Sydney Girls' High School

2008
MATHEMATICS EXTENSION 1

YEAR 11
HALF-YEARLY EXAMINATION

Time Allowed: 60 minutes

TOPICS: Harder 2U, Functions and Trigonometry

Directions to Candidates

- There are four (4) questions.
- Attempt ALL questions.
- Questions are of equal value.
- Start each question on a new page.
- Write on one side of the paper only.
- Show all necessary working. Marks will be deducted for careless or badly arranged work.
- Diagrams are NOT drawn to scale.
- Board-approved calculators may be used.

Total: 60 marks

QUESTION 1 (15 marks) — Marks

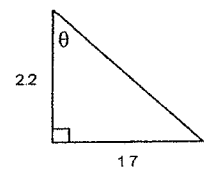
a) Solve $(x+4)^2 = 3(x+4)$ — 3

b) Find the exact value of $\cos 315^\circ$ — 1

c) State the domain of the function $g(x) = \sqrt{2+x} + \sqrt{2-x}$ — 2

d) Solve $|2x+1| = 5$ — 3

e) Find θ (to the nearest minute) in the triangle below — 2



f) Find the exact value of $\cot 270^\circ$ — 1

g) Solve for x $\dfrac{1}{2x-3} > 1$ — 3

QUESTION 2 (15 marks)

Sketch the following graphs on separate number planes showing all relevant features.

a) $y = 2$ — 1

b) $x + y = 2$ — 2

c) $y = -2^x$ — 2

d) $y = \dfrac{1}{x+2}$ — 2

e) $y = |2+x|$ — 2

f) $y = |x^2 - 1|$ — 2

g) $y = -\dfrac{2}{x^2}$ — 2

h) $f(x) = \begin{cases} 2x-1 & \text{if } x > 2 \\ x^2 & \text{if } x \leq 2 \end{cases}$ — 2

QUESTION 3 (15 marks) Marks

a) Find the exact value of

$$\tan 60^\circ \times \frac{1}{\sin 45^\circ} \times (\cos 45^\circ)^2$$ 2

b) If $f(x) = 6 - 2x$

(i) find $f(-3)$ 1
(ii) solve $f(t) = 3t$ 2
(iii) solve $f(2-t) \geq 2 - t$ 3

c) If $\sin A = \frac{12}{13}$ and A is acute find the exact value of cot A 3

d) On the same graph sketch the following for $0^\circ \leq \theta \leq 360^\circ$

(i) $y = \cos x$ 2

(ii) $y = \sec x$ 2

QUESTION 4 (15 marks)

a) Given $x = \sqrt{5} + 2$, find b if $x + \frac{1}{x} = 2\sqrt{b}$ 3

b) Solve $2^{3x-1} = \frac{1}{4}$ 2

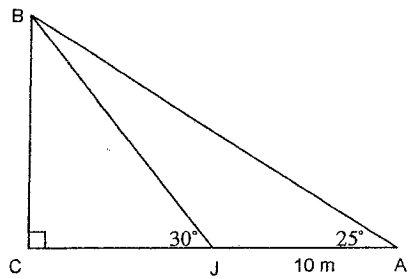
c) State the range of $y = \frac{1}{2x+3} - 3$ 1

d) On the same diagram shade the following region for which

$y \leq x^2$, $y \geq 0$ and $y \leq \sqrt{9 - x^2}$ are simultaneously true. 4

QUESTION 4(continued) Marks

e) Angela measures the angle of elevation to the top of the tower as 25° and Jessica measures the angle of elevation as 30°. Jessica is standing 10metres closer to the tower than Angela.



(i) Copy the diagram onto your exam paper
(ii) Find the height of the tower. 5

THE END

YEAR 11 - HALF-YEARLY 2008
EXTENSION 1 — SOLUTIONS

Question 1

a) $(x+4)^2 = 3(x+4)$

$x^2 + 8x + 16 = 3x + 12$

$x^2 + 5x + 4 = 0$

$(x+4)(x+1) = 0$

$x = -4$ or $x = -1$ (3)

b) $\cos 315^\circ = \cos 45^\circ$

$= \frac{1}{\sqrt{2}}$ (1)

c) $2 + x \geq 0$

$x \geq -2$

and $2 - x \geq 0$

$-x \geq -2$

$x \leq 2$

$\therefore$ Domain: $-2 \leq x \leq 2$ (2)

d) $|2x+1| = 5$

$2x+1 = 5$ or $2x+1 = -5$

$2x = 4$ $\quad 2x = -6$

$x = 2$ $\quad x = -3$ (3)

e) $\tan\theta = \frac{1.7}{2.2}$

$\theta = 37^\circ 42'$ (2)

f) $\cot 270^\circ = 0$ (1)

g) $\frac{1}{2x-3} > 1$

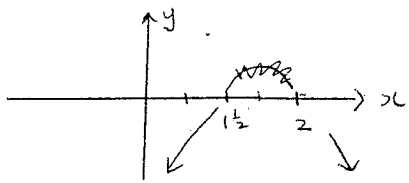
$\frac{1}{2x-3} - 1 > 0$ (3)

