SYDNEY GIRLS HIGH SCHOOL

Year 11

HALF-YEARLY EXAMINATION

MAY 2008 MATHEMATICS

TIME ALLOWED 75 MINUTES

Instructions

There are 4 questions:

Q1 – 20 Marks

Q2 – 20 Marks

Q3 – 17 Marks

Q4 – 18 Marks

Attempt all questions

Show all necessary working, marks may be deducted for badly arranged work.

Start each question on a new page. Write on only one side of the paper.

Question 1 (20 MARKS)

a) Evaluate $\sqrt[3]{\frac{5.67\times10^{15}}{3.5\times10^{3}}}$ expressing your answer in scientific notation correct to 3 significant figures. **2**

b) What number is halfway between $\frac{1}{3}$ and $\frac{1}{5}$? **1**

c) Evaluate $|7|-|-3|\times-4$ **1**

d) Write $0.0\dot{3}\dot{6}$ as a simple fraction. **2**

e) 110% of a number is 84.7. What is the number? **1**

f) Express with a rational denominator $\frac{3}{5\sqrt{2}}$ **1**

g) Express with a rational denominator $\frac{\sqrt{5}-1}{\sqrt{5}+1}$ **2**

h) Simplify fully $10\sqrt{20}+5\sqrt{45}-3\sqrt{125}$ **2**

i) Simplify $\frac{8x^2\times2x}{4x^{-3}}$ **2**

j) Express with a rational index $\sqrt[5]{xy^7}$ **1**

k) If $a=\sqrt{\frac{b^2c}{2}}$ find b when $a=3\sqrt{2}$ and $c=4$ **2**

l) If $\frac{x+3y}{x-y}=6$ then $\frac{x}{y}=?$ **2**

Question 2 (20 MARKS)

a) Expand and simplify

i. $(2x+3)(x+7)$ **2**

ii. $-(x^2+2x)(x+3)$ **2**

iii. $(3x+4)^2$ **2**

b) Fully factorise the following

i. $6x^2+9x$ **2**

ii. $3x^2+11x+10$ **2**

iii. $ab+2ac+3bd+6cd$ **2**

iv. x^3+27 **2**

v. $x^2-(y-2)^2$ **2**

c) Simplify

i. $\frac{2}{x+3}+\frac{2}{x}$

ii. $\frac{x+7}{(x+4)(x-1)} \div \frac{x^2+11x+28}{x-1}$ **2**

Question 3 (17 MARKS)

a) Solve the following

i. $\frac{2x}{3}=11-\frac{x}{4}$ **2**

ii. $2x^2+3x-7=0$ (Leave your answers in exact form) **2**

iii. $\left|\frac{3x+2}{4}\right|=5$ **2**

iv. $|x-3|=2x$ **3**

b) Solve the following equations simultaneously

$2x+y=8$
$x^2+y=23$

c) Solve the following inequalities. Graph each solution on a number line.

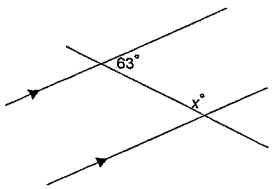
i. $2x+3<7+5x$ **3**

ii. $|2x+4|<7$ **3**

Question 4 (18 Marks)

a) Find the value of the pronumerals, giving reasons for your answer **8**

i)



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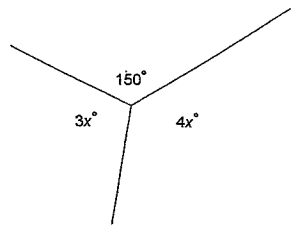

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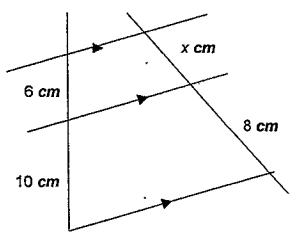

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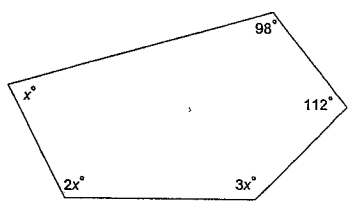
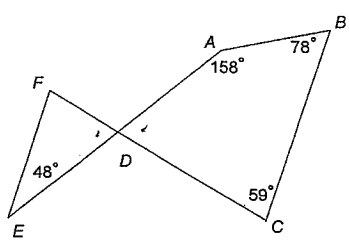
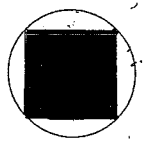


b) Find the size of $\angle DFE$ **2**

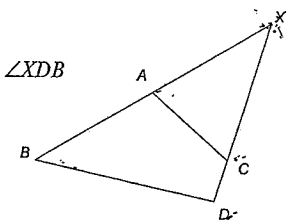


c) If the exterior angles of a regular polygon are each 8^o, how many sides are in the polygon **2**

d) If the circle has a radius of 1cm, find the shaded area of the following figure correct to one decimal place. **2**



