SYDNEY GIRLS HIGH SCHOOL

YEAR 11

HSC First Assessment Task

December 2002

MATHEMATICS 4 Unit

Time allowed: 90 minutes

Topics: Curve Sketching, Circular Motion

Instructions:

- There are Eleven questions. The questions are not of equal value
- Marks for each question are shown.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work.
- Write on one side of the paper only.
- Questions are not necessarily written in order of difficulty
- Use g = 10 ms^{-2}

Question 1. (16 marks)

Sketch the following curves, showing any important points.

a) $y = \log_3 x$

b) $x = \sqrt{9 - y^2}$

c) $x^2 + y^2 - 4x + 6y - 3 = 0$

d) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

e) $\frac{x^2}{9} - \frac{y^2}{4} = 1$

f) $y = 2\cos^{-1}(x-1)$

g) $y = 2\sin\left(x + \frac{\pi}{6}\right)$

h) $y = (x-2)^2(x-4)^3$

Question 2 (8 marks)

a) Sketch the curve $f(x) = \frac{2x-1}{x}$.

b) Find $f'(x)$ and explain why f(x) has an inverse function

c) Determine the inverse function $y = f^{-1}(x)$ and sketch the curve

d) State any common points between $y = f(x)$ and $y = f^{-1}(x)$

Question 3 (6 marks)

On the one sets of axes sketch the curves for the domain $0 \le x \le 2\pi$

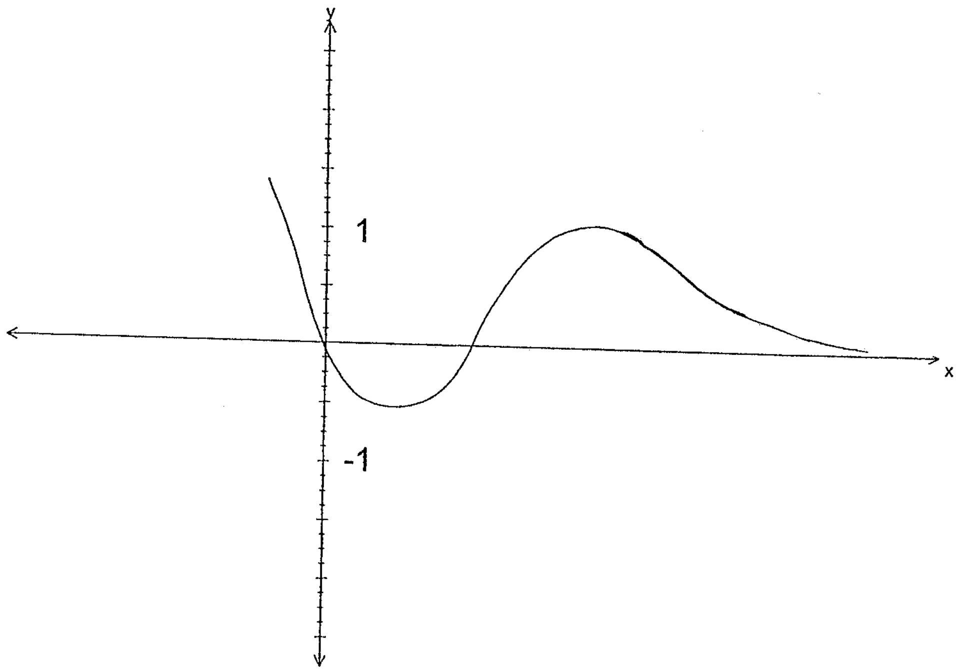
a) $y = \cos 2x$

b) $y = -2\sin x$

c) Add the ordinates of the above curves to sketch $y = \cos 2x - 2\sin x$

Question 4

(10 marks)



The diagram shows a graph of the function y = f(x). The graph has an asymptote at y = 0. Draw separate sketches of the graphs of the following functions:

a) $y = |f(x)|$

b) $y = f|x|$

c) $y^2 = f(x)$

d) $y = \frac{1}{f(x)}$

Question 5

(10 marks)

Show any roots or asymptotes and sketch the curves

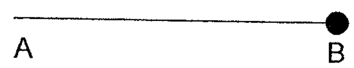
a) $y = \frac{(2x+1)^2}{2x}$

b) $y = \frac{(x-1)^2}{x(x+2)^2}$

Question 6

(5 marks)

A 5 metre piece of string AB has a mass of 4 kg attached at point B



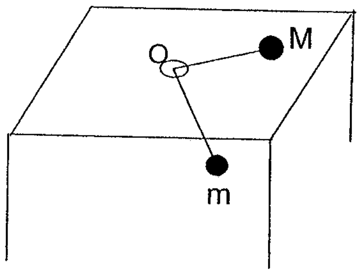
If the string breaks when the speed of rotation reaches 6 rad/ sec;

a) Find the breaking strain in the string.

b) If the 4 kg mass at B is replaced by a 2 kg mass at the same position, and a 2 kg mass 3 m from A, find the new maximum speed of rotation.

