Sydney Girls High School

YEAR 11

MATHEMATICS

Half Yearly Examination 2010

Time Allowed: 60 minutes

Total Marks: 60

Topics: Basic Arithmetic and Algebra, Plane Geometry

Instructions:

- Attempt ALL questions
- There are 4 questions, each worth 15 marks.
- Show all necessary working. Full marks may not be awarded for careless or incomplete working.
- Begin each question on a new page.
- Diagrams are NOT to scale.

Name:_____ **Class:** _____

QUESTION 1 Marks

a) Find, correct to 3 significant figures, the value of: 2

$$\frac{(8.7-3.9)^2}{\sqrt{4.375}}$$

b) Evaluate: $4\left|-2\right|^3+6\left|\frac{-1}{2}\right|$ 1

c) Find the exact value of: $49^{-\frac{1}{2}}\times 27^{\frac{2}{3}}$ 1

d) Expand and simplify: $\left(2x^3-3\right)^2$ 1

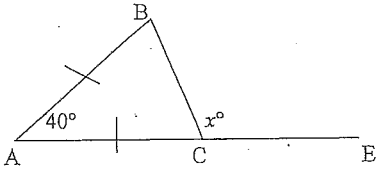
e) Simplify: $-9x^3y^5\div 36x^3y^2$ 1

f) Simplify fully: $\sqrt{98}-3\sqrt{48}-\sqrt{12}$ 2

g) Express xy^n in scientific notation where $x=5.5\times10^3$, $y=4.0\times10^{-2}$ and $n=3$. 2

h) Express $2.5\dot{4}$ as a fraction in its simplest form. Show all working. 2

i) In the diagram, AB = AC and $\angle BAC = 40°$, find $x°$, giving reasons. 3



NOT TO SCALE

QUESTION 2 (Begin a new page) Marks

a) Solve: $\frac{2x+3}{3}-\frac{3x-1}{4}=-3$ 2

b) Factorise completely:

i. $xy+27-3x-9y$ 2

ii. $8x^3-125y^3$ 2

iii. $4x^2-8x+3$ 2

c) Solve the following, for all real values of x:

i. $2-3(x-2)\leq 11$ 2

ii. $|2x-5|=5x+1$ 3

d) Three of the angles of a pentagon are 98°, 112° and 114°. If the other two angles are equal, what is their size? Show all working. 2

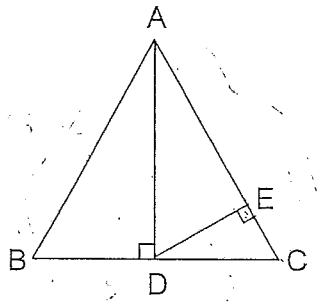
QUESTION 3 (Begin a new page) Marks

a) Simplify: $\frac{3}{x^2-4}-\frac{2}{x^2-3x+2}$ 4

b) Solve: $\left|\frac{3x-1}{2}\right|>1$ and graph the solution on a number line. 3

c) Solve the following equation by *completing the square*: $2x^2+7x=4$. 3

d) In the diagram $AB=AC$, $AD\perp BC$ in ΔABC and $DE\perp AC$ in ΔDAC.



NOT TO SCALE

i. Show that ΔABD and ΔDCE are similar. Give reasons. 3

ii. If $AB=5$ and $BD=CD=3$, find the length of AE. 2

QUESTION 4 (Begin a new page) Marks

a) Simplify $\sqrt[3]{\frac{-8m^6}{27m^{12}}}$ 2

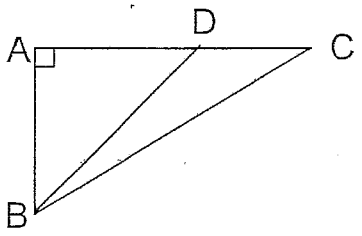
b) Given $a=\frac{\sqrt{5}+1}{\sqrt{5}-1}$, evaluate $\left(a+\frac{1}{a}\right)$, without the use of a calculator. 3

c) i. Express $x^{-1}+y^{-1}$ as a single fraction 2

ii. Hence show that: $\frac{x^2-y^2}{x^{-1}+y^{-1}}=xy(x-y)$ 2

d) Solve simultaneously: $\begin{cases} y=x^2+4 \\ 4x+y=1 \end{cases}$ 3

e) 3



NOT TO SCALE

Triangle ABC is right angled at A
D is a point on AC such that AD = 2DC.

Prove that $5DC^2 = BC^2 - BD^2$

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Year 11 Mathematics Half Yearly Examination 2010 SOLUTIONS

Question 1

a) 11.0

b) $4\times 8+3=35$

c) $\frac{1}{\sqrt{49}}\times\left(\sqrt[3]{27}\right)^2$

$=\frac{1}{7}\times 9$

$=\frac{9}{7}$

d) $4x^6-12x^3+9$

e) $-\frac{y^3}{4}$

f) $7\sqrt{2}-12\sqrt{3}-2\sqrt{3}$

$=7\sqrt{2}-14\sqrt{3}$

g) 3.52×10^{-1}

h) $x = 2.54444\ldots$

$10x = 25.44444\ldots$

$100x = 254.4444\ldots$

$90x = 229$

$x=\frac{229}{90}$

i) $\angle ABC=\angle ACB=y$ (base $\angle$s of an isosceles Δ)

$2y+40=180$ ($\angle$ sum of Δ)

$2y=140$

$y=70$

$x=70+40$ (Ext. $\angle$ = opposite interior $\angle$ sum)

