

# SYDNEY GIRLS HIGH SCHOOL



## MATHEMATICS

YEAR 10

HALF-YEARLY EXAMINATION

2006

Time Allowed: 75 minutes

### INSTRUCTIONS:

- There are FIVE (5) Questions of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each Part on a new page.
- Write on one side of paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your name and Maths class clearly at the top of each question and clearly number each question.

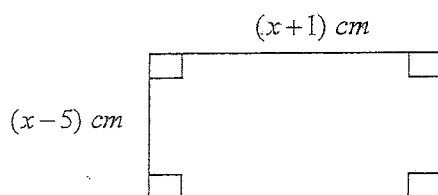
### QUESTION 1 (20 Marks)

MARKS

1. Solve the following:

- |     |  |   |
|-----|--|---|
| (a) | $(x-2)(x+5) = 0$                                 | 1 |
| (b) | $a^2 - 64 = 0$                                   | 1 |
| (c) | $y^2 + y - 20 = 0$                               | 2 |
| (d) | $2x^2 + 7x + 6 = 0$                              | 2 |
| (e) | $(m+5)^2 = 7$ (leave your answer in surd form)   | 2 |
| (f) | $x^2 + 7 = 15 - 6x$ (answer to 2 decimal places) | 3 |

2. If the area of the rectangle drawn below is  $27 \text{ cm}^2$ :



- |     |                                   |   |
|-----|-----------------------------------|---|
| (a) | find the value of $x$ .           | 3 |
| (b) | find the length of the rectangle. | 1 |
3. A right-angled triangle is drawn so that the hypotenuse is twice the length of the shortest side, and the other side is 1 cm longer than the shortest side. Draw a diagram and then form an algebraic equation to represent the information given. (Let  $x$  = the shortest side). Solve the equation and find the length of the hypotenuse to 2 decimal places. 5

QUESTION 2 (20 Marks)

MARKS

1. Jenny places \$620 in a term deposit account. If the bank pays her 5.2% p.a. simple interest for 6 months, how much interest does Jenny receive at the end of 6 months? 3
  
2. Catherine buys a new car valued at \$24 500 but decided to buy it on terms. She pays a \$5000 deposit and then pays \$700 monthly instalments over three years. 3
  - (a) How much has Catherine paid for the car?
  - (b) What is the amount of interest she paid on the car?
  
3. If Selina paid \$2350 in interest over two years on a loan of \$29 000, what was the rate per annum of simple interest charged? 4
  
4. \$35 000 is invested and earns compound interest at a rate of 5.4% p.a. Find the interest earned after two years if it compounded monthly. 4
  
5. A new plasma TV depreciates by 25% per year. If it costs \$1900 new, what will the TV be worth in five years? 3
  
6. A yacht now worth \$120 000 has been depreciating at a rate of 8% p.a. for the last five years. What was its value five years ago? 3

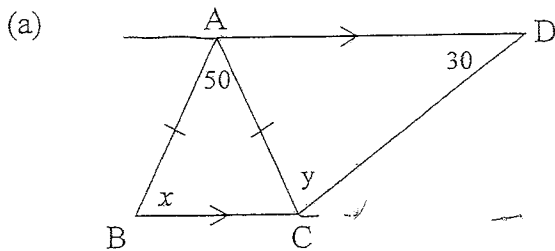
QUESTION 3 (20 Marks)

MARKS

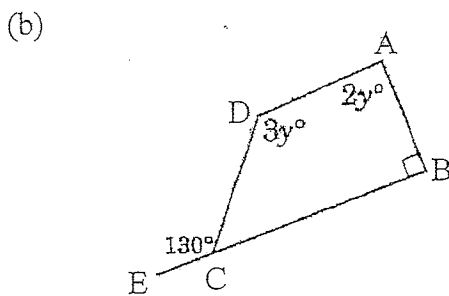
1

1. When comparing the properties of a rectangle and a rhombus, state one way in which the diagonals are different for both quadrilaterals.

