SYDNEY GIRLS HIGH SCHOOL

YEAR 10 MATHEMATICS

Common Test 1

March 2003

Time allowed: 60 minutes

Topics: Geometry,Probability and Quadratic equations

Instructions:

- There are Five(5) questions. Questions are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work.
- Start each question on a new page. Write on one side of the paper only.

Name: ....

QUESTION ONE (12 Marks)

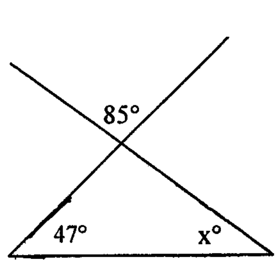
a) Solve the following equations

i) $(y-3)(y+1)=0$
ii) $1-x^2=0$
iii) $3a^2=a$

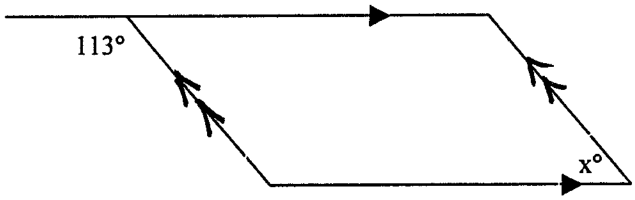
b) In an international tennis competition , there are 5 Australian competitors in a field of 16. If the names for the first round matches are drawn randomly, what is the probability that two Australians will be drawn first.

c) Find the value of x in each of the following, reasoning not required.

i)



ii)



QUESTION TWO (12 Marks)

a) Solve the following equations

i) $x^2-2x-15=0$

ii) $10x^2+3x-4=0$

iii) $y+\frac{2}{y}=\frac{9}{2}$

b) In a class of 30 students, there are 21 who like geometry and 16 who like trigonometry. If 6 students don't like either , draw a Venn diagram and find the probability that a student likes both.

c) The probability that three students, Adele, Cathy and Lili will pass their year 10 Science exam is $\frac{6}{7},\frac{3}{4}$ and $\frac{2}{3}$ respectively.

i) What is the probability that all three will pass?
ii) What is the probability that only Cathy passes the exam?

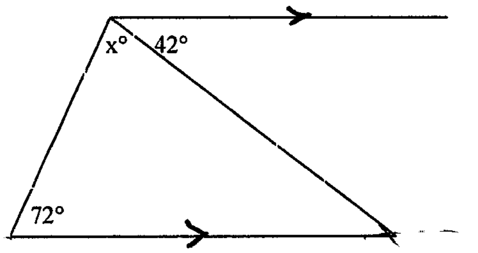
QUESTION THREE (12 Marks)

a) Solve the following equation by completing the square.

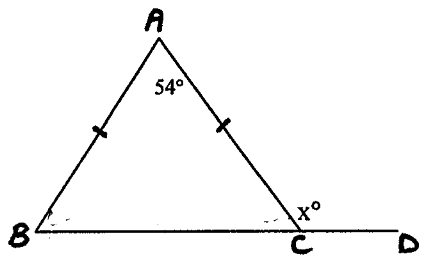
$$x^2 - 2x = 5$$

b) Find the value of x in each of the following , giving reason.

i)



ii)



c) Use the quadratic formula to solve $-5x^2 + 7x = -3$. Give your correcto two decimal places.

QUESTION FOUR (12 Marks)

a) A bag contains 4 red balls and 6 green balls.A ball is selected at random.

i) What is the probability that the ball is green?

ii) What is the probability that the ball is red or green?

iii) What is the probability that the ball is not green?

iv) The first ball is removed and replaced with a ball of the other colour, and then a second ball is randomly selected. Draw a probability tree diagram to show the outcomes.

v) What is the probability that both balls selected are red?

