Sydney Girls High School

2003

Year 10

MATHEMATICS

Half Yearly Examination

Time allowed – 75 minutes

Instructions:

- There are 5 questions . Each of equal value (20 marks)
- Show all necessary working
- Marks will be deducted for untidy or badly arranged work.
- Start each question on a new page.
- Total marks : 100

Question 1. (20 marks)

(a) Calculate the simple interest on a loan of $5,460 at 8¼% p.a. for 3 years.

(b) What sum of money must I invest today at 6% p.a. compounded monthly which will amount to $20,000 in 10 years time?

(c) My car which cost $38,000 new depreciates by 10% p.a. What will it be worth in 5 years time?

(d) My home loan of $100,000 has an interest rate of 1% per month on the balance owing. How much do I owe after 2 months have passed and my monthly repayment is $1,500?

(e) Over a period of 4 years the kangaroo population at Shady Creek has dropped from 200 down to 170. Find the annual rate of decline in the population as a percentage to the nearest whole number .

Question 2. (20 marks)

Solve the following equations (correct to 2 decimal places if necessary)

1.

(a) $2x^2 - 72 = 0$

(b) $a^2 = 8a$

(c) $x^2 - 11x + 24 = 0$

(d) $2x^2 - x - 10 = 0$

(e) $\dfrac{3(a+1)}{a} = a$

2. Using the formula $D = ut + \frac{1}{2}at^2$
Find the positive value of t if $u = 4$, $a = 10$ and $D = 9$

3. In a right angled triangle the hypotenuse is 15cm. The second side is twice the length of the third side. Find the length of the shortest side correct to 1 decimal place.

4. Find the dimensions of a rectangle which has an area of $125m^2$ and a perimeter of 60m. (Dimensions are length and breadth).

Question 3. (20 marks)

1. My friend Sue has 3 children.

   (a) Draw a tree diagram to illustrate all the possible combinations of her children.

   (b) What is the probability that:-

   (i) She has 2 girls

   (ii) She has at least 2 boys.

2. Three cards are labelled (1) (2) (3) One card is selected and then returned and another card is then selected.

   (a) Draw a tree diagram to illustrate this experiment.

   (b) Find the probability that both cards are the same number.

   (c) Find the probability that both cards are different.

3. Out of a total of 50 sales made at a fashion shop, 20 were made on Monday and 30 on ~~Thursday~~ Tuesday.

   Of the Monday sales, 5 were made by Visa Card.

   Of the Tuesday sales, 10 were made by Visa Card.

   If one sale was selected at random, find the probability that:-

   (a) it was made by Visa Card

   (b) it was made on Tuesday

   (c) it was made by Visa Card given that it occurred on Monday

   (d) it was made by Visa Card and it occurred on Tuesday.

4. In a class of 30 girls all students do History or Geography or both. If 16 do History and 18 do Geography and we select one student at random. Find:-

   (a) The probability that she does both subjects

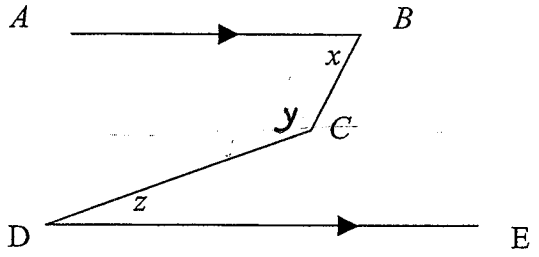
   (b) The probability that she does only Geography

   (c) The probability that she does either History or Geography but not both.

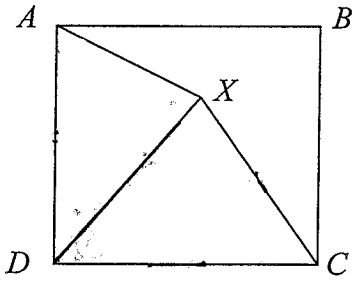
Question 4 (20 marks)

1. Find the sum of the interior angles of a regular polygon with 8 sides.
Hence find the size of each angle.

2. In figure $AB \parallel DE$
Find the value of $(x + y - z)$
(Reasons need not be given).

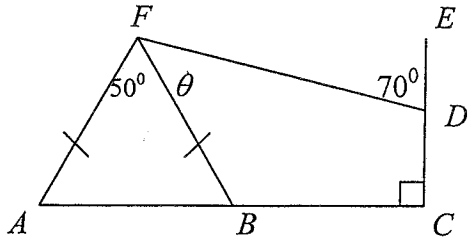


3. $ABCD$ is a square and CDX is an equilateral triangle
AX has been joined. Find the size of $\angle XAD$
(Reasons need not be given).