2. Determine the value of the pronumeral(s) in each of the following. Give reasons for your answers.



5



4

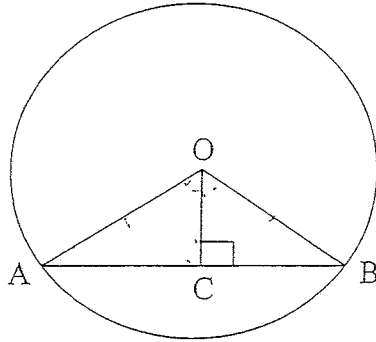
QUESTION 3 continued

MARKS

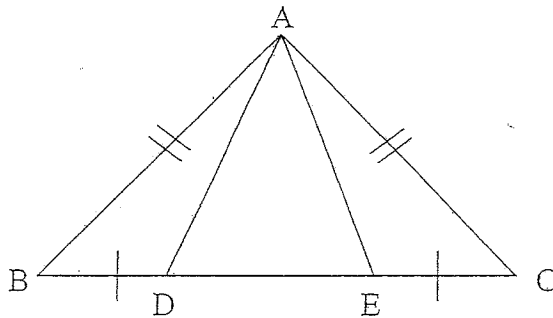
4

3. Prove  $\triangle$ s AOC and BOC are congruent.

Note: O is the centre of the circle and OC is drawn perpendicular to AB.



4.  $\triangle$ ABC is an isosceles triangle.  $AB = AC$  and  $BD = CE$ .



(a) Prove  $\triangle$ s ABD and ACE are congruent.

4

(b) Hence show that  $\triangle$ ADE is isosceles.

2

QUESTION 4 (20 Marks)

MARKS

1. A traffic light shows red for 1 minute 20 seconds, amber for 20 seconds and green for 1 minute. At any given time, what is the probability that the light is:

8

- (a) amber?
- (b) not red?
- (c) amber or red?
- (d) green or blue?

2. Four cards, [one black (B), one red (R), one yellow (Y), one green (G)], are placed in a bag. Two cards are chosen at random without replacing the cards.

- (a) Construct a tree diagram and list all the possible pairs chosen.
- (b) What is the probability of getting a black card?
- (c) What is the probability of getting a red and a blue card?

3

1

1

3. The following data of blood pressure readings was collected for a number of children.

Blood Pressure	Frequency (f)
115-119	0
120-124	2
125-129	9
130-134	6
135-139	2
140-145	0

If a child is selected at random, find the probability that the child has blood pressure:

2

- (a) less than 125
- (b) between 130-139.

4. There are 80 employees in a certain large supermarket. 35 are check-out operators, 20 are shelf stackers. Of the employees, 11 work as both check-out operator and shelf stacker.

- (a) Draw a Venn diagram to display this information.
- (b) If an employee is chosen at random, what is the probability that the employee:
  - (i) is a check-out operator but not a shelf stacker?
  - (ii) is neither a check-out operator nor a shelf stacker?

3

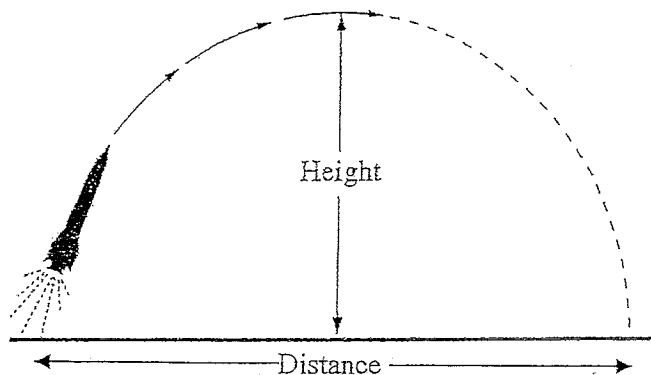
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QUESTION 5 (20 Marks)