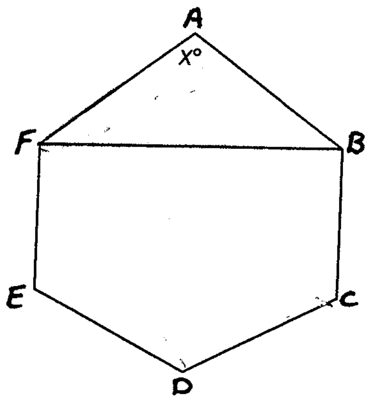
vi) Find the probability that the second ball selected is red.

b)The sum of the squares of two consecutive even positive integers is 452. Find the integers. (let the first number be n)

QUESTION FIVE (12 Marks)

a) Solve $\frac{18}{2x+1}=\frac{5+x}{x}$

b) A diagonal of this regular hexagon has been drawn. Find the value of x and y. Give reasons.



c) In triangle ABC, angle B equals to angle C. A perpendicular is drawn from A to BC, meeting it at D.

i) Draw a neat sketch of these information

ii) Prove that triangles ABD and ACD are congruent.

iii) Hence or otherwise show that the perpendicular AD, bisects BC.

Question 1 (12 marks)

1(a)(i) $(y-3)(y+1)=0$

$y-3=0$ or $y+1=0$

$y=3$ or $y=-1$

(ii) $1-x^2=0$

$(1-x)(1+x)=0$

$1-x=0$ or $1+x=0$

$1=x$ or $x=-1$

$\therefore x=\pm 1$

(iii) $3a^2=a$

$3a^2-a=0$

$a(3a-1)=0$

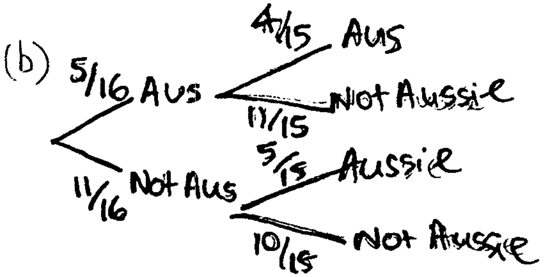
$a=0$ or $3a-1=0$

$3a=1$

$a=\frac{1}{3}$

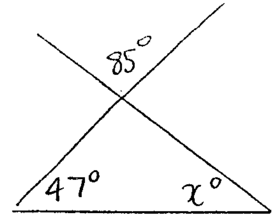
$\therefore a=0$ or $a=\frac{1}{3}$

(b)



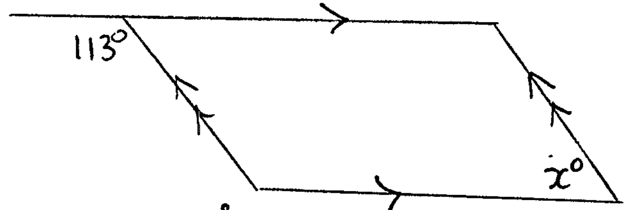
$P(\text{Aussie, Aussie}) = \frac{5}{16} \times \frac{4}{15}$ $= \frac{1}{4}\times\frac{1}{3}$

1(c)



$x° + 47° + 85° = 180°$

$x = 48°$



$x° + 113° = 180$ $\therefore x = 67°$

Question 2 (12 marks)

(a)(i) $x^2-2x-15=0$

$(x-5)(x+3)=0$

$x-5=0$ or $x+3=0$

$x=5$ or $x=-3$

2

(ii) $10x^2+3x-4=0$ $\quad -40$

Think of two numbers whose:

Product $= -40$

Sum $= 3$

the numbers are 8 and -5

$\frac{(10x+8)(10x-5)}{10}=0$

$2(5x+4)5(2x-1)=0$

2(a)(ii) (cont)

$\frac{\cancel{10}(5x+4)(2x-1)}{\cancel{10}}=0$

$(5x+4)(2x-1)=0$

$5x+4=0$ or $2x-1=0$

$5x=-4$ or $2x=1$

$x=\frac{-4}{5}$ $\quad x=\frac{1}{2}$

2

(iii) $y+\frac{2}{y}=\frac{9}{2}$

multiply both sides of eqn by y

$y^2+2=\frac{9y}{2}$

Now multiply both sides by 2

$2y^2+4=9y$

$2y^2-9y+4=0$ $\quad 8$

Think of two numbers whose:

Product $= 8$ Sum $= -9$

the numbers are -8 and -1

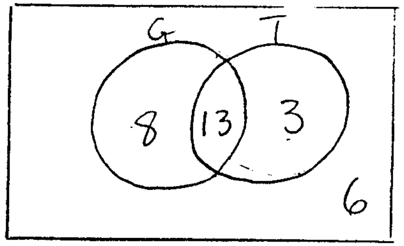
$\frac{(2y-8)(2y-1)}{2}=0$

$\frac{\cancel{2}(y-4)(2y-1)}{\cancel{2}}=0$

$(y-4)(2y-1)=0$

2

Q2(b)



1 mark

G: Geometry T: Trigonometry

$30 - 6 = 24$

$21 + 16 = 37$

$37 - 24 = 13$

$P(\text{student likes both}) = \frac{13}{30}$

1 mark

(c)(i) $P(\text{all 3 pass}) = \frac{6}{7} \times \frac{3}{4} \times \frac{2}{3}$

$= \frac{3}{7}$ 2 marks

Note: 1 mark given if you had written $\frac{6}{7} + \frac{3}{4} + \frac{2}{3}$

(ii) P(only Cathy Passes)

$= P(\bar{A}\, C\, \bar{L}) = \frac{1}{7} \times \frac{3}{4} \times \frac{1}{3}$

$= \frac{1}{28}$ 2 marks

Question 3 (12 marks)

(a) Solve by completing the square

$x^2 - 2x = 5$

$x^2 - 2x + (-1)^2 = 5 + (-1)^2$

Note: half of the co-efficient of x is -1.

$x^2 - 2x + 1 = 6$

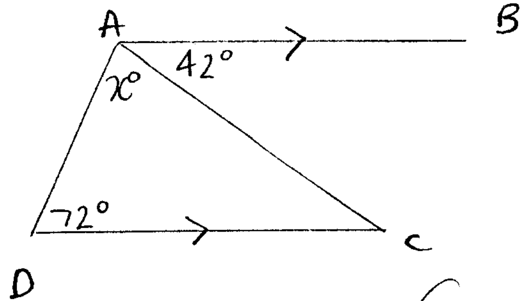
$(x-1)^2 = 6$

$x - 1 = \pm\sqrt{6}$

$x = 1 \pm \sqrt{6}$

3 marks

(b)(i)



3 marks

One possible method:

$x° + 42° + 72° = 180°$ (co-interior ∠'s and parallel lines) i.e, AB ∥ DC

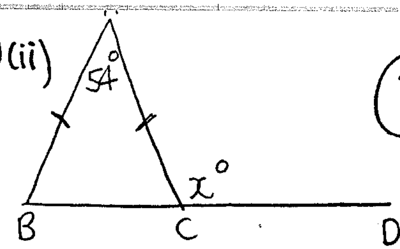
$x = 180 - 114$

$= 66°$

Another method: $\angle ACD = 42°$ (alternate ∠'s and AB ∥ DC)

$x° + 42° + 72° = 180$ (angle sum of

(b)(ii)



3 marks

$\angle ABC = \angle ACB = y$ (base angles of isosceles Δ)

$2y° + 54° = 180°$ (angle sum of Δ)

$2y = 180° - 54°$

$= 126°$

$y = 63°$

$y° + x° = 180°$ (supplementary ∠'s)

$x = 180 - 63$

$= 117°$

OR/ $\angle ABC = \angle ACB = y$ (base angles of isosceles Δ)

$2y + 54 = 180$ (angle sum of Δ)

$y = 63°$

$63° + 54° = x°$ (exterior angle of Δ)

$117 = x$

(c) $-5x^2 + 7x = -3$

$-5x^2 + 7x - 3 = 0$ $a = -5$ $b = 7$ $c = -3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 3 marks