Question 7 (7 marks)

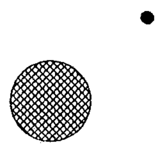
A mass M moves in circular motion about O with angular velocity w, and a mass m attached to M through a hole in the table rotates with angular velocity $2w$. If the ratio of M : m = 5 : 2, find what fraction of the string lies above the table. (use $g = 10ms^{-2}$)



Question 8 (12 marks)

The gravitational force F between two masses M and m is given by $F = \frac{GMm}{r^2}$, where $G = 6.67 \times 10^{-11}$ and r is the distance between the objects. If M represents the mass of the earth $(5.97 \times 10^{24} kg)$ and m the mass of a satellite;

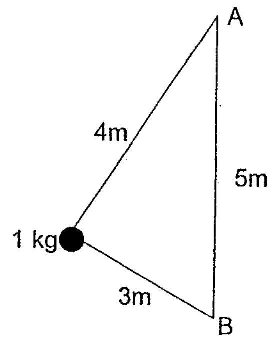
a) Show that the time for the satellite to orbit the earth is given by $T = 2\pi\sqrt{\frac{r^3}{GM}}$
b) Find the radius of the orbit if the satellite is to stay above a certain position
c) Find the acceleration due to gravity at the height of the satellite



d) Find the speed of the satellite

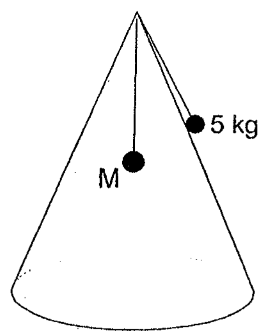
Question 9 (8 marks)

Point B is 5metres directly below A. A mass of 1 kg is attached to A and B by strings of length 4m and 3m respectively. The mass rotates in a horizontal circle about the vertical line AB. Find the angular velocity of the mass if the tension in the two strings is to be equal.



Question 10. (8 marks)

An inverted cone has a hole in the apex and a 5 kg mass rotating around the outside of the cone at 4 radians per second. The semi-vertical angle is 30°
If the length of the string from the apex to the rotating mass is 2 metres, what must be the value of the stationary mass M for the system to remain in equilibrium. (Write answer correct to 3 significant figures)



Question 11 (10 marks)

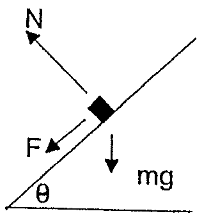
The above diagram illustrates a mass travelling
Around a corner of radius r at an angle of $\theta°$

Given: Vertically: $N\cos\theta = F\sin\theta + mg$

Horizontally: $N\sin\theta + F\cos\theta = \frac{mv^2}{r}$

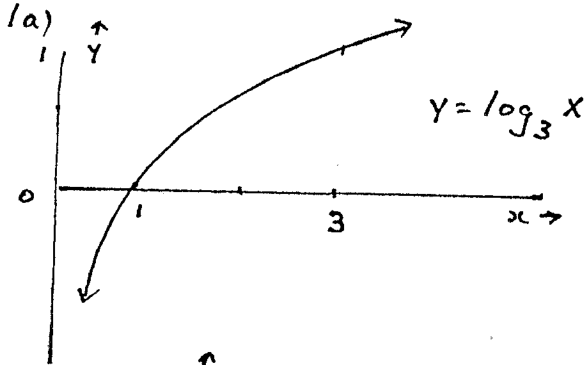
a) A train turning a corner causes the same lateral force down the slope When it travels at 72 km/hr, as it does up the slope when it travels at 144 km/hr. If the radius of the corner is 500metres, find the angle at which the track is Banked. (Answer to the nearest minute).

