SYDNEY GIRLS H.S.

Year 11 Calculus Test (1)

1. If f(x) = $2x^2 - 3x - 4$, find the derivative by first principles
2. Find $\frac{dy}{dx}$ in the following:

a) $y = 3x^4 - 4x^3$ b) $y = \sqrt{x}$ c) $y = \frac{1}{x^3}$

d) $y = (2x-5)^3$ e) $y = \sqrt{3-4x}$ f) $y = (2x-1).\sqrt{x}$

g) $y = \frac{x^2 - 4}{x}$ h) $y = \frac{2x-1}{x+3}$ i) $y = \frac{x^2+2}{1-3x}$

3. Find $\frac{d^2y}{dx^2}$ in the following

a) $y = 2x^2 + 3x + 1$ b) $y = x\sqrt{x}$

4. Find the equation of the tangent and normal to $y = \sqrt{2x+1}$ at the point (4,3)

5. Find the points on the curve $y = x^4 - 4x^3$ where

a) $y = 0$

b) $\frac{dy}{dx} = 0$

c) $\frac{d^2y}{dx^2} = 0$

6. Explain why the gradinet of the curve $y = x^3 + 1$ is always positive

7. Find $\frac{dy}{dx}$ in simplest form

a) $y = (2x-3)^3(3x+1)^2$ b) $y = \frac{x+1}{\sqrt{x}}$

SYDNEY GIRLS H.S.

Year 11 Calculus Test (2)

1. If $f(x) = x^3 - 6x^2 - 15x + 9$, find any stationary points and their nature, any inflexion points and sketch the curve.

2. State any asymptotes and sketch the curves:

   a) $y = \dfrac{2x - 1}{x + 3}$ b) $y = \dfrac{x + 2}{2x - 4}$

3. Find the dimensions of the rectangle of maximum area if the perimeter is 20cms

4. Find the maximum volume of a cone if the sum of the radius and height is 30 cms

   (Volume of a cone is $V = \dfrac{1}{3}\pi r^2 h$

5. In a book, the pages are to have an area of 338 square centimetres
   A margin of 1 cm is left at either side and a margin of 2cm is left at the top and bottom. Inside the margins is the print area
   Find the dimensions of the page so that the print area is a maximum

4, 11 Calculus Test 1

1. If $f(x) = 2x^2 - 3x - 4$

$$f(x+h) = 2(x+h)^2 - 3(x+h) - 4$$
$$= 2(x^2 + 2xh + h^2) - 3x - 3h - 4$$
$$= 2x^2 + 4xh + 2h^2 - 3x - 3h - 4$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{2\cancel{x^2} + 4xh + 2h^2 - 3\cancel{x} - 3h - \cancel{4} - 2\cancel{x^2} + 3\cancel{x} + \cancel{4}}{h}$$
$$= \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h}$$
$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}} \quad 4$$

$$\frac{dy}{dx} = 4x - 3$$

2. (a) $y = 3x^4 - 4x^3$

$$\frac{dy}{dx} = 12x^3 - 12x^2 \quad 1$$

(b) $y = \sqrt{x}$

$$y = x^{1/2}$$
$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2}$$
$$= \frac{1}{2\sqrt{x}} \quad 2$$

(c) $y = \frac{1}{x^3} \qquad y = x^{-3}$

$$\frac{dy}{dx} = -3x^{-4} \quad 2$$
$$= \frac{-3}{x^4}$$

(d) $y = (2x-5)^3$

$$\frac{dy}{dx} = 3(2x-5)^2 . 2 \quad 2$$
$$= 6(2x-5)^2$$

(e) $y = \sqrt{3-4x}$

$$y = (3-4x)^{1/2}$$
$$\frac{dy}{dx} = \frac{1}{2}(3-4x)^{-1/2} . -4$$
$$= \frac{-2}{\sqrt{3-4x}} \quad 2$$

(f) $y = (2x-1)\sqrt{x}$

$$y = (2x-1). x^{1/2}$$
$$y = 2x^{3/2} - x^{1/2}$$
$$\frac{dy}{dx} = 2 \cdot \frac{3}{2} x^{1/2} - \frac{1}{2} x^{-1/2}$$
$$= 3\sqrt{x} - \frac{1}{2\sqrt{x}} \quad 3$$

(g) $y = \frac{x^2 - 4}{x} = \frac{x^2}{x} - \frac{4}{x}$

$$y = x - 4x^{-1}$$
$$\frac{dy}{dx} = 1 + 4x^{-2} \quad 3$$
$$= 1 + \frac{4}{x^2}$$

