Sydney Girls High School

2010
MATHEMATICS EXTENSION 1
[May 2010]

YEAR 11
HALF YEARLY

Time Allowed: 60 minutes

Name: _____ **Teacher:** _____

Topics: Functions , Trigonometry, Harder 2 Unit Topics

General Instructions:

- There are FOUR (4) Questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your student number clearly at the top of each question and clearly number each question.

Total: 60 marks

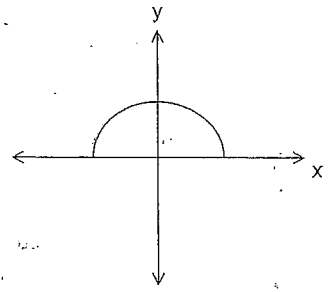
Marks

QUESTION 1 (15 MARKS)

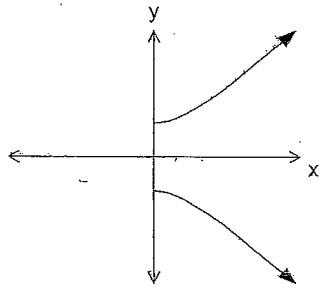
a. Find the exact value of $\sec 300°$ .. 2

b. State whether the following are functions: 2

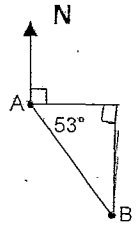
i.



ii.



c. State the bearing of A from B.



d. Sketch $2x+3y-6=0$ showing any intercepts with the coordinate axes. 2

Marks

e. If $\cos x = \frac{2}{5}$ and $270° \le x \le 360°$ find the exact value of $\tan x$. 2

f. Simplify $\frac{\cos\theta}{\cos(90-\theta)}$ 1

g. Simplify $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$. 2

h. On the same axes sketch $y = \sin x$ and $y = \text{cosec}x$ for $0° \le x \le 360°$ 3

QUESTION 2 (15 MARKS)

a. Prove that $\tan^2\theta + \cot^2\theta = \sec^2\theta \text{cosec}^2\theta - 2$ 2

b. Sketch the following function: 3

$$f(x) = \begin{cases} x+1 & \text{for} \quad x < 0 \\ -x^2 & \text{for} \quad 0 \le x < 2 \\ 4 & \text{for} \quad x \ge 2 \end{cases}$$

c. Sketch each of the following showing all essential features:

i. $y = x^2 + x - 12$ 3

ii. $y = \frac{2}{3-x} + 1$ 3

iii. $y = 3^{-x} + 2$ 2

iv. $y = \left|\frac{x}{2} + 2\right|$ 2

Marks

QUESTION 3 (15 MARKS)

a. If $f(x) = x^4 + 3x^2 - 7$, show whether $f(x)$ is odd, even or neither. 2

b. Solve for x, $\tan(x-15) = \cot(2x+35)$. 2

c. Find θ if $\tan\theta = -\frac{1}{\sqrt{3}}$ for $0° \le \theta \le 360°$. 2

d. Solve $4\cos^2\theta - 3 = 0$ for $0° \le \theta \le 360°$. 3

e. State the domain and range of the following:

i. $y = x^2 + 3$ 2

ii. $y = 4^{-x} + 1$ 2

iii. $y = \frac{1}{\sqrt{4-x^2}}$ 2

QUESTION 4 (15 MARKS)

a. For the circle given by $x^2 - 2x + y^2 + 2y = 7$:

i. Find the centre and radius. 2

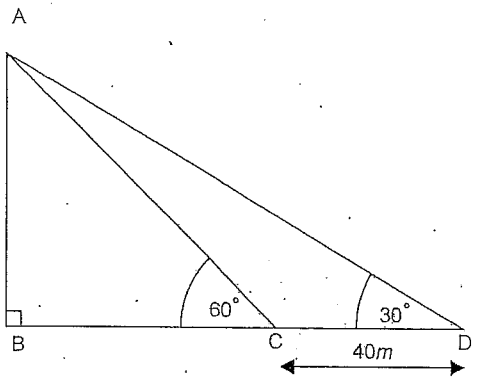
ii. Sketch the circle. 1

b. Using Pascal's triangle or otherwise expand $\left(2 + \sqrt{x}\right)^4$. 3

c. Solve $\frac{2x-3}{4x-5} + 2 < 0$. 3

Marks

d.