e) If $AX.XB = CX.XD$, prove that $\angle XAC = \angle XDB$ **4**



Q1 191157148

a) 11744.60292
= 11 700 (to 3 sig figs.) ✓
(= 1.17×10^4) × $\frac{1}{2}$

b) $\dfrac{\frac{1}{3}+\frac{1}{5}}{2}$

$= \dfrac{\frac{5+3}{15}}{2}$

$= \frac{8}{15} \div 2$
$= \frac{8^4}{15} \times \frac{1}{2_1}$
$= \frac{4}{15}$ ✓

c) $|7| - |-3| \times -4$
$= |7| - (3 \times -4)$
$= 7 - (-12)$
$= 19$ ✓

d) Let $x = 0.0363636$
$10x = 0.363636$
$1000x = 36.363636$
$\therefore 990x = 36$
$x = 36/990$
$= 2/55$ //

e) $110\%\ x = 84.7$
$x = 84.7 \div \frac{110}{100}$
$= 84.7 \times \frac{100}{110}$
$= 77$ ✓

1	$17\frac{1}{2}$	19
2	18	20
3	15	17
4	10	18
5		
6		
7		
8		
Tot	$66\frac{1}{2}$	

Q1 191157148

f) $\dfrac{3}{5\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$
$= \dfrac{3\sqrt{2}}{10}$ ✓

g) $\dfrac{\sqrt{5}-1}{\sqrt{5}+1} \times \dfrac{\sqrt{5}-1}{\sqrt{5}-1}$
$= \dfrac{5-\sqrt{5}-\sqrt{5}+1}{4}$
$= \dfrac{5-2\sqrt{5}+1}{4}$
$= \dfrac{6-2\sqrt{5}}{4}$ ✓ $= \dfrac{3-\sqrt{5}}{2}$.

h) $10\sqrt{20} + 5\sqrt{45} - 3\sqrt{125}$
$= 10\sqrt{4} \times \sqrt{5} + 5\sqrt{9} \times \sqrt{5} - 3\sqrt{25} \times \sqrt{5}$
$= 20\sqrt{5} + 15\sqrt{5} - 15\sqrt{5}$
$= 20\sqrt{5}$ ✓

i) $8x^2 \times 2x$
$= 16x^3$
$= 16x^3 \times x^3$
$= 16x^6$

j) $\dfrac{8x^2 \times 2x}{4x^{-3}}$
$= \dfrac{16x^3}{4 \times \frac{1}{x^3}}$
$= \dfrac{16x^3}{\frac{4}{x^3}}$
$= 16x^3 \times \dfrac{x^3}{4}$
$= 4x^6$ ✓

Q1 19157148

i) $\sqrt[5]{xy^7}$

$= \sqrt[5]{x} \times \sqrt[5]{y^7}$

$= x^{1/5} \times y^{7/5}$ ✓

$= x^{1/5} \times y^{1\frac{2}{5}}$

ii) $3\sqrt{2} = \sqrt{\frac{b^2 \times 4}{2}}$

$\left(3\sqrt{2}\right)^2 = \left(\sqrt{\frac{4b^2}{2}}\right)^2$

$18 = \frac{4b^2}{2}$

$4b^2 = 36$

$b^2 = 9$

$\therefore b = \pm\sqrt{9}$

$b = \pm 3$ //

iii) $\frac{x+3y}{x-y} = 6$

$x + 3y = 6x - 6y$ ✓

$9y = 5x$

$\therefore x = 9, y = 5$

$\frac{x}{y} = ?$ X

17 1/2

Q2 19157148

a) i) $(2x+3)(4x+7)$

$= 2x^2 + 14x + 3x + 21$

$= 2x^2 + 17x + 21$ ✓ 2

ii) $-(x^2+2x)(x+3)$

$= (-x^2 - 2x)(x+3)$

$= -x^3 - 3x^2 - 2x^2 - 6x$ ✓ 2

$= -x^3 - 5x^2 - 6x$

iii) $(3x+4)^2$

$= 9x^2 + 24x + 16$ ✓ 2

18/20

b) i) $6x^2 + 9x$

$= 3x(2x+3)$ ✓ 2

ii) $3x^2 + 11x + 10$

$= 3x^2 + 6x + 5x + 10$

$= 3x(x+2) + 5(x+2)$ ✓ 2

$= (3x+5)(x+2)$

iii) $ab + 2ac + 3bd + 6cd$

$= a(b+2c) + 3d(b+2c)$

$= (a+3d)(b+2c)$ ✓ 2

iv) $x^3 + 27$

$= x^3 + 3^3$ ✓ 2

$= (x+3)(x^2 - 3x + 9)$

v) $x^2 - (y-2)^2$

$= x^2 - (y^2 - 4y + 4)$

$= x^2 - y^2 + 4y - 4$ X

$= (x+y)(x-y) + 4(y-1)$

Q2

c) i) $\frac{2}{x+3} + \frac{2}{x}$

$= \frac{2x + 2x+6}{x(x+3)}$

$= \frac{4x+6}{x(x+3)}$

$= \frac{2(2x+3)}{x(x+3)}$ ✓ 2

ii) $\frac{x+7}{(x+4)(x-1)} \times \frac{x-1}{(x^2+11x+28)}$

$= \frac{x+7}{(x+4)(x-1)} \times \frac{x-1}{(x+7)(x+4)}$

$= \frac{1}{(x+4)^2}$ ✓ 2

Q3

a)i) $\frac{2x}{3} = \frac{11}{1} - \frac{x}{4}$

$\frac{2x}{3} = \frac{44 - x}{4}$

$132 - 3x = 8x$

$132 = 11x$

$\therefore x = 12.$ ✓ 2

ii) $2x^2 + 3x - 7 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-3 \pm \sqrt{3^2-(4 \times 2 \times -7)}}{2(2)}$

