Sydney Girls High School

Year 10

MATHEMATICS

Yearly Examination 2007

Time Allowed: 90 minutes

Instructions:

- There are FIVE (5) questions, of equal value.
- Full marks may not be awarded for careless or incomplete working.
- Start each question on a new page.

TOTAL: 100 marks

QUESTION 1: (20 marks)

a) For the scores:

12 17 11 12 15 16 16 19

i. Find the range. **1**

ii. Find the mode. **1**

iii. Find the median. **1**

iv. Find the mean (to 2 decimal places). **2**

v. Calculate the standard deviation (to 1 decimal place). **2**

vi. Find the interquartile range. **2**

vii. Draw a box and whiskers plot. **3**

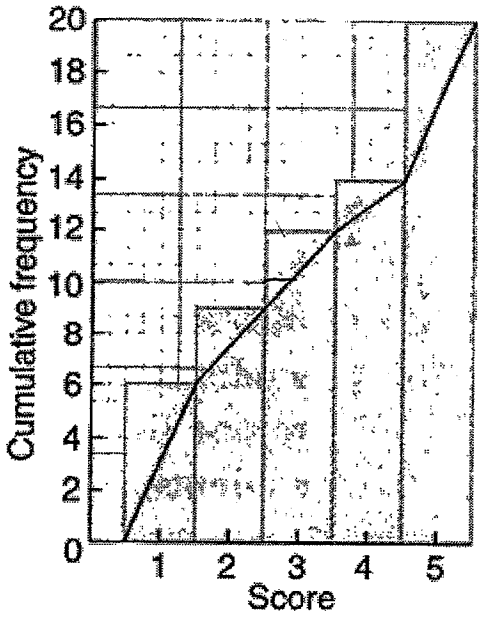
b) If a score of 13 is added to the sample below, which measure (median, range, mode or mean) will change? **1**

Score	*Frequency*
11	5
12	4
13	1
14	6
15	4

c) Find the median of the scores below: **2**

Stem	*Leaf*
1	0 3 6
2	4 4 6 7 9
3	3 5 6 7 8 9
4	6
5	5

d)



Using the graph above:

i. find the median. **1**

ii. find the interquartile range. **2**

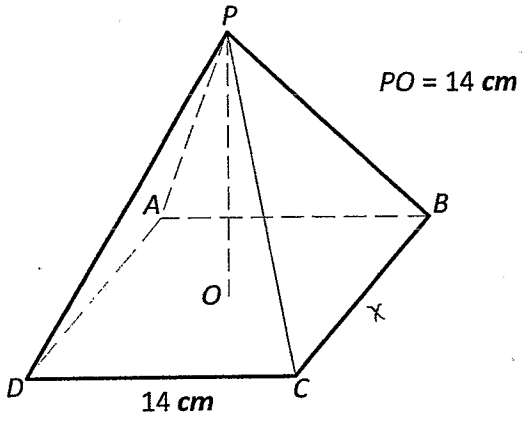
e) The statistics for Meredith's class test marks in three subjects are as follows:

Test	*Meredith's mark*	*Mean*	*Standard deviation*
English	80	75	5
Maths	80	55	15
Science	80	60	10

In which test did Meredith perform best, compared to the class? Give reasons. **2**

QUESTION 2: (20 marks)

a) Find the surface area of the square pyramid below, to 1 decimal place: **3**



b) The surface area of a closed cylinder is 120π cm^2 and the radius is 4 cm. Find the exact height of the cylinder. **2**

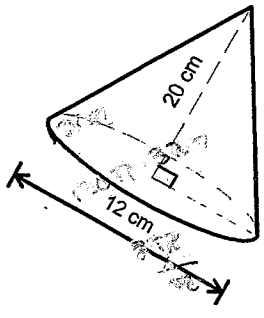
c) A chocolate orange with a radius of 4 cm is to be covered in silver foil. Alex thinks that this could be done using a rectangular piece of paper measuring 16 cm by 12 cm. Explain, using calculations, whether the piece of foil is large enough. **3**

d) A cone has a diameter of 12 cm and a perpendicular height of 8 cm. Find:

i. the slant height of the cone; **2**

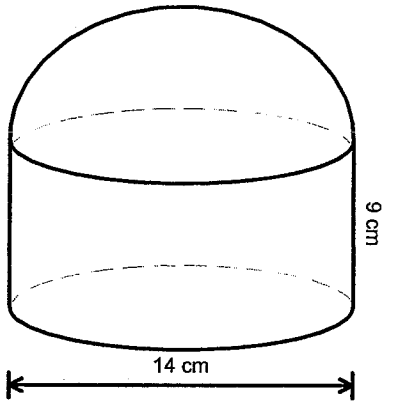
ii. the surface area of the cone, to 2 decimal places. **2**

e) Find the exact volume of the cone below: **2**



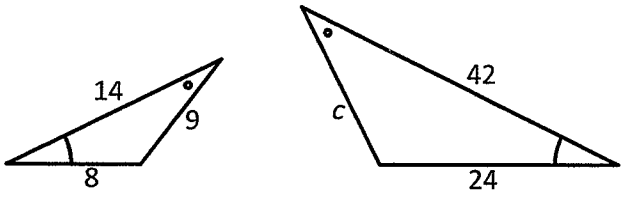
f) A square pyramid has a height of 6 cm and a volume of 50 cm^3. Find the side length of the base, to 1 decimal place. **2**

g) Calculate the volume of the solid below, correct to the nearest cm^3: **4**

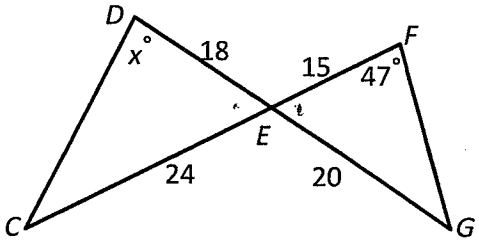


QUESTION 3: (20 marks)

a) The triangles below are similar. Find the value of *c*. All lengths are in mm. **2**



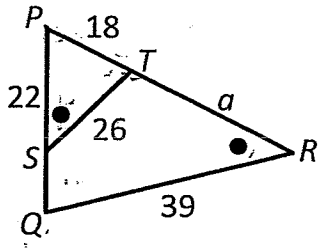
b)



i. Prove that $\Delta CDE \mid\mid\mid \Delta GFE$. **3**

ii. Hence, find the value of *x*. **1**

c)

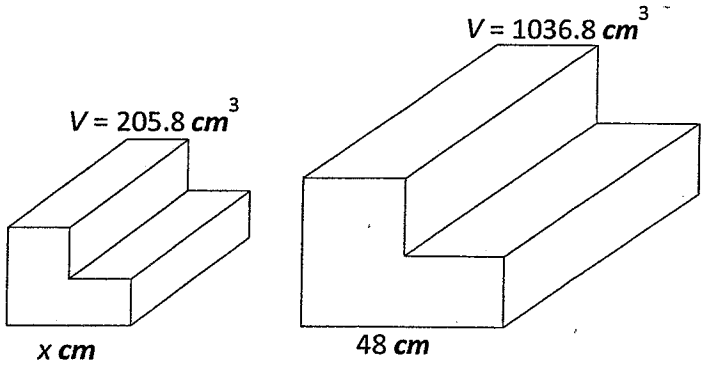


i. Prove that ΔPST is similar to ΔPRQ. **3**

ii. Hence, find the value of a. **2**

d) A building 20 m tall casts a shadow of length 5 m when the sun is almost directly overhead. At the same time, a tree of height 12 m casts a shadow in the same line as the shadow cast by the building. Find the length of the shadow cast by the tree. **4**

e) The solids below are similar:



i. Find the ratio of the sides in its simplest form. **3**

ii. Find the value of x. **2**

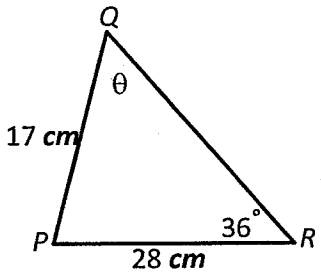
QUESTION 4: (20 marks)

a) Find all possible values of θ, where $0° \le \theta \le 180°$, correct to the nearest minute if:

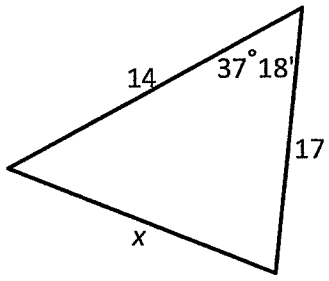
i. $\tan\theta = -0.641$ **2**

ii. $\sin\theta = 0.837$ **2**

b) Find the size of the acute angle θ, correct to the nearest minute: **3**



c) Find the value of x, correct to 1 decimal place. Measurements are in cm. **3**



d) In an isosceles triangle ΔPQR, $PQ = QR$, $\angle PQR = 40°$ and $PR = 37$ cm. Find the length of the equal sides, correct to the nearest centimetre. **3**

e) Addison drove 37 km from A to B on a bearing of $062°$. She then turned and drove for 54 km on a bearing of $112°$ to C. Find the distance AC. **4**

f) In ΔLMN, $\angle M$ is obtuse, MN = 13 cm and ML = 8 cm. The area of the triangle is 13 cm^2. Find the angle θ, correct to the nearest degree. **3**

QUESTION 5: (20 marks)

a) The weight, W of a man varies inversely as the square of his distance D from the centre of the Earth. A man weighs 70 kg on the surface of the Earth. Take the radius of the Earth to be 6 400 km.

i. Find a formula relating W and D. **2**

ii. How much would this man weigh when 200 km above the surface of the Earth? Give the answer correct to 2 decimal places. **2**

b) Find the rule for the quadratic relationship below: **4**

x	1	2	3	4
y	2	7	14	23

c) Solve these literal equations for a:

i. $K = n\sqrt{\frac{a}{p}}$ **3**

ii. $L = \frac{3a}{a - m}$ **3**

d) Solve by substitution for x: $9^x - 12\left(3^x\right) + 27 = 0$ **2**

e)

i. Factorise $x^2 - 6x + 9$. **1**

ii. Hence factorise fully $x^4 - x^2 + 6x - 9$ **3**

END OF TEST

LEVEL: _____ YEAR: 10

EXAM TYPE: Yearly (2007)

Question 1:

M5 Mas

a)

i) Range = 19 − 11

= 8

ii) Mode = 12 and 16

iii)

11, 12, 12, 15, 16, 16, 17, 19

Median = $\frac{15+16}{2}$

= 15·5

iv) $\bar{x}$ = 14·75

v) σ = 2·6 (1 dec. pl)

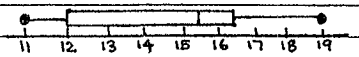
vi) Q_1 = 12 Q_3 = 16·5

I.Q.R = $Q_3 - Q_1$

= 16·5 − 12

= 4·5

vii)



b)

Median

c) Median = $\frac{29+33}{2}$

= $\frac{62}{2}$

= 31

d)

i) Median = 3

ii) I.Q.R = 5 − 1

= 4

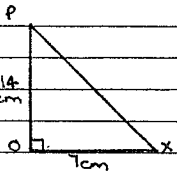
e)

Science. She is 2 standard deviations above the mean. The more standard deviations above the mean, the better the mark is.

Question 2:

a) Area base = 14 × 14

= 196 cm²



$PX^2 = 14^2 + 7^2$

= 196 + 49

= 245

$PX = \sqrt{245}$ cm

Area $\Delta DPC = \frac{1}{2} \times 14 \times \sqrt{245}$

$= 7\sqrt{245}$ cm²

∴ S.A pyramid = $196 + 4 \times 7\sqrt{245}$

$= 196 + 28\sqrt{245}$

= 634·3 cm²

(1 dec. pl)

b) S.A = $2\pi r^2 + 2\pi rh$

$120\pi = 2\pi \times 4^2 + 2\pi \times 4 \times h$

$= 32\pi + 8\pi h$

15 = 4 + h

h = 11 cm

∴ height of cylinder is 11 cm.

c)

S.A = $4\pi r^2$

$= 4 \times \pi \times 4^2$

$= 64\pi$

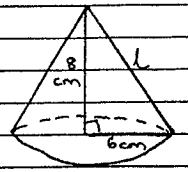
= 201·1 cm² (1 dec. pl)

Area rectangle = 16 × 12

= 192 cm²

∴ Foil is not large enough to cover the chocolate orange.