$\frac{1 - 1(2x-3)}{2x-3} > 0$

$\frac{1 - 2x + 3}{2x-3} > 0$

$(2x-3)^2 \frac{4-2x}{(2x-3)} > 0 \ (2x-3)^2$

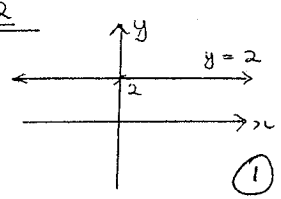
$(2x-3)(4-2x) > 0$



$1\frac{1}{2} < x < 2$

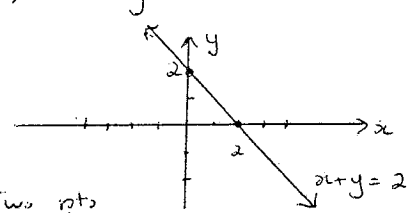
Question 2

a) $y = 2$



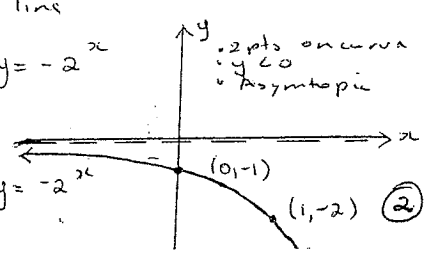
(1)

b) $x + y = 2$



Two pts on line (2)

c) $y = -2^x$

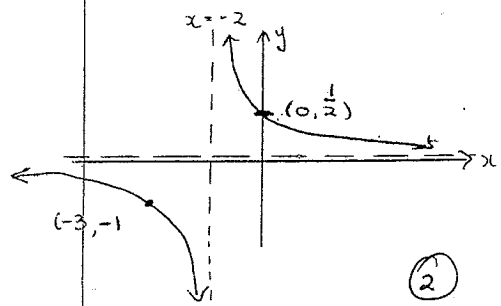


• 2 pts on curve
• $y < 0$
• Asymptote (2)

Question 2 (con't)

d) $y = \frac{1}{x+2}$

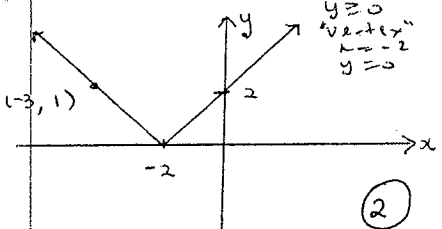
• Asymptote $x = -2$
• pt on branch



(2)

e) $y = |2 + x|$

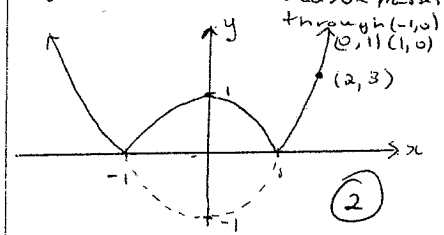
• Show a pt on each branch or right $\angle$.
• $y \geq 0$
• "Vertex" $x = -2$, $y = 0$



(2)

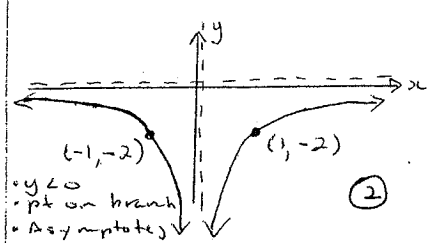
f) $y = |x^2 - 1|$

• $y \geq 0$
• curve passes through $(-1,0)$ $(0,1)$ $(1,0)$



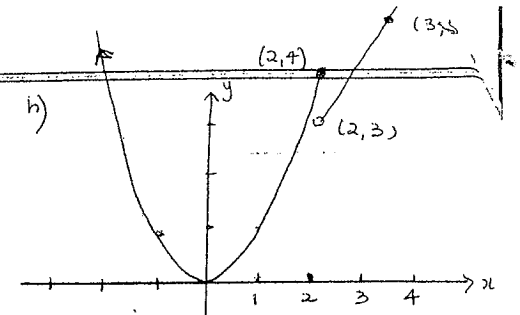
(2)

g) $y = -\frac{2}{x^2}$



• $y < 0$
• pt on branch
• Asymptotes (2)

h)



• graph discont;
• clearly indicate pts of discontinuity
• Show 2 pts on line
• Show parabola continues on LHS.