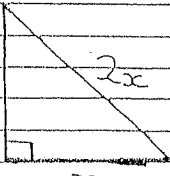
MARKS

1. For the parabola  $y = x^2 + 4x - 5$ , find the following information:
- |  |   |
|--|---|
| (a) the y-intercept  | 1 |
| (b) the x-intercepts   | 2 |
| (c) the vertex.  | 3 |
| (d) Hence or otherwise, sketch the parabola showing all relevant features. | 2 |
2. Find the equation of the parabola with x-intercepts of  $(-2,0)$  and  $(4,0)$  and y-intercept of  $(0,-16)$ . 3
3. Sketch the following on separate graphs, showing all relevant features 6
- (a)  $y = 3^x$
- (b)  $xy = 5$
- (c)  $x^2 + y^2 = 9$
4. A rocket fired from Earth travels in a parabolic path represented by the equation  $y = -\frac{x^2}{20} + 3x$ , where  $y$  is the vertical height in kilometres above the Earth's surface and  $x$  is the horizontal distance travelled in kilometres.
- Find the maximum height that the rocket reaches. 3

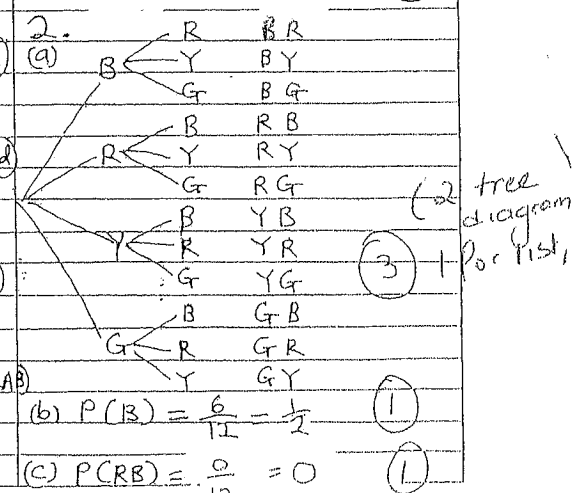


End of Paper

# Year 10 Half-Yearly 2006 (Solutions)

Question 1	
a) $x = 2, -5$ (1)	$(2x)^2 = x^2 + (x+1)^2$ (1)
b) $a^2 = 64$	$4x^2 = x^2 + x^2 + 2x + 1$ (1)
$a = \pm 8$ ( $\frac{1}{2}$ each) (1)	$2x^2 - 2x - 1 = 0$ (1)
	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
c) $(y+5)(y-4) = 0$	$= \frac{2 \pm \sqrt{4 - 4 \times 2 \times -1}}{4}$
$y = -5, 4$ (2)	$x = \frac{2 \pm \sqrt{12}}{4}$ (1)
d) $\frac{(2x+4)(2x+3)}{2} = 0$	$\frac{\text{Hypotenuse} = 2x}{4}$ (1)
$(x+2)(2x+3) = 0$	$= 2x \times (2 + \sqrt{12})$ (1)
$x = -2, -\frac{3}{2}$ (2)	$= \frac{2 + \sqrt{12}}{2}$ (1)
e) $m+5 = \pm\sqrt{7}$	$\approx 2.73 \text{ cm}$ (1)
$m = -5 \pm \sqrt{7}$ (2)	
f) $x^2 + 6x - 8 = 0$	
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
$= \frac{-6 \pm \sqrt{36 - 4 \times -8}}{2}$	
$= \frac{-6 \pm \sqrt{68}}{2}$	
$x \approx 1.12, -7.12$ (3)	
2.	
a) $(x+1)(x-5) = 27$	<b>Question 2</b>
$x^2 - 4x - 5 = 27$	1. $I = PRT$
$x^2 - 4x - 32 = 0$	$= \$620 \times 0.052 \times 0.5$ (3)
$(x-8)(x+4) = 0$	$= \$16.12$
$x = 8, -4$	2. (a) Payment = $\$5000 + 700 \times 3 \times 12$
$x$ must be positive $\therefore x = 8$ (2)	$= \$30200$ (1)
b) Length = $x+1$	(b) Interest paid = $\$30200 - \$24500$
$= 8+1$	$= \$5700$ (2)
$= 9 \text{ cm}$ (1)	3. $I = PRT$
	$2350 = 29000 \times R \times 2$
	$R = \frac{2350}{58000}$
	$= 0.04052$
	$\therefore \text{Rate} = 4.05\% \text{ p.a.}$
	4. $A = P(1+r)^n$
	$= 35000 \left(1 + \frac{0.054}{12}\right)^{2 \times 12}$ (3)
	$= \$38982.23$
3.	Interest earned
	$= \$38982.23 - \$35000$
	$= \$3982.23$ (1)

