Sydney Girls High School

Mathematics Department

Year 10 Half Yearly Examination

2008

Time Allowed: 75 minutes

Instructions:

- There are FIVE questions, of equal value.
- All questions are to be attempted.
- Board-approved calculators may be used in all parts of the test.

QUESTION 1: (*20 marks*)

a) Solve:

i. $(x-2)(2x+1)=0$ 2

ii. $x^2=3x$ 2

b) A card is drawn at random from a standard pack of playing cards. What is the probability that the card is:

i. a spade? 1

ii. the 8 of spades? 1

iii. a spade *or* an 8? 2

c) Find the simple interest earned if \$600 is invested for 5 months at 6% p.a. 2

d) Find the compound interest earned on \$4000 if it is invested for 5 years at 5% p.a. 2

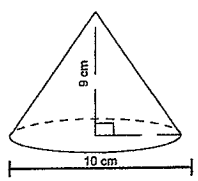
e) Sketch each graph on a separate number plane, showing essential features:

i. $y=9-x^2$ 2

ii. $xy=4$ 2

f) Find the exact volume of a sphere whose radius is 5 cm, in terms of π. 2

g) Find the exact volume of the cone below, in terms of π. 2



QUESTION 2: *(20 marks)*

a) Solve the following quadratic equations, giving your answers to 1 decimal place where necessary:

i. $x^2+11x+30=0$ **2**

ii. $12m^2-m-20=0$ **3**

iii. $3x^2+7x-2=0$ **2**

b) The diagram shows the choices of two electives made by a Year 10 class.

	Art	Music
French	7	4
Japanese	6	11

i. How many students are in the class? **1**

A student is chosen at random. What is the probability that the student:

ii. did *not* choose Japanese or Art? **1**

iii. chose Music? **1**

c) For the parabola $y=x^2-2x-8$:

i. find the y intercept; **1**

ii. find the x-intercepts; **2**

iii. find the equation of the axis of symmetry; **2**

iv. find the minimum value of y; **2**

v. sketch the parabola, clearly showing all key points. **3**

QUESTION 3: *(20 marks)*

a) Anne borrowed \$16 000 at 12% p.a. simple interest for 5 years so that she could buy a second hand car. At the end of the 5 years, both the interest and loan had been repaid.

i. How much interest was charged? **2**

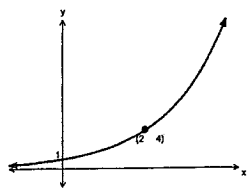
ii. How much was paid back altogether? **2**

iii. If the loan was repaid in equal monthly payments over the 5 years, calculate the amount of each repayment. **2**

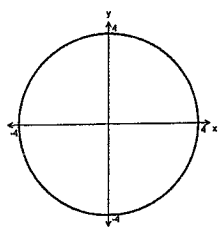
b) After having depreciated at a rate of 9% p.a, a car is now worth \$15 000. What was its value 6 years ago? **3**

c) Find the equation of the graphs below:

i. **2**



ii. **2**



Question 3 continued...

d) A coin is tossed and a die is thrown.

i. List all possible outcomes. 2

ii. What is the probability of a tail and a three? 1

e) Find the surface area of a square pyramid with a base of length 8 cm and perpendicular height 10 cm. Give your answer correct to 2 decimal places. 4

QUESTION 4: *(20 marks)*

a) Sketch the graph of $y = 3^{-x}$. 2

b) Find the equation of the circle with its centre at the origin that passes through the point (5, -12). 2

c) The height of a triangle is 5 cm less than its base. If the area is 7 cm^2, find the length of the base. 4

d) Three marbles – one red, one green and one blue – are placed in a bag. A marble is drawn out at random and its colour is noted. The marble is not returned to the bag before the second marble is drawn out.

i. Using a tree diagram, list all possible outcomes. 2

ii. What is the probability that the blue marble is selected first? 1

iii. What is the probability that the green and red marbles are selected, in any order? 2

e) Find the surface area of a cone of base radius 5 cm and 3

QUESTION 5: *(20 marks)*

a) The sum of a number and its reciprocal is $2\frac{1}{6}$. Find the number. 4

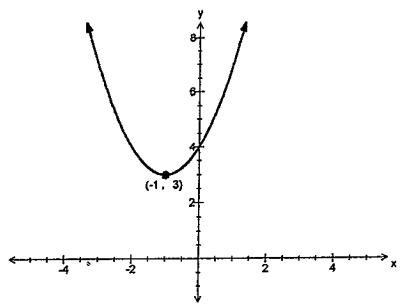
b) Each student in a class of 30 must learn at least one instrument. If 19 students are learning how to play the recorder and 17 are learning how to play the piano, find the probability that a student selected at random:

i. only plays the recorder; 2

ii. plays both instruments 2

c) Sam is on a salary of $80 000 and he estimates that inflation will cause his salary to increase by 5% every year. How long will it take for his salary to double? 4

d) Find the equation of the parabola below: 4



QUESTION 1:

a) i. $x = 2$ or $x = -\frac{1}{2}$ ii. $x = 0$ or $x = 3$

b) i. $P(\text{spade}) = \frac{1}{4}$ ii. $P(8 \text{ of spades}) = \frac{1}{52}$

iii. $P(\text{spade or 8}) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

$= \frac{4}{13}$

c) $I = 600 \times \frac{5}{12} \times \frac{6}{100}$

$= \$15$

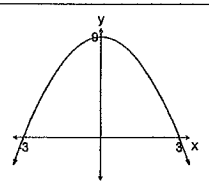
d) $A = P(1+r)^n$

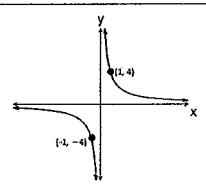
$4000(1.05)^5$

$= \$5105.13$ (nearest cent)

$I = \$5105.13 - \$4000$

$= \$1105.13$ (nearest cent)

e) i. 

ii. 

f) $A = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 5^3$

$= \frac{500\pi}{3}$ cm^3

g) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 5^2 \times 9$

$= 75\pi$ cm^3

QUESTION 2:

a) i. $x^2 + 11x + 30 = 0$

$(x+6)(x+5) = 0$

$x = -6$ or $x = -5$

ii. $12m^2 - m - 20 = 0$

$12m^2 - 16m + 15m - 20 = 0$

$4m(3m-4) + 5(3m-4) = 0$

$(4m+5)(3m-4) = 0$

$m = -\frac{5}{4}$ or $m = \frac{4}{3}$

iii. $3x^2 + 7x - 2 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-7 \pm \sqrt{49+24}}{6}$

$= \frac{-7 \pm \sqrt{73}}{6}$

$x \approx 0.26$ or $x = -2.59$ (to 2 dec. pl)

b) i. 28 ii. $\frac{4}{28} = \frac{1}{7}$ iii. $\frac{15}{28}$

c) i. y intercept: $y = -8$

ii. $x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$ or $x = -2$

iii. $x = -\frac{b}{2a}$

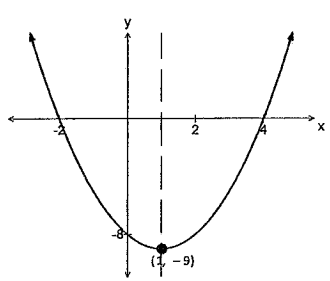
$= -\frac{2}{2}$

$= 1$

iv. When $x = 1$:

$y = (1)^2 - 2(1) - 8$

$= -9$



QUESTION 3:

a) i. $I = 16000 \times \frac{12}{100} \times 5$

$= \$9600$

ii. Total repaid $= \$16000 + \$9600$

$= \$25600$

iii. Monthly repayment $= \frac{25600}{5 \times 12}$

$= \$426.67$

b) $A = P(1-r)^n$

$15000 = P(1-0.09)^6$

$= P(0.91)^6$

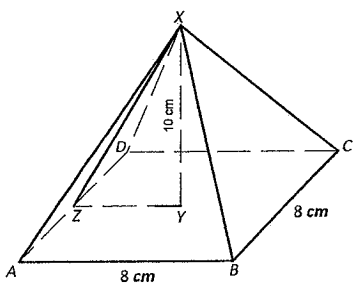
$P = \$26414.53$ (nearest cent)

Car was worth \$26414.53 6 years ago.

c) i. $y = 2^x$ ii. $x^2 + y^2 = 16$

d) i. $\{H1, H2, H3, H4, H5, H6\}$
$\{T1, T2, T3, T4, T5, T6\}$ ii. $P(T3) = \frac{1}{12}$

e)



Area of base $= 8^2$

$= 64$ cm^2

$XZ^2 = 4^2 + 10^2$

$= 116$

$XZ = 2\sqrt{29}$ cm

Area of Δs $= 4 \times \left(\frac{1}{2} \times 8 \times 2\sqrt{29}\right)$

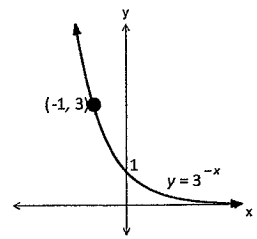
$= 32\sqrt{29}$ cm^2

S.A $= 32\sqrt{29} + 64$

≈ 236.33 cm^2

QUESTION 4:

a)

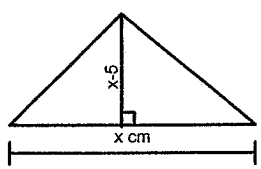


b) $r^2 = 5^2 + (-12)^2$

$= 13$

$x^2 + y^2 = 169$

c)



Let the length of the base be x.

