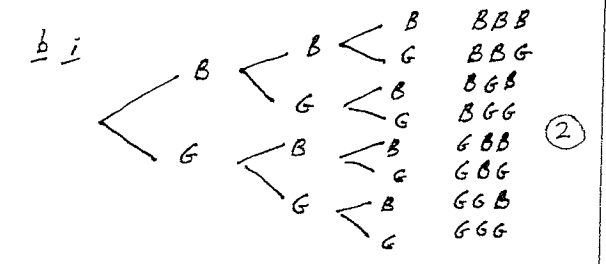
ii  $(2x+3)^2 = (x+4)(x+3)$   
 $4x^2 + 12x + 9 = x^2 + 7x + 12$   
 $3x^2 + 5x - 3 = 0$   
 $x = \frac{-5 \pm \sqrt{25+36}}{6}$  (2)  
 $= 0.47$  or  $-2.14$

i  $h = 20 + 8x - x^2$   
 Let  $h = 0$  to hit ground  
 $0 = 20 + 8x - x^2$   
 $x^2 - 8x - 20 = 0$   
 $(x-10)(x+2) = 0$  (2)  
 $\therefore x = 10$  or  $-2$   
 $\therefore$  It hits ground 10 m from the cliff base.

ii Find maximum height  
 Let  $x = 4$  ( $= \frac{10-2}{2}$ )  
 $h = 20 + 32 - 16$  (2)  
 $= 36$   
 $\therefore$  Max height = 36 m.

Question 2

i  $P(K \text{ or Club}) = \frac{16}{52} = \frac{4}{13}$  (1)  
 ii  $P(4 \text{ and Red}) = \frac{2}{52} = \frac{1}{26}$  (1)  
 i  $P(Q \text{ given Heart}) = \frac{1}{13}$  (1)  
 i  $P(\text{Heart given Jack}) = \frac{1}{4}$  (1)



ii  $P(2G) = \frac{3}{8}$  (1)  
 iii  $P(\text{at least 1B}) = \frac{7}{8}$  (1)  
 iv  $P(\text{at most 1G}) = \frac{4}{8} = \frac{1}{2}$  (1)  
 v  $P(\text{all same sex}) = \frac{2}{8} = \frac{1}{4}$  (1)



$P(W \text{ or } D \text{ or both}) = \frac{50}{100} = \frac{1}{2}$  (1)  
 $P(W \text{ or } D \text{ not both}) = \frac{30}{100} = \frac{3}{10}$  (1)  
 $P(\bar{W} \text{ and } \bar{D}) = \frac{50}{100} = \frac{1}{2}$  (1)

i

	Male	Female	Totals
FOR	250	100	350
AGAINST	50	100	150
TOTALS	300	200	500

i  $P(\text{Fem and AG}) = \frac{100}{500} = \frac{1}{5}$  (1)  
 ii  $P(\text{FOR}) = \frac{350}{500} = \frac{7}{10}$  (1)

Question 3  
 a S.I =  $2600 \times \frac{10\frac{1}{2}}{100} \times \frac{9}{12}$   
 $= \$199.88$

b  $A = P(1+r)^n$   
 $= 200,000 \times (1.2)^4$   
 $= \$414,720$

c  $A = P(1-r)^n$   
 $7000 = 11,000 \times (0.86)^n$   
 $(0.86)^n = \frac{7}{11}$   
 Try  $n = 3$   
 $(0.86)^3 = 0.636 < \frac{7}{11}$   
 $\therefore 3$  years

d Loan = \$9000  
 Total repayments  
 $= \$300 \times 4 \times 12$   
 $= \$14,400$   
 $\therefore$  S.I = \$5,400

$5400 = 9000 \times R \times 4$   
 $R = 0.15$   
 $\therefore$  Rate p.a = 15% p.a.

e For 15 years  
 Interest =  $1529.04 \times 180$   
 $- 160000$   
 $= 115227.20$

For 20 years  
 Interest =  $1338.30 \times 240$   
 $- 160,000$   
 $= 161,992.00$   
 $\therefore$  Extra Interest = \$45,964.80

$P(\text{FOR given Fem}) = \frac{100}{200} = \frac{1}{2}$  (1)

i  $\angle DCB = 45^\circ$  (alt int  $\angle$   $AB \parallel DC$ )

$x + 45 = 82$  (ext  $\angle$   $EF \parallel DC$ )

$\therefore x = 37^\circ$

(3)

ii  $\angle EDC = 45^\circ$  (diag bisects  $\angle D$ )