i. Show that $BC = \dfrac{40\tan 30}{\tan 60 - \tan 30}$. 2

ii. Hence find BC. 1

e. Solve $3\sin\theta\cos\theta = 2 - \cos^2\theta$ for $0° \le \theta \le 360°$ 3

End of Test

Question One

38/60 = 63.3% Average 0

(a). $\sec 300 = \dfrac{1}{\cos 300}$

$= \dfrac{1}{\cos 60}$

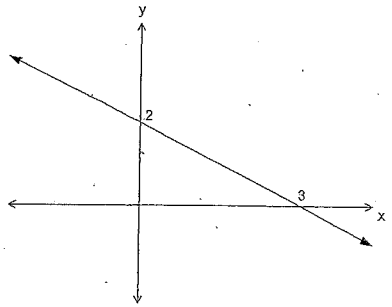
$= \dfrac{1}{\left(\frac{1}{2}\right)}$

$= 2$

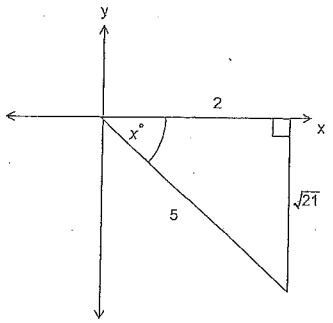
(b). (i). Yes (a function) (ii). No (not a function)

(c). $N\,37°W$ or $323°T$

(d).



(e).



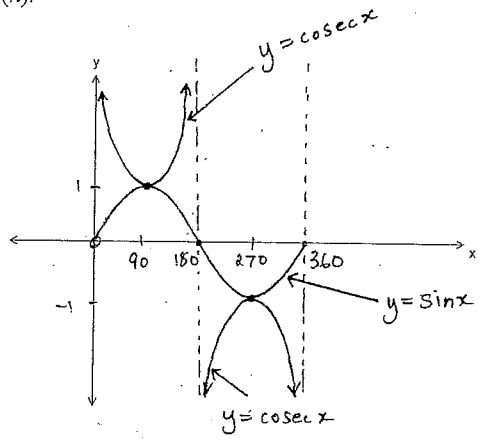
$\tan x = -\dfrac{\sqrt{21}}{2}$

(f). $\dfrac{\cos\theta}{\sin\theta} = \cot\theta$

(g). $\sec^2\theta - \tan^2\theta = \tan^2\theta + 1 - \tan^2\theta$

$= 1$

(h).



Please note : No half marks have been awarded throughout question one.

Question 2

a.

$LHS = \tan^2\theta + \cot^2\theta$

$= \dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta}$

$= \dfrac{\sin^4\theta + \cos^4\theta}{\cos^2\theta\sin^2\theta}$

$= \dfrac{\sin^2\theta(1-\cos^2\theta) + \cos^2\theta(1-\sin^2\theta)}{\cos^2\theta\sin^2\theta}$

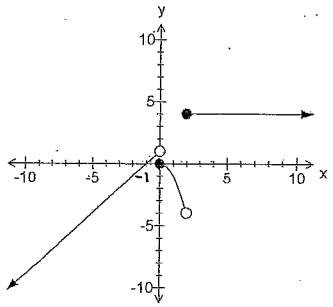
$= \dfrac{1 - 2\cos^2\theta\sin^2\theta}{\cos^2\theta\sin^2\theta}$

$= \dfrac{1}{\cos^2\theta\sin^2\theta} - \dfrac{2\cos^2\theta\sin^2\theta}{\cos^2\theta\sin^2\theta}$

$= \sec^2\theta\,\mathrm{cosec}^2\theta - 2$

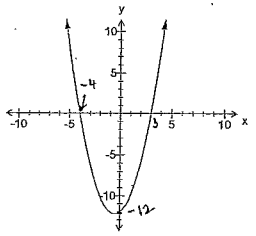
$= RHS$

b.

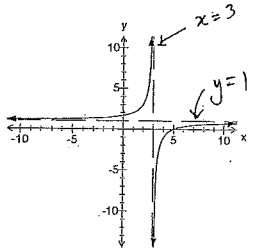


c.

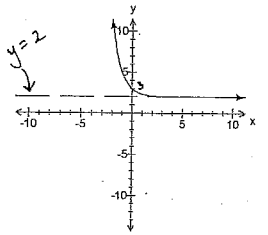
i.