$= \frac{-3 \pm \sqrt{65}}{4}$ ✓ 2

$\therefore x = \frac{-3+\sqrt{65}}{4}$ and $x = \frac{-3-\sqrt{65}}{4}$

iii) $\left| \frac{3x+2}{4} \right| = 5$

$\frac{3x+2}{4} = 5$ OR $\frac{-3x-2}{4} = 5$

$3x+2 = 20$ $-3x-2 = 20$

$3x = 18$ $-3x = 22$

$x = 6$ $x = -22/3.$

$(\therefore x = 6, x = -22/3)$ ✓ 2

Check: $x = 6$

$\left| \frac{18+2}{4} \right| = 5$

$|5| = 5$

$\therefore$ true

Check: $x = -22/3$

$\left| \frac{-22+2}{4} \right| = 5$

$|-5| = 5$

$\therefore$ true

(∴ both $x = 6$, $x = -22/3$ are solutions)

Q3

19157148

a) iv) $|x-3| = 2x$

$x-3 = 2x$ or $-x+3 = 2x$

$-3 = x$ $3 = 3x$

$x = 1$

Check $x=-3$

$|-6| = 2(-3)$

$|-6| \neq -6$ ∴ false

Check $x=1$

$|-2| = 2$

∴ true

∴ only $x=1$ is the solution

3

b) $2x+y = 8$ ... ①

$x^2+y = 23$ ... ②

from ① $y = 8-2x$

from ② $y = 23-x^2$

$\therefore 8-2x = 23-x^2$

$x^2-2x-15 = 0$

$(x-5)(x+3) = 0$

$\therefore x=5$ or $x=-3$ ✓

$y=?$ $y=?$

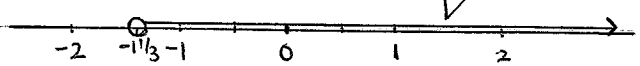
1

c) i) $2x+3 < 7+5x$

$-4 < 3x$

$-4/3 < x$

$x > -4/3$



3

ii) $|2x+4| < 7$

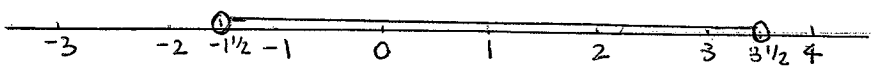
$-7 < 2x+4 < 7$

$-7-4 < 2x+4-4 < 7-4$

$-3 < 2x < 7$

$-1\frac{1}{2} < x < 3\frac{1}{2}$ ✗

2



Q4

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a) i) $x° = 180° - 63°$ (cointerior ∠s ; // lines) ✓

$= 117°$

ii) $3x° + 4x° + 150° = 360°$ (∠s at a point)

$7x° + 150° = 360°$

$7x° = 210°$ ✓

$x° = 30°$

iii) $\frac{6}{10} = \frac{x}{8}$ (proportional intercept theorem)

$10x = 48$ ✓

$x = 4.8$

iv) $x° + 2x° + 3x° + 112° + 98° = 360°$ (∠ sum of quadrilateral)

$6x° + 210° = 360°$

$6x° = 150°$

$x° = 25°$

1

b) $\angle ADC = 360° - 158° - 78° - 59°$ (∠ sum of quadrilateral)

$= 65°$

$\angle FDE = \angle ADC$ (vertically opp ∠s)

$\therefore \angle FDE = 65°$ ✓

1

c) Exterior ∠ sum of polygon $= 360°$

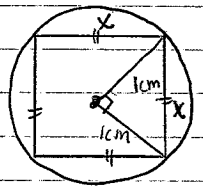
$\frac{360°}{8°}$ = sides in polygon ✓

∴ there are 45 sides in the polygon

Q 4

19157148

d)



$x^2 = 1^2 + 1^2$ (the angle is a right ∠, we know because the diagonals of a square bisect each other at 90°)

∴ We can use Pythag. theorem

$x^2 = 1^2 + 1^2$

$x = \sqrt{2}$

∴ Area of square $= x^2$

$= (\sqrt{2})^2$

$= 2\,cm^2$ ✓

[illegible]

e) $AX \cdot XB = CX \cdot XD$

can also be written as:

$\frac{AX}{XD} = \frac{CX}{XB}$

In $\triangle XAC$ and $\triangle XDB$:

$\angle AXC = \angle DXB$ (common ∠)

$\frac{AX}{XD} = \frac{CX}{XB}$ (given) ✓

∴ $\triangle XAC \,|||\, \triangle XDB$ (two sides in equal ratio and included angle equal) ✓

∴ $\angle XAC = \angle XDB$ (corresponding ∠s in similar triangles)