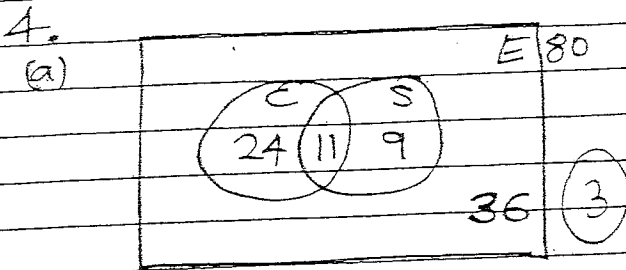
Question 2 continued.		4.	
5. $A = P(1-r)^n$ (3)	$= 1900(1-0.25)^5$	(a) In $\Delta$ s ABD and ACE: (4)	
	$= \$450.88$	1. $AB = AC$ (given)	
		2. $BD = CE$ (given)	
		3. $\angle ABD = \angle ACE$ (base $\angle$ s)	
6. $A = P(1-r)^n$	$120000 = P(1-0.08)^5$ (3)	$\therefore \Delta ABD \equiv \Delta ACE$ (SAS)	
	$P = \$182071.56$	(b) $AD = AE$ (equal corresp. sides in cong. $\Delta$ s ABD, ACE)	
		$\therefore \Delta ADE$ is isosceles (2 equal sides)	
<b>Question 3</b>			
1. Diagonals $\perp$ in rhombus but not in rectangle (1)			
OR			
Diagonals bisect $\angle$ 's through which they pass in rhombus but not in rectangle		<b>Question 4</b>	
OR		1. (a) $P(\text{amber}) = \frac{20}{160}$	
Diagonals are equal in rectangle but not in rhombus		$= \frac{1}{8}$ (2)	
		(b) $P(\text{not red}) = 1 - P(\text{red})$	
		$= 1 - \frac{80}{160}$	
		$= \frac{1}{2}$ (2)	
		(c) $P(\text{amber or red}) = \frac{20+80}{160}$	
		$= \frac{100}{160}$	
		$= \frac{5}{8}$ (2)	
2.		(d) $P(\text{green or blue}) = \frac{60+40}{160}$	
a) $x+x+50 = 180$ ( $\angle$ sum isosc. $\Delta$ ABC)		$= \frac{100}{160}$	
$2x = 130$		$= \frac{5}{8}$ (2)	
$\therefore x = 65$			
$\angle CAD = \angle BCA$ (Alt. $\angle$ 's, AD//BC)			
$= 65$			
$\angle CAD + y + 30 = 180$ ( $\angle$ sum $\Delta$ APC)			
$65 + y + 30 = 180$			
$\therefore y = 85$ (4)			
b) $\angle DCB + 130 = 180$ (str. $\angle$ )			
$\angle DCB = 50$			
$50 + 3y + 2y + 90 = 360$ ( $\angle$ sum quad ABCD)			
$5y = 220$			
$y = 44$			
3. In $\Delta$ s AOC and BOC: (4)			
1. $OA = OB$ (equal radii)			
2. $OC$ is common side			
3. $\angle OCA = \angle OCB = 90^\circ$ (given $OC \perp AB$ )			
$\therefore \Delta AOC \equiv \Delta BOC$ (RHS)			
(b) $P(B) = \frac{6}{11} = \frac{1}{2}$ (1)			
(c) $P(CB) = \frac{0}{12} = 0$ (1)			