$= \frac{-7 \pm \sqrt{49 - 4(-5 \times -3)}}{-10}$

$= \frac{-7 \pm \sqrt{109}}{-10}$

(i) $P(\text{Green}) = \frac{6}{10}$

$= \frac{3}{5}$

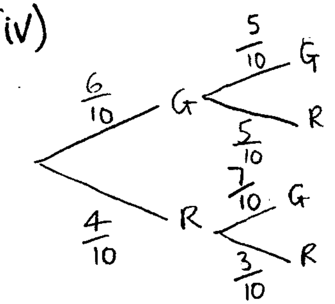
(ii) $P(\text{red or green}) = \frac{4}{10} + \frac{6}{10}$

$= \frac{10}{10} = 1$

(iii) $P(\text{not green}) = \frac{4}{10}$

$= \frac{2}{5}$

(iv)



GG

GR

RG

RR

(v) $P(\text{both red}) = P(RR)$

$= \frac{4}{10} \times \frac{3}{10}$

$= \frac{12}{100}$

$= \frac{3}{25}$

(vi) $P(2^{nd} \text{ ball is red}) = P(GR \text{ or } RR)$

$= \left(\frac{6}{10} \times \frac{5}{10}\right) + \left(\frac{4}{10} \times \frac{3}{10}\right)$

$= \frac{3}{10} + \frac{3}{25}$

let n be the first number and $n+2$ be the second number

$(n)^2 + (n+2)^2 = 452$

$n^2 + n^2 + 4n + 4 = 452$

$2n^2 + 4n - 448 = 0$

$n^2 + 2n - 224 = 0$

$(n+16)(n-14) = 0$

$n+16=0$ or $n-14=0$

$n=-16$ $\quad n=14$

n must be positive

$\therefore$ the numbers are

$n = 14$

and $n+2 = 14+2$

$= 16$

i.e. 14 and 16

Question 5 (12 marks)

(a) Solve $\frac{18}{2x+1} = \frac{5+x}{x}$

$18x = (5+x)(2x+1)$

$18x = 10x + 5 + 2x^2 + x$

$0 = 2x^2 + 11x - 18x + 5$

$0 = 2x^2 - 7x + 5$

Think of two numbers whose:

product $= 10$

sum $= -7$

the numbers are -5 and -2

$\frac{(2x-5)(2x-2)}{2} = 0$

$\frac{(2x-5)\cancel{2}(x-1)}{\cancel{2}} = 0$

$(2x-5)(x-1) = 0$

$2x-5=0$ or $x-1=0$

$2x=5$ or $x=1$

$x=\frac{5}{2}$

(b) angle sum of a hexagon $= (6-2) \times 180°$

$= 4 \times 180°$

$= 720°$

$\therefore x = \frac{720°}{6}$

$= 120°$

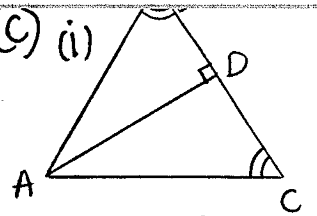
$\angle AFB = \angle ABF = y°$ (base angles of isosceles Δ)

$2y + 120 = 180°$ (angle sum of Δ)

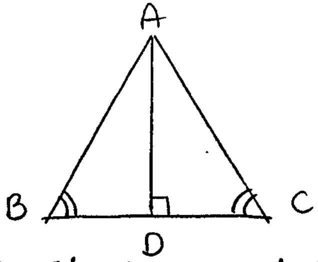
$2y = 60°$

$y = 30°$

(c) (i)



or



In Δ's ABD and ACD

(ii) $\angle ABD = \angle ACD$ (given)

AD is common

$\angle ADB = \angle ADC = 90^\circ$ ($AD \perp BC$)

$\therefore \Delta ABD \equiv \Delta ACD$ (AAS)

iii) Hence or otherwise show that the perpendicular AD bisects BC

$BD = DC$ (corresponding sides of congruent Δ's)

$\therefore$ AD bisects BC