b) Find the speed in kilometres per hour at which the lateral force F is zero. (Answer to nearest kilometre per hour)

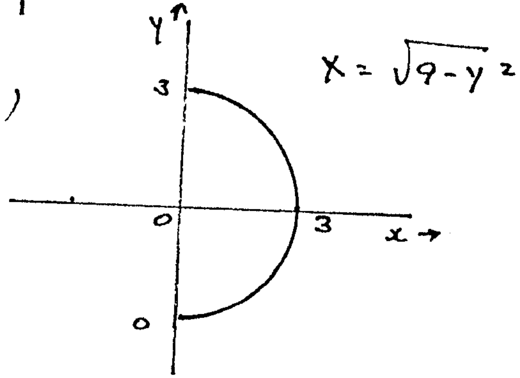


END OF EXAM

1a)



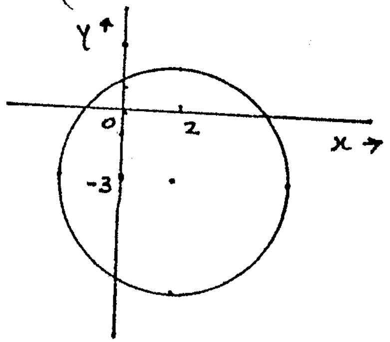
b)



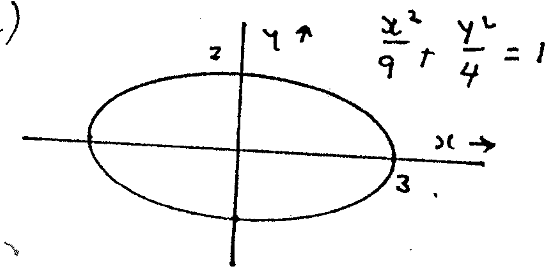
c) $x^2+y^2-4x+6y-3=0$

$x^2-4x+4+y^2+6y+9=16$

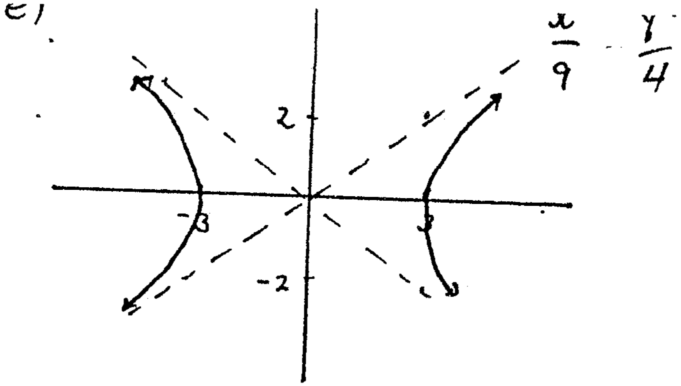
$(x-2)^2+(y+3)^2=4^2$



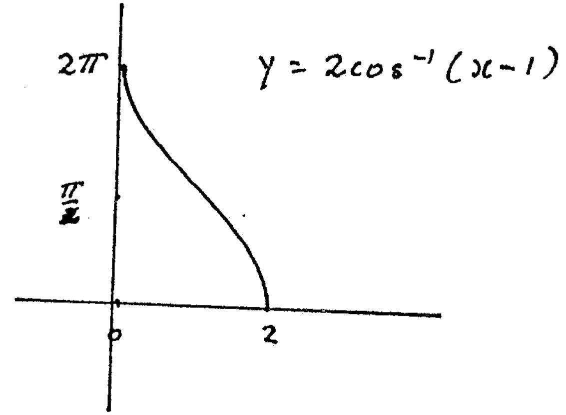
d)



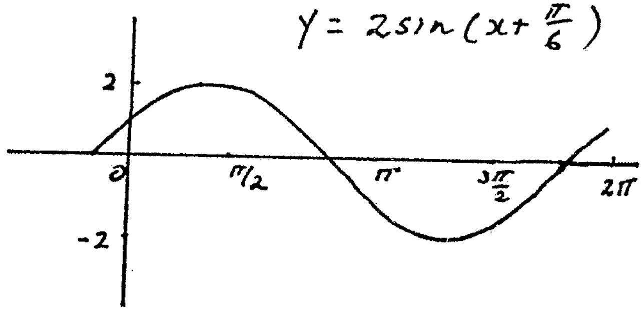
e)