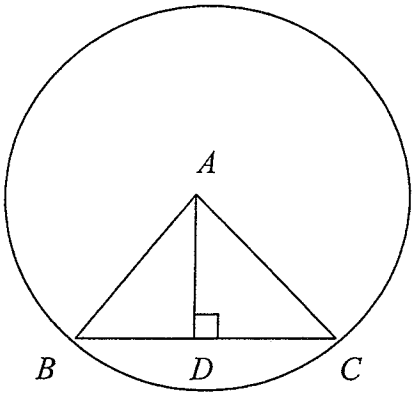
4. (i) Copy this diagram

(ii) Find the value of θ. Give reasons.



5. (i) Copy this diagram

(ii) Prove that the line AD, draws from the centre of the circle (A) to meet the chord BC at right angles at point D bisects BC. Give reasons.



Question 5 **(20 marks)**

1. Sketch the following curves marking one critical point on each curve.

   (a) $x^2 + y^2 = 4$

   (b) $xy = 4$

   (c) $y = 2^{-x}$

   (d) $y = 2 - x^2$

2. A parabola has x intercept $= 2$ and y intercept $= 4$. Find its equation.

3. For the parabola $y = 2x^2 - 8x - 10$. Find:-

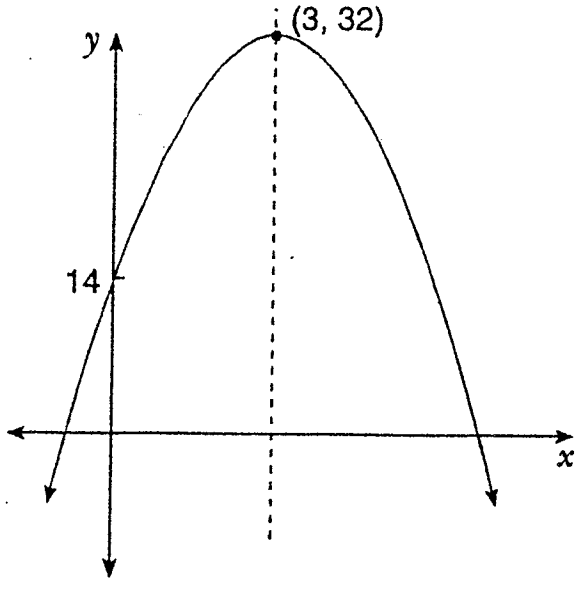
   (a) the axis of symmetry

   (b) the vertex

   (c) the concavity type (up or down)

   (d) the minimum value

4. Find the equation of the parabola below.



- End of Exam -

Sydney Girls High School

Stephanie Pham
Excellent work!
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2003

SOLUTIONS

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Instructions:

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SOLUTIONS

Question 1. (20 marks)

(a) Calculate the simple interest on a loan of \$5,460 at 8¼% p.a. for 3 years.
$I = Prn = \$1,351.35$ ✓

(b) What sum of money must I invest today at 6% p.a. compounded monthly which will amount to \$20,000 in 10 years time?
$A = P(1+r)^n$ $20000 = 1.8P$
$20000 = P(1.005)^{120}$ $P = \$10,992.65$ ✓

(c) My car which cost \$38,000 new depreciates by 10% p.a. What will it be worth in 5 years time? $A = P(1-r)^n$
$= 38000\ (0.9)^5$
$= \$22,438.62$ ✓

(d) My home loan of \$100,000 has an interest rate of 1% per month on the balance owing. How much do I owe after 2 months have passed and my monthly repayment is \$1,500?
$I_1 = 0.01 \times 100\,000$ $I_2 = 0.01 \times 99\,500$ ✓ $= \$998.995$ ✓
$= \$1000$ ✓ $= \$995$.
$A_1 = \$99\,500$ ✓ $A_2 = 100\,495 - 1500$ ✓