d)



i) $l^2 = 8^2 + 6^2$

= 64 + 36

= 100

l = 10 cm

∴ slant height = 10 cm

ii) $A = \pi r^2 + \pi rl$

$= \pi \times 6^2 + \pi \times 6 \times 10$

$= 36\pi + 60\pi$

$= 96\pi$ cm²

= 301·6 cm² (1 dec. pl)

e) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6^2 \times 20$

$= 240\pi$ cm³

f) $V = \frac{1}{3}Ah$

$50 = \frac{1}{3} \times A \times 6$

2A = 50

A = 25 cm²

As $A = s^2$

∴ $s^2 = 25$

s = 5 cm

∴ side length of base is 5cm

g) Vol. cylinder = $\pi r^2 h$

$= \pi \times 7^2 \times 9$

= 441 cm³

Vol. hemisphere = $\frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times 7^3$

$= \frac{686\pi}{3}$ cm³

∴ Vol. solid = $441\pi + \frac{686\pi}{3}$

= 2104 cm³

(nearest cm³)

Question 3:

a) $\frac{c}{9} = \frac{42}{14}$

$\frac{c}{9} = 3$

c = 27 mm

b)

i) $\frac{DE}{FE} = \frac{18}{15} = \frac{6}{5}$ (given)

$\frac{CE}{GE} = \frac{24}{20} = \frac{6}{5}$ (given)

$\angle DEC = \angle FEG$ (vert. opp)

$\therefore \triangle CDE \,|||\, \triangle GFE$ (pair of corres. sides in the same ratio and their included angles are equal)

ii) $x = 47°$ (corres. $\angle$s of similar triangles are equal).

c)

i) In $\triangle PST$ and $\triangle PRQ$:

$\angle SPT = \angle RPQ$ (common)

$\angle PST = \angle PRQ$ (given)

$\therefore \triangle PST \,|||\, \triangle PRQ$ (equiangular)

ii) $\frac{PT}{PQ} = \frac{PS}{PR} = \frac{ST}{QR}$ (corres. sides of similar triangles in the same ratio)

$\frac{22}{18+a} = \frac{26}{39}$

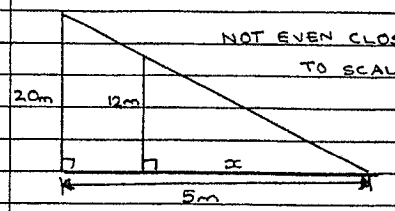
$\frac{22}{18+a} = \frac{2}{3}$

$66 = 2(18+a)$

$33 = 18+a$

$a = 15$

d)



Let tree's shadow = x

$\frac{x}{5} = \frac{12}{20}$

$20x = 60$

$x = 3$

$\therefore$ tree's shadow is 3m long.

e)

i) $\frac{V_S}{V_L} = \frac{205 \cdot 8}{1036 \cdot 8}$

$= \frac{343}{1728}$

$\therefore$ Ratio of sides $= \frac{\sqrt[3]{343}}{\sqrt[3]{1728}}$

$= \frac{7}{12}$

ii) $\frac{x}{48} = \frac{7}{12}$

$12x = 336$

$x = 28$

Question 4:

a)

i) $\tan\theta = -0 \cdot 641$

if $\tan\theta = 0 \cdot 641$

$\theta = 32° 40'$ (nearest minute)

but $\tan\theta < 0$

$\therefore \theta = 180° - 32° 40'$

$= 147° 20'$

ii) $\sin\theta = 0 \cdot 837$

$\theta = 56° 49', 123° 11'$

b) $\frac{\sin\theta}{28} = \frac{\sin 36°}{17}$

$\sin\theta = \frac{28 \sin 36°}{17}$

$\theta = 75° 30'$

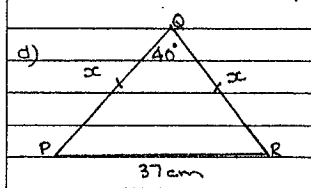
c)

$c^2 = a^2 + b^2 - 2ab\cos C$

$x^2 = 14^2 + 17^2 - 2 \times 14 \times 17 \times \cos 37° 18'$

$x = 10 \cdot 3$ cm (1 dec. pl)

d)