$x=110$

Question 2

a) $\frac{2x+3}{3}-\frac{3x-1}{4}=-3$

$4(2x+3)-3(3x-1)=-3\times 12$

$8x+12-9x+3=-36$

$-x+15=-36$

$x=51$

b)

i. $xy+27-3x-9y$

$=x(y-3)-9(y-3)$

$=(y-3)(x-9)$

ii. $8x^3-125y^3$

$=(2x-5y)(4x^2+10xy+25y^2)$

iii. $4x^2-8x+3$

$=4x^2-6x-2x+3$

$=2x(2x-3)-(2x-3)$

$=(2x-3)(2x-1)$

c)

i. $2-3(x-2)\le 11$

$2-3x+6\le 11$

$-3x\le 3$

$x\ge -1$

ii. $|2x-5|=5x+1$

$2x-5=5x+1$ or $-2x+5=5x+1$

$\therefore x=-2$ $\therefore x=\frac{4}{7}$

Check: $|-4-5|\neq 5(-2)+1$

Check: $\left|2\times\frac{4}{7}-5\right|=5\times\frac{4}{7}+1$

$\therefore$ Only solution is $x=\frac{4}{7}$

d) Angle sum of pentagon $=3\times 180^\circ=540^\circ$

$\therefore 98+112+114+2x=540$

$2x=216$

$x=108^\circ$

Question 3

a) $\dfrac{3}{x^2-4}-\dfrac{2}{x^2-3x+2}$

$=\dfrac{3}{(x-2)(x+2)}-\dfrac{2}{(x-2)(x-1)}$

$=\dfrac{3(x-1)-2(x+2)}{(x-1)(x-2)(x+2)}$

$=\dfrac{x-7}{(x-1)(x-2)(x+2)}$

b) $\left|\dfrac{3x-1}{2}\right|>1$

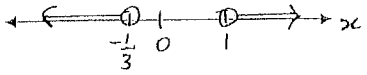
$-\left(\dfrac{3x-1}{2}\right)>1$

$\dfrac{3x-1}{2}>1$ or $\dfrac{3x-1}{2}<-1$

$3x-1>2$ or $3x-1<-2$

$3x>3$ or $3x<-1$

$x>1$ or $x<-\dfrac{1}{3}$



c) $2x^2+7x=4$

$x^2+\dfrac{7}{2}x=2$

$x^2+\left(\dfrac{7}{2}\right)x+\left(\dfrac{7}{2}\times\dfrac{1}{2}\right)^2=2+\left(\dfrac{7}{2}\times\dfrac{1}{2}\right)^2$

$\left(x+\dfrac{7}{4}\right)^2=\dfrac{81}{16}$

$x+\dfrac{7}{4}=\pm\dfrac{9}{4}$

$x=-\dfrac{7}{4}\pm\dfrac{9}{4}$

$x=\dfrac{1}{2},-4$

d)

i.

In ΔABD and ΔDCE

$\angle ABD=\angle DCE$ (base $\angle$s of an isosceles ΔABC)

$\angle ADB=\angle DEC=90°$ (given)

$\therefore \Delta ABD \,///\, \Delta DCE$ (equiangular)

ii.

$\dfrac{CE}{BD}=\dfrac{DC}{AB}$

$\therefore CE=\dfrac{3\times3}{5}=\dfrac{9}{5}$

$AE=AC-CE$

$=5-\dfrac{9}{5}$

$=\dfrac{16}{5}$

Question 4

a) $\sqrt[3]{\dfrac{-8m^6}{27m^{12}}}=\left(\dfrac{-8m^6}{27m^{12}}\right)^{1/3}$

$=\dfrac{-2m^2}{3m^4}$

$=\dfrac{-2}{3m^2}$

b) If $a=\dfrac{\sqrt{5}+1}{\sqrt{5}-1}$

Then $a+\dfrac{1}{a}$

$=\dfrac{\sqrt{5}+1}{\sqrt{5}-1}+\dfrac{\sqrt{5}-1}{\sqrt{5}+1}$

$=\dfrac{\left(\sqrt{5}+1\right)^2+\left(\sqrt{5}-1\right)^2}{5-1}$

$=\dfrac{5+2\sqrt{5}+1+5-2\sqrt{5}+1}{5-1}$

$=\dfrac{12}{4}$

$=3$

c) i.

$x^{-1}+y^{-1}=\dfrac{1}{x}+\dfrac{1}{y}$

$=\dfrac{x+y}{xy}$

ii.

$\dfrac{x^2-y^2}{x^{-1}+y^{-1}}=\dfrac{(x+y)(x-y)}{1}\times\dfrac{xy}{x+y}$

$=xy(x-y)$

d) $\begin{cases} y=x^2+4 \\ 4x+y=1 \end{cases}$

$y=x^2+4$.....1

$4x+y=1$

$y=1-4x$.....2

$1=2$

$1-4x=x^2+4$

$0=x^2+4x+3$

$0=(x+3)(x+1)$

$\left.\begin{matrix} x=-3 \\ y=13 \end{matrix}\right\}$, $\left.\begin{matrix} x=-1 \\ y=5 \end{matrix}\right\}$

e) In ΔABC: $AB^2=BC^2-AC^2$.....1

In ΔABD: $AB^2=BD^2-AD^2$.....2

From 1 and 2

$BC^2-AC^2=BD^2-AD^2$.....3

Now

$AD=2DC$ and $AC=3DC$

$\therefore AD^2=4DC^2$ $\therefore AC^2=9DC^2$

Substitute in 3

$BC^2-9DC^2=BD^2-4DC^2$

$BC^2-BD^2=5DC^2$