19

Q2 (h) $y = \dfrac{2x-1}{x+3}$ $\dfrac{u}{v}$ let $u = 2x-1$ $v = x+3$

$\dfrac{du}{dx} = 2$ $\dfrac{dv}{dx} = 1$

$$\frac{dy}{dx} = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

$$= \frac{2(x+3) - 1(2x-1)}{(x+3)^2}$$

$$= \frac{2x+6-2x+1}{(x+3)^2}$$

3

$$= \frac{7}{(x+3)^2}$$

(i) $y = \dfrac{x^2+2}{1-3x}$ $\dfrac{u}{v}$ let $u = x^2+2$ $v = 1-3x$

$\dfrac{du}{dx} = 2x$ $\dfrac{dv}{dx} = -3$

$$\frac{dy}{dx} = \frac{2x(1-3x) + 3(x^2+2)}{(1-3x)^2}$$

$$= \frac{2x - 6x^2 + 3x^2 + 6}{(1-3x)^2}$$

3

$$= \frac{-3x^2 + 2x + 6}{(1-3x)^2}$$

3. (a) $y = 2x^2 + 3x + 1$

$\dfrac{dy}{dx} = 4x + 3$

$\dfrac{d^2y}{dx^2} = 4$

2

(b) $y = x\sqrt{x}$

$y = x^1 . x^{1/2} = x^{3/2}$ 1

$\dfrac{dy}{dx} = \dfrac{3}{2}x^{1/2} = \dfrac{3}{2}\sqrt{x}$ 1

$\dfrac{d^2y}{dx^2} = \dfrac{3}{2}.\dfrac{1}{2}x^{-1/2}$

$= \dfrac{3}{4\sqrt{x}}$ 1

11

4. $y = (2x+1)^{1/2}$ at $P(4,3)$ $\quad x_1\, y_1$

$\frac{dy}{dx} = \frac{1}{2}(2x+1)^{-1/2} \cdot 2$

$= \frac{1}{\sqrt{2x+1}}$ at $x = 4$

$= \frac{1}{\sqrt{9}}$

$m = \frac{1}{3}$ $\quad 2$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{3}(x-4)$

$3y - 9 = x - 4$

$3y - x - 5 = 0$ $\quad 2$

Equation of normal

$m_1 = \frac{1}{3} \quad \therefore \quad m_2 = -3$

$y - y_1 = m(x - x_1)$

$y - 3 = -3(x-4)$

$y - 3 = -3x + 12$

$y + 3x - 15 = 0$

3

5. $y = x^4 - 4x^3$

(a) $y = 0$

$x^4 - 4x^3 = 0$

$x^3(x-4) = 0$ $\quad 3$

$\therefore x = 0$ or $x = 4$ $\quad P_2(4,0)$

$P_1(0,0)$ $\quad y = 0 \quad\quad y = 0$

(b) $\frac{dy}{dx} = 4x^3 - 12x^2$

$\therefore$ at $\frac{dy}{dx} = 0$

$4x^3 - 12x^2 = 0$ $\quad 3$

$4x^2(x-3) = 0$

$x = 0$ or $x = 3$

$P_1(0,0)$ $\quad y = 0 \quad\quad y = -27$ $\quad P_2(3,-27)$

(c) $\frac{d^2y}{dx^2} = 12x^2 - 24x$

at $\frac{d^2y}{dx^2} = 0$

3

$12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0$ or $x = 2$

$y = 0 \quad\quad y = -16$

$P_1(0,0) \quad\quad P_2(2,-16)$

6. $y = x^3 + 1$

2

$\frac{dy}{dx} = 3x^2$ $\quad \therefore$ no matter what value of x

$\frac{dy}{dx}$ is always positive because of the x^2

13

7. (a) $y = (2x-3)^3(3x+1)^2$

let $u = (2x-3)^3$ $\quad v = (3x+1)^2$

$$\frac{du}{dx} = 3(2x-3)^2 \cdot 2 \qquad \frac{dv}{dx} = 2(3x+1)^1 \cdot 3$$

$$= 6(2x-3)^2 \qquad = 6(3x+1)$$

$$\therefore \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= 6(3x+1)(2x-3)^3 + 6(2x-3)^2(3x+1)^2$$

$$= 6(3x+1)(2x-3)^2\left[(2x-3)+(3x+1)\right]$$

$$= 6(3x+1)(2x-3)^2(5x-2) \qquad 4$$

(b) $y = \dfrac{x+1}{\sqrt{x}}$

$$y = (x+1)x^{-1/2}$$

$$y = x^{1/2} + x^{-1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-3/2}$$

$$= \frac{1}{2}x^{-1/2}\left(1 - x^{-1}\right)$$

$$= \frac{1}{2\sqrt{x}}\left(1 - \frac{1}{x}\right)$$

3

let $u = x+1 \quad v = x^{1/2}$

$$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \frac{1}{2}x^{-1/2}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{\sqrt{x} - (x+1)\frac{1}{2\sqrt{x}}}{x}$$

$$= \frac{\sqrt{x} - \frac{1}{2\sqrt{x}} - \frac{\sqrt{x}}{2}}{x}$$

$$= \left(\frac{2x - 1 - x}{2\sqrt{x}}\right) \div x$$

$$= \frac{\cancel{2x}\,x - 1}{2\sqrt{x}} \times \frac{1}{x}$$

7 $$= \frac{x-1}{2x^{3/2}}$$

Year 11 Calculus Test (2)

