SYDNEY GIRLS HIGH SCHOOL

MATHEMATICS

YEAR 10

## YEARLY EXAMINATION

2002

Time Allowed: 75 minutes

INSTRUCTIONS:

- Show all necessary working
- Marks will be deducted for careless or badly arranged work.
- There are five(5) questions of equal value.
- Attempt all questions.
- Start each part on a new page and write on only one side of the paper.
- Diagrams are not to scale.
- Board approved calculators may be used

QUESTION 1 (15 Marks)

Marks

a. Find the slope of the line $2x + y - 5 = 0$ (1)

b. A letter is chosen at random from the word ALGEBRA
Find the probability of

   (i) an A (1)

   (ii) a consonant (1)

c. Find the value of the pronumeral.
Give reasons.
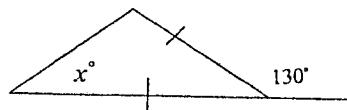
(3)

d. Solve

   (i) $x^2 = 8x$ (ii) $x^2 + x - 42 = 0$ (4)

e. The mean of 4,1,6,3 and x is 5. Find x. (2)

f. A square has an area of $169cm^2$. What is its perimeter? (3)

Calculate the simple interest earned
on $4000 at 5% p.a. for 18 months.   (2)

Calculate the exact area of triangle ABC
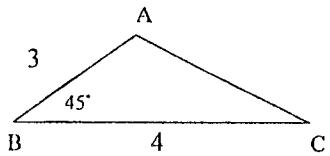
                                       (3)

The mean and standard deviation of an English exam and a Maths exam
are as follows :

    English: $\bar{x} = 59, \quad \sigma = 8$
    Maths : $\bar{x} = 65, \quad \sigma = 6.$

In which exam did Norman have a better mark and why if he scored

    83 in Maths and
    83 in English ?   (2)

Find the equation of the quadratic graph that passes through
the point (2,15) and has its vertex at P(-1,-12).   (3)

e.   This box and whisker plot summaries the number of hours spent watching T.V.
during one week by 30 year 7 students.
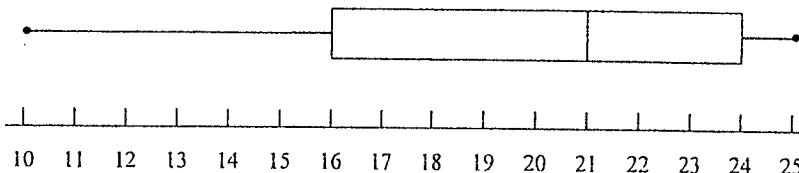
(i)   What is the median number of hours spent watching T.V.?   (1)

(ii)   What is the lower quartile?   (1)

(iii)   Find the interquartile range   (1)

        Find the percentage of students who watched T.V.

(iv)   more than 16 hours per week   (1)

(v)   between 21 and 24 hours per week.   (1)

QUESTION 3 (20 Marks)

a. Tom earns $168 for working an 8 hour day.
  (i)   How much does he earn in a 36 hour week?
  (ii)   How long will it take him to earn $420?   (4)

b. In a group of 20 people 14 like to watch the news on television
and 17 like to watch old movies.
Everybody watches one or the other or both.

  (i)   Draw a Venn diagram to show all information given   (2)

If I choose one person at random find the probability
that the person likes watching:

  (ii)   both the news and old movies   (2)
  (iii) only the news.   (1)

c. Match each equation with a graph

  (i)   (ii)   (iii)

  (iv)   (v)   (vi)
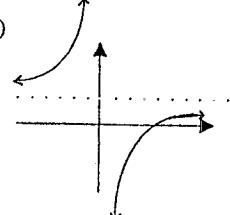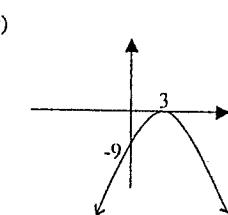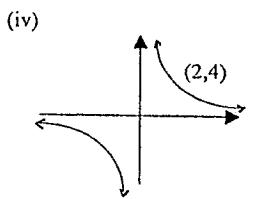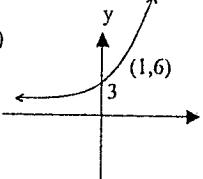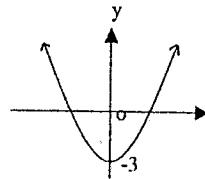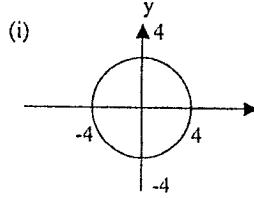
                                                   (6)