(e) Over a period of 4 years the kangaroo population at Shady Creek has dropped from 200 down to 170. Find the annual rate of decline in the population as a percentage to the nearest whole number .
$\frac{30}{200} \times 100 = 15$ ∴ annual rate = 4% (nearest whole no.) ✓
$15 \div 4 = 3.75$ ✓

Question 2. (20 marks)

Solve the following equations (correct to 2 decimal places if necessary)

1.
(a) $2x^2 - 72 = 0$
$x^2 - 36 = 0$
$(x-6)(x+6) = 0$
$\therefore x = 6$ or -6 ✓

(b) $a^2 = 8a$
$a^2 - 8a = 0$
$(a-8)(a+0) = 0$
$\therefore a = 8$ or 0 ✓

(c) $x^2 - 11x + 24 = 0$
$(x-8)(x-3) = 0$
$\therefore x = 8$ or 3 ✓

(d) $2x^2 - x - 10 = 0$
$2x^2 - 5x + 4x - 10 = 0$
$2x(x+2) - 5(x+2) = 0$
$(2x-5)(x+2) = 0$
$\therefore x = \frac{5}{2}$ or -2 ✓

(e) $\frac{3(a+1)}{a} = a$
$3a + 3 = a^2$
$a^2 - 3a - 3 = 0$
$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$= \frac{3 \pm \sqrt{9+12}}{2}$
$= \frac{3 \pm \sqrt{21}}{2}$
$\therefore a = 3.79$ or -0.79 (2 d.p.) ✓

2. Using the formula $D = ut + \frac{1}{2}at^2$
Find the positive value of t if $u = 4$, $a = 10$ and $D = 9$
$9 = 4t + 5t^2$
$5t^2 + 4t - 9 = 0$
$5t^2 + 9t - 5t - 9 = 0$
$5t(t-1) + 9(t-1) = 0$
$(5t+9)(t-1) = 0$
$\therefore t = -\frac{9}{5}$ or 1
∴ the positive value of t is 1 ✓

3. In a right angled triangle the hypotenuse is 15cm. The second side is twice the length of the third side. Find the length of the shortest side correct to 1 decimal place.

Let the shortest side be x, $x^2 - 45 = 0$
$x^2 + (2x)^2 = 15^2$ $x^2 = 45$ ✓
$x^2 + 4x^2 - 225 = 0$ $x = 6.708$
$5x^2 - 225 = 0$ ∴ the length of the shortest side is 6.7cm (1 d.p.) ✓

4. Find the dimensions of a rectangle which has an area of $125m^2$ and a perimeter of 60m. (Dimensions are length and breadth).
5m x 25m ✓

Question 3. (20 marks)

1. My friend Sue has 3 children.

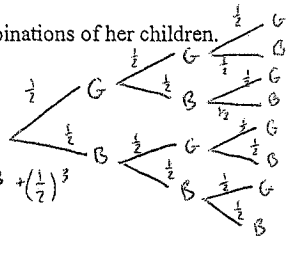
(a) Draw a tree diagram to illustrate all the possible combinations of her children.

(b) What is the probability that:-

(i) She has 2 girls $= \frac{1}{4}$ ✓

(ii) She has at least 2 boys. $= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3$

$= \frac{1}{2}$ ✓



2. Three cards are labelled (1) (2) (3) One card is selected and then returned and another card is then selected.

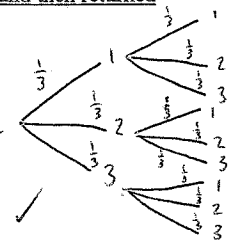
(a) Draw a tree diagram to illustrate this experiment.

(b) Find the probability that both cards are the same number. $= \frac{1}{9} \times 3$

$= \frac{1}{3}$ ✓

(c) Find the probability that both cards are different.

$= \frac{1}{9} \times 6 = \frac{2}{3}$ ✓



3. Out of a total of 50 sales made at a fashion shop, 20 were made on Monday and 30 on Thursday. Tuesday.

Of the Monday sales, 5 were made by Visa Card.

Of the Tuesday sales, 10 were made by Visa Card.