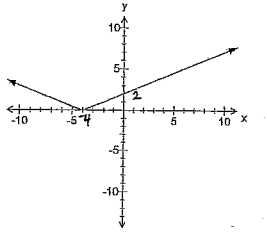
ii.



iii.



iv.



$= x^4 + 3x^2 - 7$ ✓

$= f(x)$

$\therefore f(x)$ is even ✓

b. $x - 15 = 90 - (2x + 35)$ ✓

$x - 15 = 55 - 2x$

$3x = 70$

$x = 23\frac{1}{3}$ ✓

c. $\theta = 180° - 30°$ or $360° - 30°$ ✓

$= 150°$ or $330°$ ✓

d. $4\cos^2\theta = 3$

$\cos^2\theta = \frac{3}{4}$ ✓

$\cos\theta = \pm\frac{\sqrt{3}}{2}$ ✓

$\theta = 30°, 180° - 30°, 180° + 30°, 360° - 30°$

$= 30°, 150°, 210°, 330°$ ✓

e. i. domain: $x \in R$ ✓

range: $y \geq 3$ ✓

ii. domain: $x \in R$ ✓

range: $y > 1$ ✓

iii. $4 - x^2 > 0$

$(2 - x)(2 + x) > 0$

✓

domain: $-2 < x < 2$

range: $y \geq \frac{1}{2}$ ✓

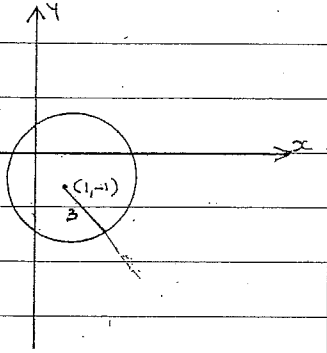
Question 4:

a) i) $x^2 - 2x + y^2 + 2y = 7$

$(x-1)^2 + (y+1)^2 = 9$

$\therefore$ centre $(1,-1)$ radius $= 3$

ii)



b) $(2+\sqrt{x})^4 = 2^4 + 4.2^3\sqrt{x} + 6.2^2(\sqrt{x})^2 + 4.2(\sqrt{x})^3 + (\sqrt{x})^4$

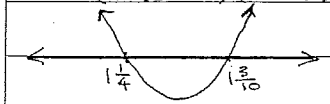
$= 16 + 32\sqrt{x} + 24x + 8x\sqrt{x} + x^2$

c) $\frac{2x-3}{4x-5} + 2 \leq 0$

$\frac{2x-3+2(4x-5)}{4x-5} \leq 0$

$\frac{10x-13}{4x-5} \leq 0$

$(10x-13)(4x-5) \leq 0$



$1\frac{1}{4} < x < 1\frac{3}{10}$

d) i) In ΔABC: $\tan 60 = \frac{AB}{BC}$

$AB = BC\tan 60$ ①

In ΔABD: $\tan 30 = \frac{AB}{BC+40}$

$AB = (BC+40)\tan 30$ ②

As ① = ②

$BC\tan 60 = (BC+40)\tan 30$

$BC\tan 60 = BC\tan 30 + 40\tan 30$

$BC\tan 60 - BC\tan 30 = 40\tan 30$

$BC(\tan 60 - \tan 30) = 40\tan 30$

$BC = \frac{40\tan 30}{\tan 60 - \tan 30}$

ii) $BC = 20m$

c) $3\sin\theta\cos\theta = 2 - \cos^2\theta$

$3\sin\theta\cos\theta = 2(\cos^2\theta + \sin^2\theta) - \cos^2\theta$

$3\sin\theta\cos\theta = \cos^2\theta + 2\sin^2\theta$

$\cos^2\theta - 3\sin\theta\cos\theta + 2\sin^2\theta = 0$

$(\cos\theta - \sin\theta)(\cos\theta - 2\sin\theta) = 0$

$\cos\theta - \sin\theta = 0$ or $\cos\theta - 2\sin\theta = 0$

$\cos\theta = \sin\theta$ $\quad 2\sin\theta = \cos\theta$

$\tan\theta = 1$ $\quad 2\tan\theta = 1$

$\theta = 45°, 225°$ $\quad \tan\theta = \frac{1}{2}$

$\theta = 26°34', 206°34'$