3. (a)  $P(<125) = \frac{2}{19}$  (1)

(b)  $P(\frac{130-139}{19}) = \frac{8}{19}$  (1)



(b)  $P(C \text{ but not } S) = \frac{24}{80}$   
 $= \frac{3}{10}$  (1)

(c)  $P(\text{not } C \text{ or } S) = \frac{36}{80}$   
 $= \frac{9}{20}$  (1)

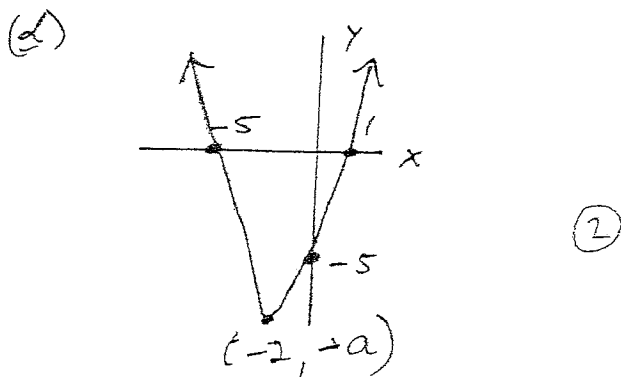
Question 5

1  $y = x^2 + 4x - 5$

(a) Let  $x = 0$   
 $y$  intercept =  $-5$  (1)

(b) Let  $y = 0$   
 $0 = (x+5)(x-1)$  (2)  
 $x = -5$  and  $1$  (x intercepts)

(c) Axis of sym  $x = \frac{-b}{2a}$   
 $x = \frac{-4}{2} = -2$   
 Sub into  $y = x^2 + 4x - 5$   
 $y = 4 - 8 - 5 = -9$   
 $\therefore$  Vertex =  $(-2, -9)$  (3)



2  $y = k(x-x_1)(x-x_2)$

$y = k(x+2)(x-4)$

Substitute  $(0, -16)$

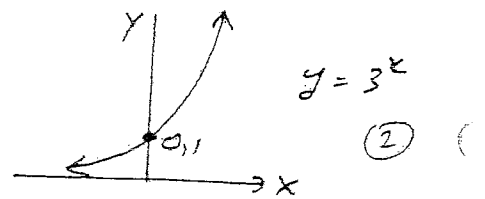
$-16 = k(2)(-4)$

$\therefore k = 2$

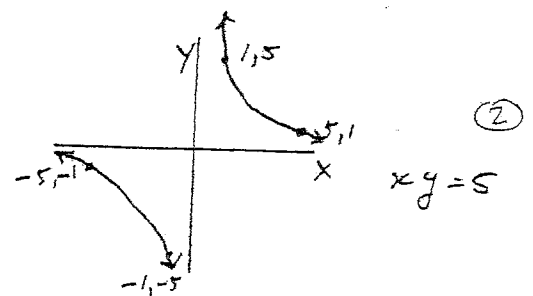
$y = 2(x+2)(x-4)$

$\therefore y = 2x^2 - 4x - 16$  (3)

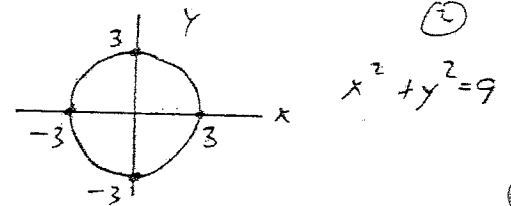
3 (a)



(b)



(c)



4

$y = -\frac{1}{20}x^2 + 3x$

Axis of sym  $x = \frac{-b}{2a}$

$x = \frac{-3}{(2 \times \frac{-1}{20})} = 30$

$y = -\frac{1}{20} \times 30^2 + 3 \times 30$

$= 45$

$\therefore$  Max. height = 45 km (3)