If one sale was selected at random, find the probability that:-

(a) it was made by Visa Card $\frac{1}{10}$ ✓

(b) it was made on Tuesday 0 ✓

(c) it was made by Visa Card given that it occurred on Monday $\frac{1}{4}$ ✓

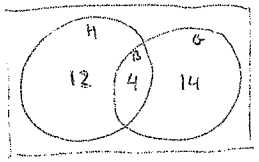
(d) it was made by Visa Card and it occurred on Tuesday. 0 ✓

4. In a class of 30 girls all students do History or Geography or both. If 16 do History and 18 do Geography and we select one student at random. Find:-

(a) The probability that she does both subjects $\frac{2}{15}$ ✓

(b) The probability that she does only Geography $\frac{7}{15}$ ✓

(c) The probability that she does either History or Geography but not both. $\frac{13}{15}$ ✓

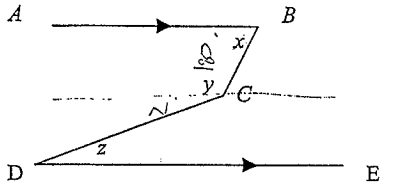


Question 4. (20 marks)

1. Find the sum of the interior angles of a regular polygon with 8 sides. 1080° ✓
Hence find the size of each angle. 135° ✓

2. In figure $AB \parallel DE$
Find the value of $(x + y - z)$
(Reasons need not be given).

180° ✓



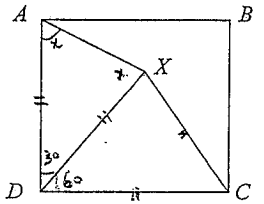
3. $ABCD$ is a square and CDX is an equilateral triangle
AX has been joined. Find the size of $\angle XAD$
(Reasons need not be given).

$180 - 30 = 150$

$DC = 150 \div 2$

$= 75^\circ$ ✓

$\therefore \angle XAD$

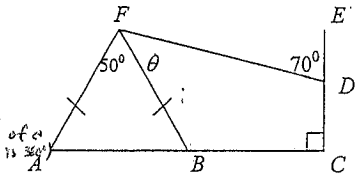


4. (i) Copy this diagram

(ii) Find the value of θ. Give reasons.

$\angle FBA = 65^\circ$ (base $\angle$s isos. $\triangle$) ✓
$\angle FBC = 115^\circ$ (supp. $\angle$)
$\angle FDC = 110^\circ$ (")

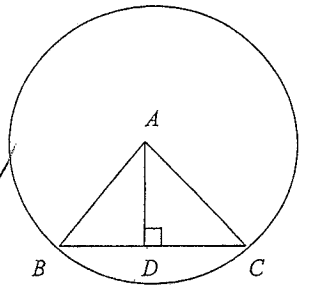
$\theta = 360 - (115 + 90 + 110)$ ($\angle$ sum of a quad. is 360°)
$= 360 - 315$
$= 45^\circ$ ✓



5. (i) Copy this diagram

(ii) Prove that the line AD, drawn from the centre of the circle (A) to meet the chord BC at right angles at point D bisects BC. Give reasons.

In $\triangle ADB$ & $\triangle ADC$,
$\angle ADB = \angle ADC$ (both 90°) ✓
AD is common
$AB = AC$ (radii of circle) ✓
$\therefore \triangle ADB \equiv \triangle ADC$ (SAS) ✓
$\therefore BD = DC$ (corr. sides in $\cong$ $\triangle$s) ✓



Question 5 (20 marks)

1. Sketch the following curves marking one critical point on each curve.

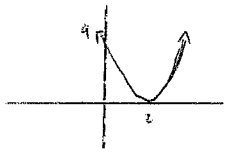
   (a) $x^2 + y^2 = 4$

   (b) $xy = 4$

   (c) $y = 2^{-x}$

   (d) $y = 2 - x^2$

2. A parabola has x intercept $= 2$ and y intercept $= 4$. Find its equation.



$y = a(x-2)^2$ $\quad$ $4 = a(0-2)^2$ $\quad$ $a = 1$ $\quad$ $\therefore$ equation is $y = 1(x-2)^2$ ✓

3. For the parabola $y = 2x^2 - 8x - 10$. Find:-

   $= 2x^2 - 10x + 2x - 10$ $\quad$ $= 2x(x+1) - 10(x+1)$ $\quad$ $= (2x-10)(x+1)$ $\quad$ $\therefore x = 5$ or -1 ✓