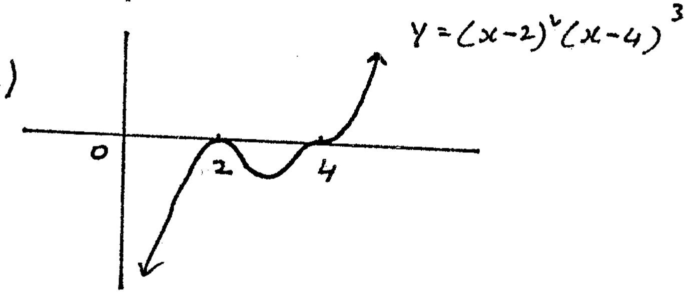
f)



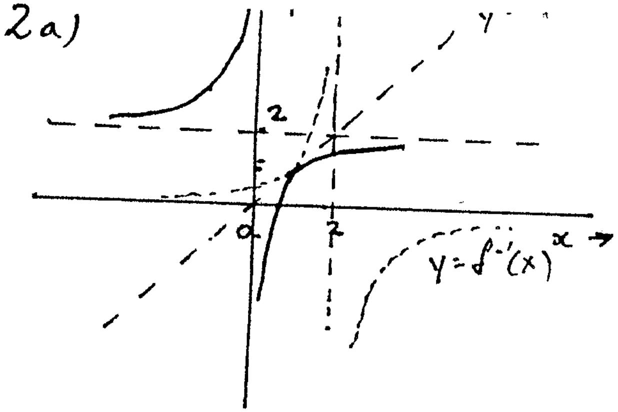
g)



h)



2a)



$f(x) = \frac{2x-1}{x} = 2 - \frac{1}{x}$

$f'(x) = +x^{-2} = \frac{1}{x^2}$

$f'(x)$ is always > 0 since numerator is a constant & denominator a square

∴ no turning pts

∴ an inverse exists

$x = \frac{2y-1}{y}$

$xy = 2y-1$

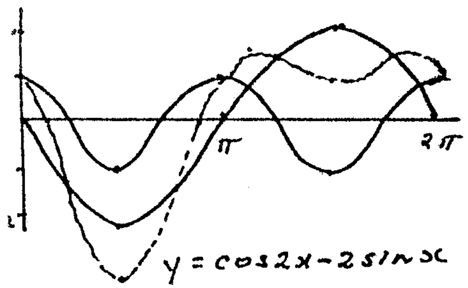
$xy - 2y = -1$

$\therefore y(x-2) = -1$

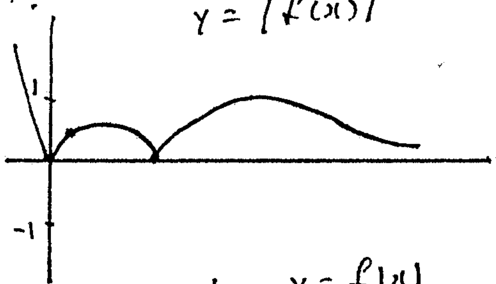
$\therefore y = \frac{-1}{x-2} = \frac{1}{2-x}$

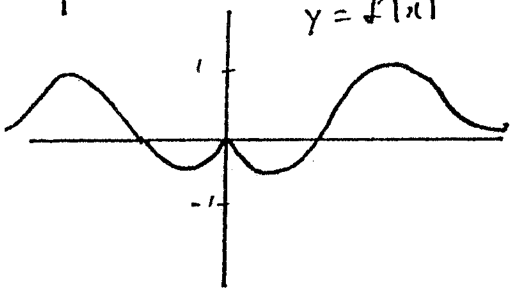
$\therefore f^{-1}(x) = \frac{1}{2-x}$

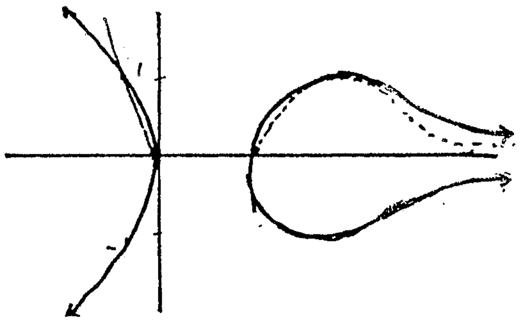
d) The common point is (1,

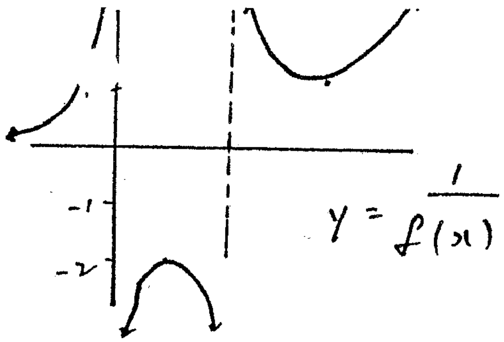


4.