Let $PQ = QR = x$

$c^2 = a^2 + b^2 - 2ab\cos C$

$37^2 = x^2 + x^2 - 2x^2\cos 40°$

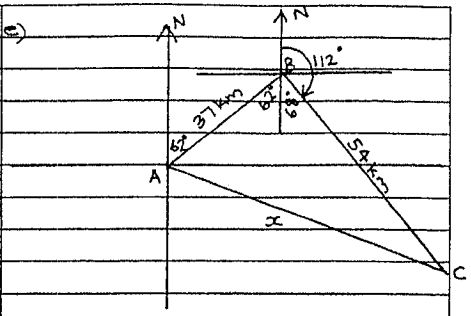
$1369 = 2x^2 - 2x^2\cos 40°$

$= 2x^2(1 - \cos 40°)$

$x^2 = \frac{1369}{2(1-\cos 40°)}$

$x = 54$cm (nearest cm)

e)



Let $AC = x$

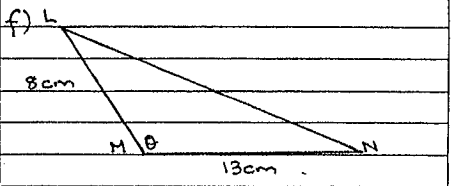
$c^2 = a^2 + b^2 - 2ab\cos C$

$x^2 = 37^2 + 54^2 - 2 \times 37 \times 54 \times \cos 130°$

$x = 83$ km (nearest km)

$\therefore$ AC is 83 km

f)



$A = \frac{1}{2}ab\sin C$

$13 = \frac{1}{2} \times 8 \times 13 \times \sin\theta$

$\sin\theta = \frac{13}{52}$

$\theta = 14°$ (nearest degree)

but θ is obtuse

$\therefore \theta = 180° - 14°$

$= 166°$ (nearest degree)

Alternate Q4d)

$\frac{x}{\sin 70°} = \frac{37}{\sin 40°}$

$x = \frac{37}{\sin 40°} \times \sin 70°$

$= 54$cm (nearest cm)

Question 5:

a)

i) $W \propto \frac{1}{D^2}$

$W = \frac{k}{D^2}$

$70 = \frac{k}{6400^2}$

$k = 2.8672 \times 10^9$

$\therefore W = \frac{2.8672 \times 10^9}{D^2}$

ii) when $D = 6600$km

$W = \frac{2.8672 \times 10^9}{6600^2}$

≈ 65.82 kg (2 dec. pl)

b) $y = ax^2 + bx + c$

x	1	2	3
y	$a+b+c$	$4a+2b+c$	$9a+3b+c$
	$3a+b$	$5a+b$	$7a+b$
		$2a$	$2a$

x	1	2	3	4
y	2	7	14	23
	5	7	9	
		2	2	

$2a = 2$

$a = 1$

$3a + b = 5$

$3 \times 1 + b = 5$

$b = 2$

$a + b + c = 2$

$1 + 2 + c = 2$

$c = -1$

$y = x^2 + 2x - 1$

c i) $K = n \times \sqrt{\frac{a}{P}}$

$\frac{K}{n} = \sqrt{\frac{a}{P}}$

$\frac{K^2}{n^2} = \frac{a}{P}$

$a = \frac{K^2 P}{n^2}$

ii) $L = \frac{3a}{a - m}$

$La - Lm = 3a$

$La - 3a = Lm$

$a(L - 3) = Lm$

$a = \frac{Lm}{L - 3}$

d) $9^x - 12(3^x) + 27 = 0$

$(3^x)^2 - 12(3^x) + 27 = 0$

Let $m = 3^x$

$m^2 - 12m + 27 = 0$

$(m - 9)(m - 3) = 0$

$m = 9 \quad m = 3$

$3^x = 9 \quad 3^x = 3$

$x = 2 \quad x = 1$

4
$16a + 4b + c$

e i) $x^2 - 6x + 9 = (x - 3)^2$

ii) $x^4 - (x^2 - 6x + 9)$

$= x^4 - (x - 3)^2$

$= [x^2 - (x - 3)][x^2 + (x - 3)]$

$= (x^2 - x + 3)(x^2 + x - 3)$