$x = 40 + 45$  (ext  $\angle$  of  $\Delta$ )

$= 85^\circ$

(3)

i  $(n-2)180 = 1980$

$n-2 = 11$

$\therefore n = 13$  sides

(2)

ii Each int angle

$= \frac{1980}{13}$

$= 152^\circ 18'$

(2)

$\frac{360}{n} = 18$

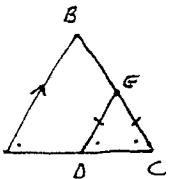
$\therefore n = 20$

Sum of interior angles

$= (20-2) \times 180$

$= 3240^\circ$

(3)



Aim: Prove  $AB = BC$

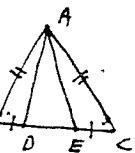
Proof:  $\angle C = \angle EDC$  (Base  $\angle$ s Isos  $\Delta$ )

$\angle A = \angle EDC$  (Corr  $\angle$ 's  $AB \parallel DE$ )

$\therefore \angle A = \angle C$

Hence  $AB = BC$  (Isos  $\Delta$  eq base  $\angle$ 's)

(3)



Aim: Prove  $AD = AE$

Proof:  $\angle B = \angle C$  (Base  $\angle$ s Isos  $\Delta$ )

In  $\Delta$ s  $ABD$  and  $\Delta AEC$

$AB = AC$  (Given)

$BD = EC$  (Given)

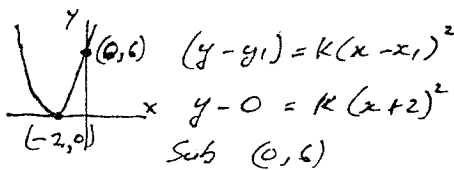
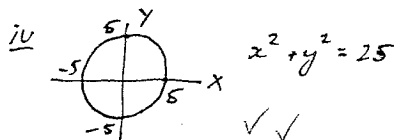
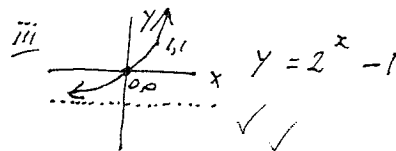
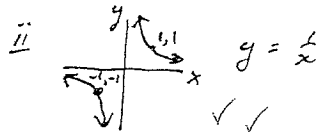
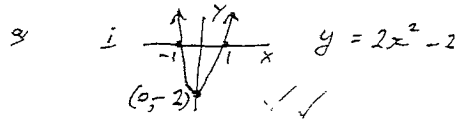
$\angle B = \angle C$  (Base  $\angle$ s Isos  $\Delta$   $ABC$ )

$\therefore \Delta ABD \cong \Delta AEC$  ( $\angle$   $AC$ )

(4)

- A ✓
- B ✓
- C ✓
- D ✓
- E ✓
- F ✓
- G ✓
- H ✓

Question 5.



$6 = k(0 + 2)^2$   
 $k = \frac{6}{4} = \frac{3}{2}$

$\therefore y = \frac{3}{2}(x + 2)^2$

$y = 2x^2 + 4x + 2$

i Concave up since  $a > 0$

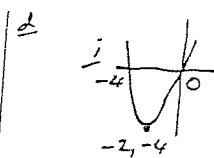
ii Axis of Sym  $x = -\frac{b}{2a}$

$\therefore x = -\frac{4}{4} \therefore x = -1$

iii Vertex Sub  $x = -1$

$y = 2 - 4 + 2 = 0$

$\therefore V = (-1, 0)$



$y - y_1 = k(x - x_1)^2$

$y + 4 = k(x + 2)^2$

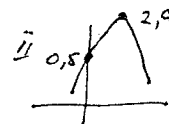
Sub  $(0, 0)$

$4 = k(0 + 2)^2$

$\therefore k = 1$

$y + 4 = 1(x + 2)^2$

$\therefore y = x^2 + 4x$



$y - y_1 = k(x - x_1)^2$

$y - 9 = k(x - 2)^2$

Sub  $0, 5$

$5 - 9 = k(0 - 2)^2$

$-4 = k(4)$

$k = -1$

$y - 9 = -1(x - 2)^2$

$\therefore y = -x^2 + 4x + 5$

iv Minimum Value = 0 ✓

v Let  $x = 0$

$\therefore Y$  intercept = 2 ✓

vi Let  $y = 0$

$0 = 2x^2 + 4x + 2$

$0 = x^2 + 2x + 1$

$0 = (x + 1)^2$  ✓

$\therefore X$  intercept = -1