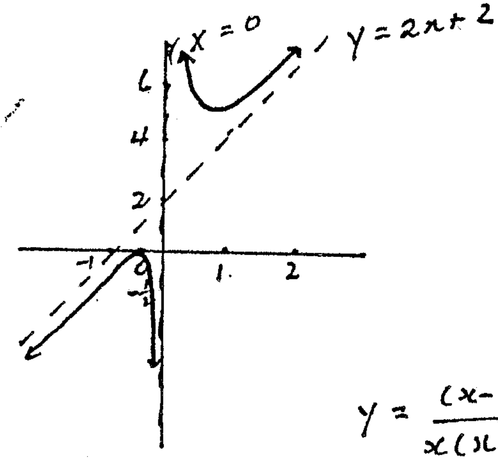




Q5. a) $y = \frac{(2x+1)^2}{2x}$

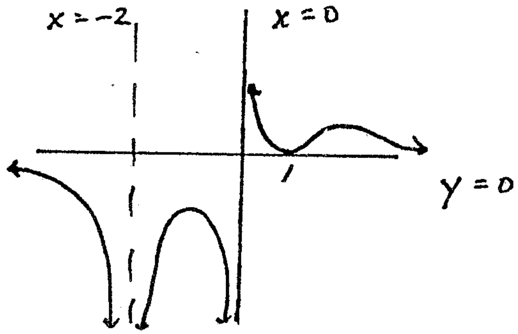
$= \frac{4x^2+4x+1}{2x}$

$= 2x + 2 + \frac{1}{2x}.$

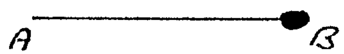


$y = \frac{(x-1)^2}{x(x+2)^2}$

b)



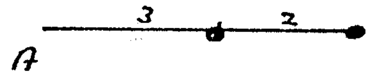
Q6.



$T = mrw^2$

$= 4 \times 5 \times 36$

$= 720\ N$



$720 = 2 \times 3 \times w^2 + 2 \times 5 \times w^2$

$= 6w^2 + 10w^2$

$= 16w^2$

$w = \sqrt{45} = 3\sqrt{5}$ rad/sec

Q7. $T = M.rw^2, \quad T = mLw_2^2$

$Mrw_1^2 = mLw_2^2$

$M : m = 5 : 2 \quad \therefore \frac{M}{m} = \frac{5}{2}$

$M = \frac{5m}{2}$

$\therefore \frac{5m}{2} r w^2 = m L . 4w^2$

$5r = 8L$

$\therefore \frac{8}{13}$ of the string lies above the hole.

8\.

a) $F = \frac{GMm}{r^2} = mr\omega^2$

$$\omega^2 = \frac{GM}{r^3}$$

$$\omega = \sqrt{\frac{GM}{r^3}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r^3}{GM}}$$

b) $T = 24$ hrs
$= 24 \times 60 \times 60$ secs

$$\therefore \left(\frac{86400}{2\pi}\right)^2 = \frac{r^3}{GM}$$

$$\therefore r \doteqdot \sqrt[3]{\left(\frac{86400}{2\pi}\right)^2 \times 6.67 \times 5.97 \times 10^{13}}$$

$$\doteqdot 4.22 \times 10^7$$

c) $F = mr\omega^2 = mg$

$$\therefore a = r\omega^2$$

$$\doteqdot 4.22 \times 10^7 \left(\frac{2\pi}{24 \times 60 \times 60}\right)^2$$

$$\doteqdot 0.223$$

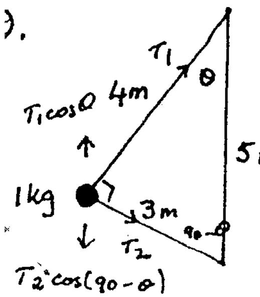
d) $a = \frac{v^2}{r}$

$$\therefore v = \sqrt{a \times r}$$

$$= \sqrt{.223 \times 4.216 \times 10^7}$$

$$\doteqdot 3066\ ms^{-1}$$

9\.