$$y = \frac{8}{x}, \quad x^2 + y^2 = 16, \quad y = x^2 - 3, \quad y = 3 \times 2^x, \quad y = \frac{1}{x} + 1, \quad y = -(x-3)^2$$

d. Change the subject of the equation to 'r' given $Y = \frac{a+r}{a-r}$ (2)

e. The base of a triangle is 5cm longer than its altitude.
If the area is 52 cm^2, find the altitude.   (3)

QUESTION 4 (15 Marks)

a. The temperature T(in degrees Celsius) of the air varies inversely with the height H (in metres) above sea level.
At 600m above sea level the temperature is 8°C.

What is the temperature at 1600m above sea level?   (4)

b. Find the compound interest earned on an investment of $3000 at 6% p.a.
over 3 years if the interest is calculated monthly.   (3)

c. For the following find

Stem	Leaf
2	1 8
3	5 6 8
4	2 3 4 4 5 6
5	1 4
6	0 2

  (i)   the mode
  (ii)   the range
  (iii) the median
  (iv) the mean
  (v)   the standard deviation

   (5)

d. The volume of a pyramid is 120 cm^2.
If the base is a square of sides 6cm, find the height of the pyramid.

                                            (3)

QUESTION 5 (20 Marks)

Calculate the surface area and volume of each solid.

(i)

(ii)

8 cm

12 cm

12 cm   12 cm

(8)
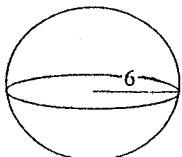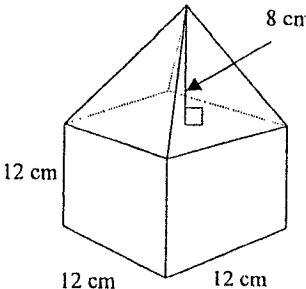
Prove   Δ ABC ||| Δ DEC   and hence find the value of x.

A

D

x

3 cm

B   5 cm   E   5 cm   C

(5)

A

h

42°   67°

B ⌊___50___⌋ C

D
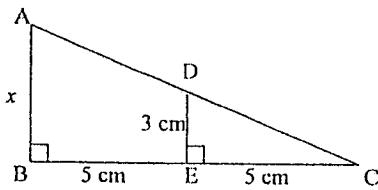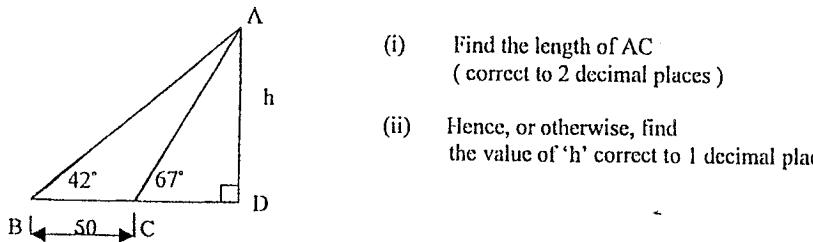
(i)   Find the length of AC
(correct to 2 decimal places)   (2)

(ii)   Hence, or otherwise, find
the value of 'h' correct to 1 decimal place

(2)

d.   A man walks 7 km on a bearing of 150° from a house. He then walks 5 km on a bearing of 025°.

(i)   Draw a diagram showing all information given.
(ii)   How far is he from the house? (correct to 1 decimal place).   (3)

THE END

Question 1

a. $2x + y - 5 = 0$

$y = -2x + 5$

$\therefore$ slope $m = -2$

b. (i) $P(A) = {}^2/_7$

(ii) $P(\text{consonant}) = {}^4/_7$

c. $2x = 130$ (exterior angle

$x = 65°$ equal to

the sum of the

two interior opposites)