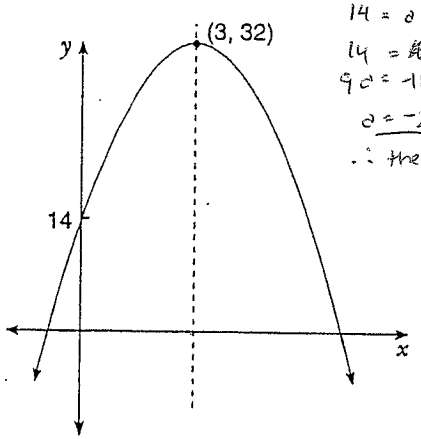
   (a) the axis of symmetry $x = 2$ ✓

   (b) the vertex $y = 2\times4 - 16 - 10 = -18$ $\quad$ vertex $= (2, -18)$ ✓

   (c) the concavity type (up or down) up ✓

   (d) the minimum value $(2, -18)$ & $y_{min} = -18$.

4. Find the equation of the parabola below.



$y = a(x-3)^2 + 32$

$14 = a(0-3)^2 + 32$ ✓

$14 = 9a + 32$

$9a = -18$

$a = -2$ ✓

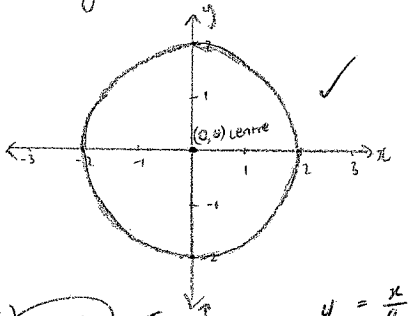
$\therefore$ the equation of the parabola is $y = -2(x-3)^2 + 32$ ✓

- End of Exam -

Question 5

Stephanie Mem

1. (a) $x^2 + y^2 = 4$

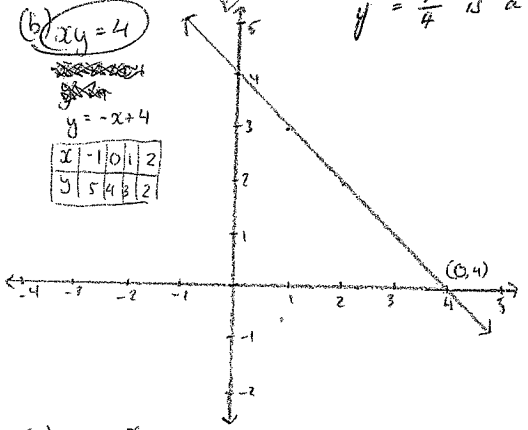


✓

(b) $xy = 4$ $\quad$ $y = \frac{x}{4}$ is a hyperbola thru origin

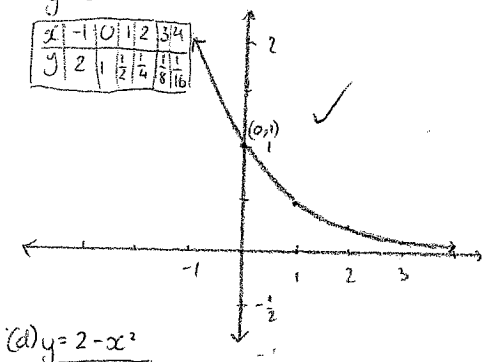
$y = -x + 4$

x	-1	0	1	2
y	5	4	3	2



(c) $y = 2^{-x}$

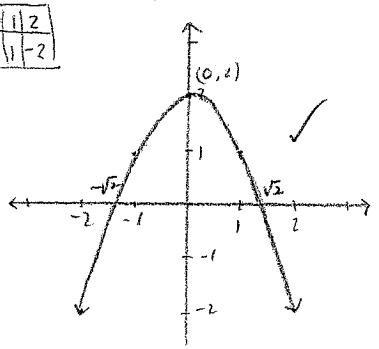
x	-1	0	1	2	3	4
y	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$



✓

(d) $y = 2 - x^2$

x	-1	0	1	2
y	1	2	1	-2



✓