$A = \frac{1}{2} \cdot x \cdot (x-5)$

$7 = \frac{1}{2} \cdot x \cdot (x-5)$

$14 = x^2 - 5x$

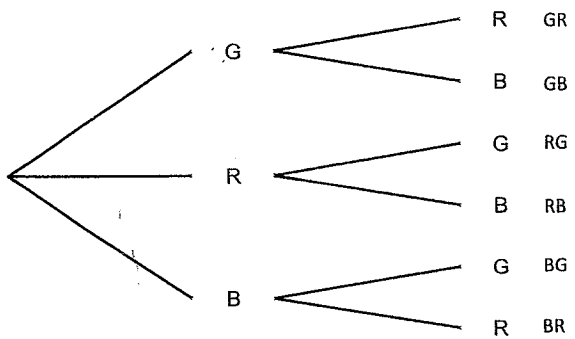
$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x = 7$ or $x = -2$ (no solution)

$\therefore$ Base is 7 cm long

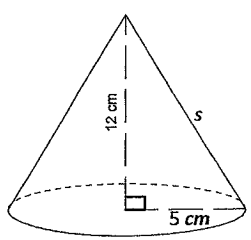
d) i.



ii. $P(\text{blue marble selected first}) = \frac{1}{3}$

iii. $P(\text{red and green in any order}) = \frac{1}{3}$

e)



$s = \sqrt{5^2 + 12^2}$

$s = 13$ cm

$SA = \pi r^2 + \pi rs$

$= \pi \times 5^2 + \pi \times 5 \times 13$

$= 25\pi + 65\pi$

$= 90\pi$ cm^2

f)

$P = \$25000$

$r = \frac{0.09}{4}$

$= 0.0225$

$n = 18 \div 3$

$= 6$

$A = 25000(1+0.0225)^6$

$= 25000(1.0225)^6$

$= \$28570.64$ (nearest cent)

QUESTION 5:

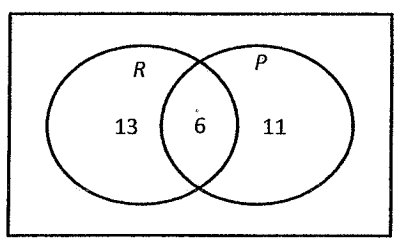
a) Let the number be x.

$$x + \frac{1}{x} = \frac{13}{6}$$
$$6x^2 + 6 = 13x$$
$$6x^2 - 13x + 6 = 0$$
$$6x^2 - 9x - 4x + 6 = 0$$
$$3x(2x-3) - 2(2x-3) = 0$$
$$(3x-2)(2x-3) = 0$$

$x = \frac{2}{3}$ or $x = \frac{3}{2}$

$\therefore$ the number is either $\frac{2}{3}$ or $\frac{3}{2}$.

b)



i. $P(\text{only plays the recorder}) = \frac{13}{30}$

ii. $P(\text{plays both instruments}) = \frac{6}{30} = \frac{1}{5}$

c) $A = P(1+r)^n$

$160000 = 80000(1+0.05)^n$

$2 = 1.05^n$

$n = 14.2067\ldots$

Sam's salary should double in his 15th year of employment.

d) $y = ax^2 + bx + 4$

Vertex at $(-1,3)$:

$$3 = a(-1)^2 + b(-1) + 4$$
$$a - b = -1 \text{ --- (1)}$$

Axis of symmetry:

$$x = -\frac{b}{2a}$$
$$-1 = -\frac{b}{2a}$$
$$b = 2a \text{ --- (2)}$$

Sub (2) into (1):

$$a - 2a = -1$$
$$-a = -1$$
$$a = 1$$

Sub $a = 1$ into (2):

$$b = 2$$

Required equation is $y = x^2 + 2x + 4$

e) $SA = 4\pi r^2$

$$= 4 \times \pi \times (7.5)^2$$
$$\approx 706.858 \text{ cm}^2$$

$$\text{No. of spheres} = \frac{10^5}{706.858}$$
$$\approx 141.471...$$

141 spheres can be covered by the material.