$T_1 \cos\theta = mg + T_2 \sin\theta$

$T_1 \sin\theta + T_2 \cos\theta = mr\omega^2$

$T_1 = T_2 = T$

$m = 1,\ g = 10,\ r = 12/5$

$\sin\theta = 3/5,\ \cos\theta = \frac{4}{5}$

$$\therefore T\left(\tfrac{4}{5}\right) = 10 + T(3/5)$$

$$T\left(\tfrac{3}{5}\right) + T(4/5) = 1 \times \tfrac{12}{5} \times \omega^2$$

$$\therefore 4T = 50 + 3T$$

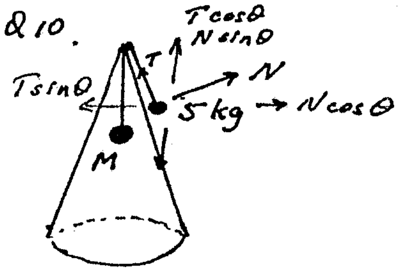
$$\therefore T = 50$$

$$\therefore 50\left(\tfrac{3}{5}\right) + 50\left(\tfrac{4}{5}\right) = \tfrac{12}{5} \times \omega^2$$

$$150 + 200 = 12\omega^2$$

$$\omega^2 = \frac{350}{12},\ \omega = \sqrt{\frac{350}{12}} \doteqdot 5.4\ \text{rad/sec}$$

Q10.



$T\cos\theta + N\sin\theta = 5g$

$T\sin\theta - N\cos\theta = mr\omega^2$

$T = Mg$

$\cos\theta = \sqrt{3}/2,\ \sin\theta = 1/2,\ g = 10$

$m = 5,\ r = l\sin\theta = 1,\ \omega = 4$

$\therefore \frac{\sqrt{3}}{2}T + \frac{N}{2} = 50 \Rightarrow \sqrt{3}T + N = 100$ (1)

$\frac{T}{2} - \frac{\sqrt{3}}{2}N = 5 \times 16 \Rightarrow T - \sqrt{3}N = 160$ (2)

$T = 10M$

From (1) $3T + \sqrt{3}N = 100\sqrt{3}$ (1a)

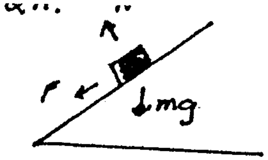
Add 1a+2: $4T = 160 + 100\sqrt{3}$

$\therefore T = 40 + 25\sqrt{3}$

& $M = \frac{40 + 25\sqrt{3}}{10} = 4 + 2.5\sqrt{3}$

$\doteqdot 8.33$ kg

Q11.



$N\cos\theta = F\sin\theta + mg$ (1)

$N\sin\theta + F\cos\theta = \frac{mv^2}{r}$ (2)

×(1) by $\sin\theta$, (2) by $\cos\theta$

$\therefore N\cos\theta\sin\theta = F\sin^2\theta + mg\sin\theta$ (1a)

$N\sin\theta\cos\theta = -F\cos^2\theta + \frac{mv^2}{r}\cos\theta$ (2a)

Subtracting

$$0 = F + mg\sin\theta - \frac{mv^2}{r}\cos\theta$$

$$\therefore F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$

72 km/hr = 20 m/s, 144 km/hr = 40
& $r = 500$

$$\therefore m\left(\frac{1600}{500}\cos\theta - 10\sin\theta\right] = -m\left(\frac{400}{500}\cos\theta - 10\sin\theta\right)$$

$$\therefore \frac{16}{5}\cos\theta - 10\sin\theta = -\frac{4}{5}\cos\theta + 10\sin\theta$$

$$\therefore 4\cos\theta = 20\sin\theta$$

$$\tan\theta = \frac{1}{5} \Rightarrow \theta \doteqdot 11°19'$$

If $F = 0$

$$\frac{v^2}{r}\cos\theta - g\sin\theta = 0$$

$$\therefore \frac{v^2}{rg} = \tan\theta$$

$$\therefore v^2 = 500 \times 10 \times \frac{1}{5}$$

$$= 1000$$

$$v = 10\sqrt{10}\ ms^{-1}$$

$$\doteqdot 114\ \text{km/hr}$$