d. (i) $x^2 - 8x = 0$

$x(x - 8) = 0$

$\therefore x = 0$ or $x = 8$

(ii) $x^2 + x - 42 = 0$

$(x - 6)(x + 7) = 0$

$x = 6$ or $x = -7$

e. Mean $= \dfrac{4 + 1 + 6 + 3 + x}{5} = 5$

$14 + x = 25$

$x = 11$

f. Area $= x^2$

$x^2 = 169$

$x = 13$ cm

$\therefore$ Perimeter $= 4x$

Perimeter $= 52$ cm

Question 2

a. $S.I = \dfrac{prt}{n}$

$= 4000 \times \dfrac{5}{100} \times 1\frac{1}{2}$

$= \$300$

b. Area $= \frac{1}{2} ac \sin B$

$= \frac{1}{2} \times 3 \times 4 \times \dfrac{1}{\sqrt{2}}$

$= 6 \times \dfrac{1}{\sqrt{2}}$

$= \dfrac{6}{\sqrt{2}}$ units^2

OR Area $= \dfrac{6}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{6\sqrt{2}}{2}$

$= 3\sqrt{2}$ units^2

c. Maths $83 = 59 + 3\sigma$

English $83 = 65 + 3\sigma$

$\therefore$ Norman's marks were

the same in each subject

as in each subject the

marks were 3σ above the

$\bar{x}$.

d. $y = a(x - k)^2 + h$ (k, h) is vertex

$y = a(x + 1)^2 - 12$

$15 = a(2 + 1)^2 - 12$

$15 = 9a - 12$

$27 = 9a$

$\therefore a = 3$

$y = 3(x + 1)^2 - 12$ OR

$y = 3x^2 + 6x - 9$

Question 2 (con't)

e. (i) 21 hours

(ii) 16 hours

(iii) $I.R. = 24 - 16$

$= 8$ hours

(iv) 75%

(v) 25%

Question 3

a. (i) $36 \div 8 = 4\frac{1}{2}$ hours

$\therefore$ Earns $= \$168 \times 4\frac{1}{2}$

$= \$756$.

OR

$\$168 \div 8 = \$21/hr$

$\therefore$ Earns $= 21 \times 36$

$= \$756$

(ii) Time $= \$420 \div 21$

$= 20$ hours

OR $= 2\frac{1}{2}$ days

b. (i)

News 3 | 11 | 6 Old + movies
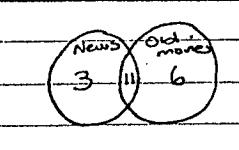
(ii) $P(\text{both N \& OM}) = \dfrac{11}{20}$

(iii) $P(\text{only news}) = \dfrac{3}{20}$

Question 3 (con't)

c. (i) $x^2 + y^2 = 16$ (ii) $y = x^2 - 3$

(iii) $y = 3x^2$ (iv) $y = \dfrac{8}{x}$

(v) $y = (x - 3)^2$ (vi) $y = \dfrac{1}{x} + 1$

d. $y = \dfrac{a + r}{a - r}$

$y(a - r) = a + r$

$ya - yr = a + r$

$ya - a = yr + r$

$a(y - 1) = r(y + 1)$

$\dfrac{a(y - 1)}{(y + 1)} = r$

$\therefore r = \dfrac{a(y - 1)}{(y + 1)}$

e.

Area $= \frac{1}{2} \cdot b \cdot h$

Area $= \frac{1}{2} (x + 5) \cdot x$

$52 = \frac{1}{2} x (x + 5)$

$104 = x^2 + 5x$
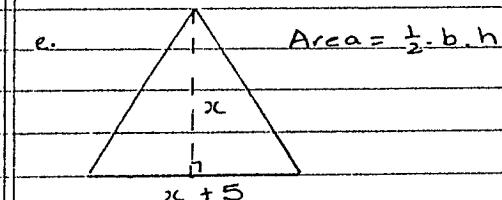
$x^2 + 5x - 104 = 0$

$(x + 13)(x - 8) = 0$

$x = -13$ or $x = 8$

$\therefore$ ONLY solution $x = 8$

$\therefore$ Altitude $= 8$ cm

Question 4

a. T varies inversely to H

$$T = \frac{k}{H}$$

$$8 = \frac{k}{600}$$

$$\therefore k = 8 \times 600$$

$$k = 4800$$

$$\therefore T = \frac{4800}{H}$$

Now, $H = 1600$

$$T = \frac{4800}{1600}$$

$$T = 3°C$$

$\therefore$ Temperature 1600m above sea level is 3°C.