(2)

Question 3

a) $\sqrt{3} \times \frac{1}{1/\sqrt{2}} \times \left(\frac{1}{\sqrt{2}}\right)^2$

$= \sqrt{3} \times \sqrt{2} \times \frac{1}{2}$

$= \frac{1}{2}\sqrt{6}$ (2)

b) (i) $f(-3) = 6 - 2(-3)$

$= 6 + 6$

$= 12$ (1)

(ii) $f(t) = 3t$

$6 - 2t = 3t$

$6 = 5t$

$\therefore t = 6/5$ (2)

(iii) $6 - 2(2-t) \geq 2 - t$

$6 - 4 + 2t \geq 2 - t$

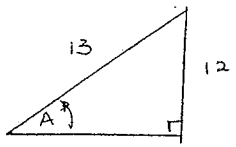
$2 + 2t \geq 2 - t$

$3t \geq 0$

$t \geq 0$ (3)

Question 3 (con't)

c) $\sin A = \frac{12}{13}$, A is acute



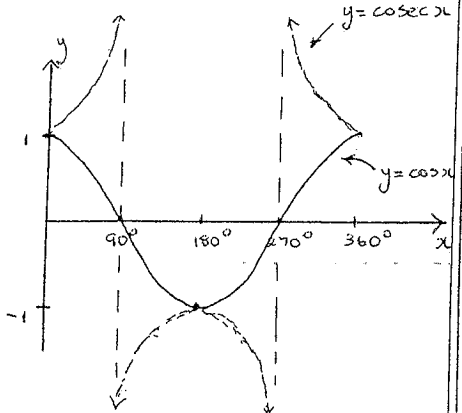
$x = \sqrt{13^2 - 12^2}$

$x = 5$ (3)

$\therefore \cot A = \frac{5}{12}$

d) (i) $y = \cos x$ (2)

(ii) $y = \operatorname{cosec} x$ (2)



Question 4

a)

$\sqrt{5} + 2 + \left(\frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}\right)$

$= \sqrt{5} + 2 + \frac{\sqrt{5}-2}{1}$

Question 4 (con't)

$= 2\sqrt{5}$ (3)

$\therefore b = 5$

b) $2^{3x-1} = 2^{-2}$

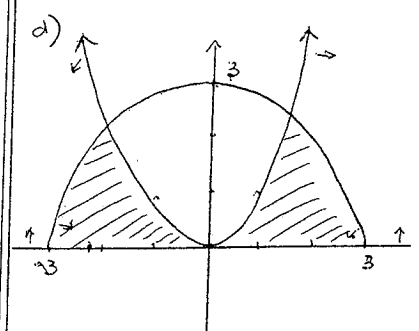
$3x - 1 = -2$

$3x = -1$ (2)

$x = -\frac{1}{3}$

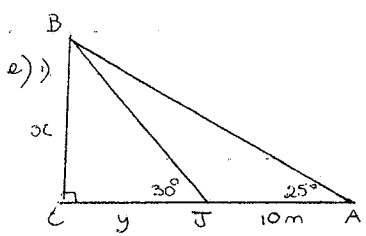
c) Range all real y, except $y = -3$. (1)

d)



(4)

e) (i)



(i) $\tan 30^\circ = \frac{x}{y}$

$y \cdot \tan 30^\circ = x \quad \ldots (1)$

$\tan 25^\circ = \frac{x}{y+10}$

$(y+10) \cdot \tan 25^\circ = x \quad \ldots (2)$

$\therefore y \cdot \tan 30^\circ = y \cdot \tan 25^\circ + 10 \cdot \tan 25^\circ$

$y(\tan 30^\circ - \tan 25^\circ) = 10 \tan 25^\circ$

$\therefore y = \frac{10 \tan 25^\circ}{\tan 30^\circ - \tan 25^\circ}$ (5)

$\therefore x = y \cdot \tan 30^\circ$

$= \frac{10 \cdot \tan 25^\circ \cdot \tan 30^\circ}{\tan 30^\circ - \tan 25^\circ}$

$= 24.25$ m