1. $f(x) = x^3 - 6x^2 - 15x + 9$

$f'(x) = 3x^2 - 12x - 15$

$= 3(x^2 - 4x - 5)$ 1

stationary pt at $f'(x) = 0$

$3(x^2 - 4x - 5) = 0$

$(x - 5)(x + 1) = 0$

$x = 5$ or $x = -1$ 2

$f(5) = -91$ $f(-1) = 17$

$f''(x) = 6x - 12$ 1

at $x = 5$

$f''(5) = 6(5) - 12$

$= 30 - 12$

$= 18$

$\therefore$ Minimum Stationary Point

at $x = -1$

$f''(-1) = 6(-1) - 12$

$= -6 - 12$ 1

$= -18$

$\therefore$ Maximum stationary point

$f''(x) = 6x - 12$

Point of Inflexion

$6x - 12 = 0$ 1

$6x = 12$

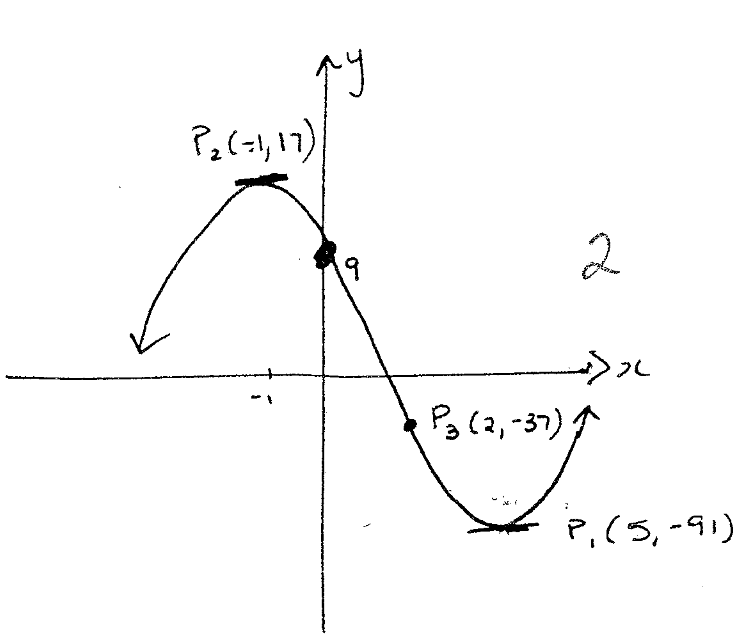
$x = 2$

x	1	2	3
$\frac{d^2y}{dx^2}$	$-$	0	$+$

$\therefore$ Point of inflexion

at $x = 2$ 1

$y = -37$



2

10

2. (a) $y = \dfrac{2x-1}{x+3}$

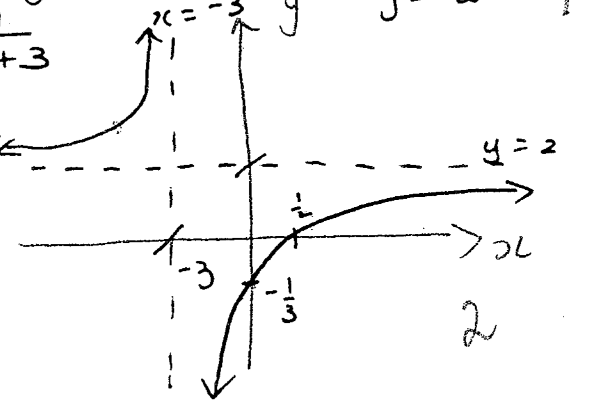
$y = \dfrac{2(x+3)}{x+3} - \dfrac{7}{x+3}$