b. Compound Interest

$$A = P(1+r)^n$$

$$P = \$3000$$

$$r = \frac{6}{100} \div 12$$

$$r = 0.005$$

$$n = 3 \times 12$$

$$n = 36 \text{ months}$$

$$\therefore A = \$3000(1+0.005)^{36}$$

$$= \$3000 \times (1.005)^{36}$$

$$= \$3590.04$$

$\therefore$ Interest = **\$590.04**

c. (i) Mode = 44

(ii) Range = 62 − 21

$$= 41$$

(iii) Median = 44

(iv) Mean $= 43.2\dot{6} = 43.3$

(v) S.D $= 10.64$

d. Volume of pyramid $= \frac{1}{3}Ah$

$$120 = \frac{1}{3}(6)^2 \cdot h$$

$$120 = \frac{1}{3} \times 36 \cdot h$$

$$120 = 12h$$

$$\therefore h = \frac{120}{12}$$

$$h = 10 \text{ cm}$$

$\therefore$ height of pyramid = 10cm

Question 5

a. (i) Surface Area of sphere

$$= 4\pi r^2$$

$$= 4\pi(6)^2$$

$$= 144\pi \text{ units}^2$$

OR/ $\doteqdot 452.4 \text{ units}^2$

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi(6)^3$$

$$= \frac{4}{3}\pi \times 216$$

$$= 288\pi \text{ units}^3$$

OR/ $\doteqdot 904.8 \text{ units}^3$

(ii) Volume $= 12^3 + \frac{1}{3}(12)^2 \cdot 8$

$$= 1728 + 384$$

$$= 2112 \text{ cm}^3$$

Question 5 (con't)

a. (ii) Surface Area

$$x^2 = 8^2 + 6^2$$

$$x^2 = 100$$

$$x = 10$$
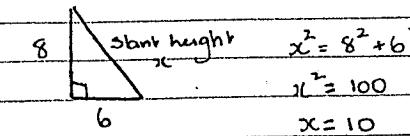
$\therefore$ SA = area of squares + area of triangles

$$= 5(12)^2 + 4\left(\frac{1}{2} \cdot 12 \cdot 10\right)$$

$$= 720 + 4(60)$$

$$= 720 + 240$$

$$= 960 \text{ cm}^2$$

b. DATA: $A\hat{B}C = D\hat{E}C = 90°$

$$DE = 3\text{cm}$$

$$BE = 5 \text{ cm}$$

$$EC = 5\text{cm}$$

Aim: Prove $\triangle ABC \parallel\mkern-10mu\parallel \triangle DEC$

PROOF: In $\triangle ABC$ and $\triangle DEC$

$A\hat{B}C = D\hat{E}C = 90°$ (given)

$B\hat{A}C = E\hat{D}C$ (corresponding angles || lines)

$\angle C$ is common

$\therefore$ By AAS Test

$\triangle ABC \parallel\mkern-10mu\parallel \triangle DEC$

hence, corresponding sides are in the same ratio.

$$\therefore \frac{AB}{DE} = \frac{BC}{EC}$$

$$\frac{x}{3} = \frac{10}{5}$$

$$x = \frac{10 \times 3}{5} = 6 \text{ cm}$$

c.

(i) $A\hat{C}B = 180 - 67$

$$= 113° \text{ (straight angle)}$$

Using, $\therefore B\hat{A}C = 180 - (113+42)$

$$= 25$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{BC}{\sin B\hat{A}C} = \frac{AC}{\sin B}$$

$$\frac{50}{\sin 25} = \frac{AC}{\sin 42°}$$

$$\frac{50 \times \sin 42}{\sin 25} = AC$$

$$AC = 79.16 \text{ units}$$

(ii) $\sin\theta = \frac{opp}{hyp}$

$$\sin 67° = \frac{h}{79.16}$$

$$(\sin 67) \times 79.16 = h$$

$$\therefore h = 72.9 \text{ units}$$

d. (i)

(ii)
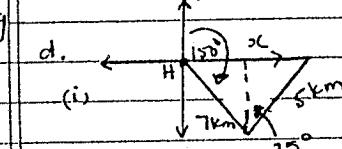
$$x^2 = a^2 + b^2 - 2ab\cos C$$

$$x^2 = 7^2 + 5^2 - 2(7)(5)\cos 55°$$

$$x^2 = 74 - 70\cos 55°$$

$$x^2 = 33.849$$

$$x = \sqrt{33.849} \doteqdot 5.8 \text{ km}$$