$y = 2 - \dfrac{7}{x+3}$

Asymptotes at $x = -3$
$y = 2$

at $x=0$, $y = -\frac{1}{3}$
at $x = \frac{1}{2}$, $y = 0$

at $x = -4$, $y = 9$



2

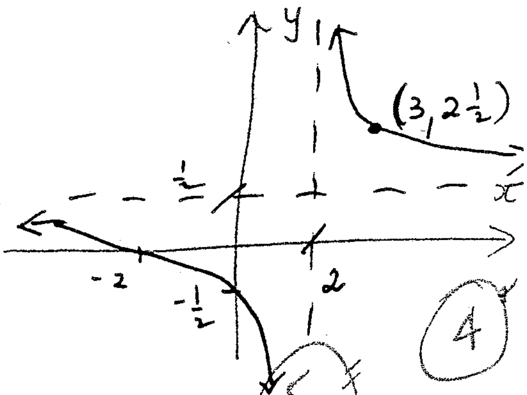
4 +1 5

(b) $y = \dfrac{x+2}{2x-4}$

$y = \dfrac{x-2}{2(x-2)} + \dfrac{4}{2(x-2)}$

$= \frac{1}{2} + \dfrac{4}{2x-4}$

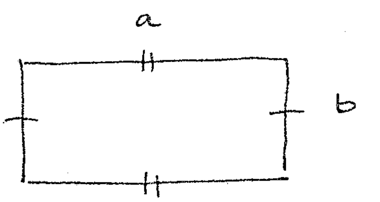
Asymptote at
$x = 2$



4 5

at $x=0$, $y = -\frac{1}{2}$
$x = -2$ $y = 0$

3.



$2a + 2b = 20$
$a + b = 10$
$a = 10 - b.$

Area $= ab$
$A = b(10-b)$
$A = 10b - b^2$

$\therefore \dfrac{dA}{db} = 10 - 2b$

stationary pt $\dfrac{dA}{db} = 0$

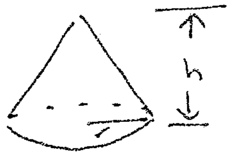
$10 - 2b = 0$
$-2b = -10$
$b = 5$

8

at $\dfrac{d^2A}{db^2} = -2$ $\therefore$ Maximum Area at $b = 5$

$\therefore a = 10 - b$
$= 10 - 5$
$a = 5$

$\therefore$ dimensions 5×5
Maximum Area

4. 

$h + r = 30$ cm   1

$h = 30 - r$ cm

$$V = \frac{1}{3}\pi r^2 h$$

1

$$= \frac{1}{3}\pi r^2 (30 - r)$$

$$V = 10\pi r^2 - \frac{1}{3}\pi r^3$$

$$\frac{dV}{dr} = 20\pi r - \pi r^2$$ 1

stationary pt $\frac{dV}{dr} = 0$

$$20\pi r - \pi r^2 = 0$$

$$\pi r (20 - r) = 0$$ 2

$$\pi r = 0 \qquad 20 - r = 0$$

$$r = 0 \qquad \underline{r = 20} \text{ cm}$$

$$\therefore h = 30 - r$$ 1

$$\underline{h = 10 \text{ cm}}$$

$$\frac{d^2V}{dr^2} = 20\pi - 2\pi r \;\Big|\; \text{at } \underline{r = 20}$$

1

$$= 20\pi - 40\pi$$

$$= -20\pi < 0 \quad \therefore \text{ Maximum Volume.}$$

$\therefore$ Maximum Volume occurs at $r = 20$ and $h = 10$.

$$\therefore V = \frac{1}{3}\pi r^2 h$$

2

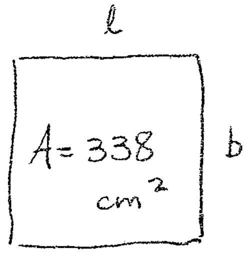
$$= \frac{1}{3}\pi (20)^2 (10)$$

$$= \frac{4000}{3}\pi$$

Volume $\doteqdot 4188.8$ cm^3

10

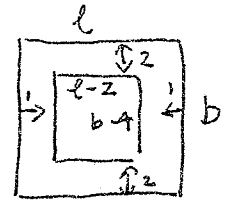
5.



$A = lb$

$lb = 338 \text{ cm}^2$

$l = \frac{338}{b}$



Print Area $= (l-2)(b-4)$

$A = lb - 4l - 2b + 8$

$= 338 - 4\left(\frac{338}{b}\right) - 2b + 8$

$= 346 - \frac{1352}{b} - 2b$

$= 173 - 676b^{-1} - b$

$\frac{dA}{db} = 676b^{-2} - 1$

$= \frac{676}{b^2} - 1$

Stationary pt $\frac{dA}{db} = 0$

$\frac{676}{b^2} - 1 = 0$

$676 - b^2 = 0$

$b^2 = 676$

$b = \sqrt{676}$

$b = 26.$

$\frac{d^2A}{db^2} = -676b^{-3}$

$= -\frac{676}{b^3}$ will always be (-ve)

$\therefore$ Maximum Area.

12

$b = 26 \quad \therefore l = \frac{338}{b}$

$= \frac{338}{26}$

$l = 13$

$\therefore$ Dimensions of page $= 26 \times 13$.

Dimension of print area $= (l-2)(b-4)$

$= (13-2)(26-4)$