SYDNEY GIRLS HIGH SCHOOL

2009

YEAR 11 HALF YEARLY EXAMINATION

Mathematics Extension 1

General Instructions

- Working time – 1 hour
- Board approved calculators may be used.
- Diagrams are NOT drawn to scale.
- All necessary working should be shown in every question.
- Start each question on a new page.

Total marks – 60

- Attempt Questions 1 – 4.
- All questions are of equal value.

Question 1: (15 marks)

a) Factorise $x^4y - xy^4$ fully. **3**

b) Solve $x^2 - 5x - 6 = 0$. **3**

c) Find the number of sides of a regular polygon if each interior angle is $172°$. **2**

d) If $f(x) = 2 - x^2$, find $f(2t)$. **2**

e) If $\sin\alpha = \frac{2}{3}$ and α is an acute angle, find the exact value of $\cos\alpha$. **3**

f) Find x if $\cos(2x+5)° = \sin(3x+10)°$. **2**

Question 2: (15 marks) *START ON A NEW PAGE.*

a) Find the domain and range of each of the following functions: **6**

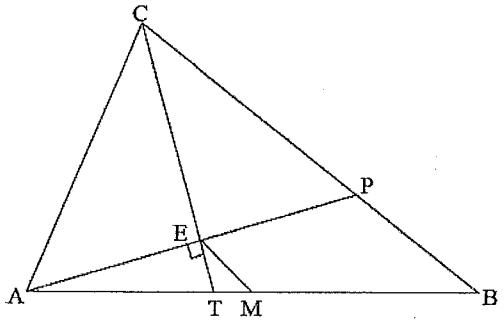
i) $4x - 3y + 2 = 0$

ii) $y = x^2 + 4$

iii) $y = \sqrt{4 - x^2}$

b) Determine whether the function $f(x) = 3x^2 - 4x$ is even, odd or neither. **3**

c) In the diagram, CT bisects $\angle ACB$. $AE \perp CT$ and M is the midpoint of AB. AE produced meets BC at point P.



i) Prove that $\Delta ACE \equiv \Delta PCE$. **3**

ii) Explain why $AE = PE$. **1**

iii) Hence show that $EM \parallel PB$. **2**

Question 3: (15 marks) *START A NEW PAGE.*

a) Draw a neat sketch of the function $y=3-\frac{1}{x+1}$, showing all key features. **3**

b)

i) Find the points of intersection of $y=x^2$ and $x-y+2=0$. **3**

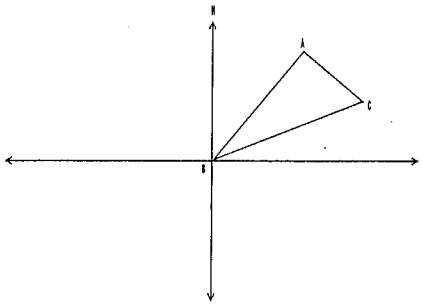
ii) Sketch both curves on the same number plane. **3**

iii) Shade the region satisfied by both $y \geq x^2$ and $x-y+2 \geq 0$. **2**

c) A, B and C are three ships. The bearing of A from B is $45°$ and the bearing of C from A is $135°$. $AB=8\text{km}$ and $AC=6\text{ km}$.

i) Copy the diagram onto your answer sheet, showing all given information. **1**

ii) Find the bearing of C from B. **3**



Question 4: (15 marks) *START A NEW PAGE.*

a) Find the centre and radius of the circle given by $x^2-4x+y^2+4y+4=0$ **2**

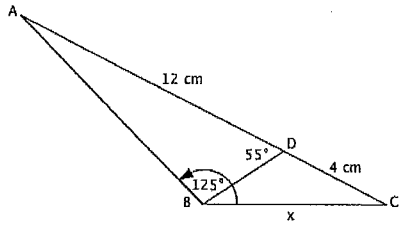
b) A function is defined as follows:

$$f(x)=\begin{cases} x^2 & \text{for } x \leq 1 \\ 2-x & \text{for } x>1 \end{cases}$$

i) Evaluate $f(-2)+f(3)$. **2**

ii) Draw the graph of $f(x)$. **2**

c) In ΔABC, $\angle ABC=125°$, $\angle ADB=55°$, $AD=12\text{cm}$ and $DC=4\text{ cm}$.



i) Show that $\Delta ABC \,|||\, \Delta BDC$. **3**

ii) Find x, the length of BC. **3**

d) Solve $\frac{4}{x+1}<3$ **3**

END OF EXAM

2009 YEAR 11 EXTENSION 1 MATHEMATICS – HALF YEARLY SOLUTIONS

Question 1:

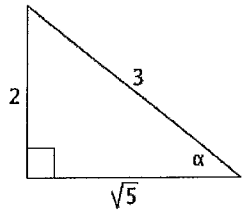
a) $x^4y - xy^4 = xy(x^3 - y^3)$
$= xy(x-y)(x^2+xy+y^2)$

b) $x^2 - 5x - 6 = 0$
$(x-6)(x+1) = 0$
$x = 6, x = -1$

c) Ext $\angle = 180^0 - 172^0$
$= 8^0$
No. sides $= 360^0 \div 8^0$
$= 45$

d) $f(x) = 2 - x^2$
$f(2t) = 2 - (2t)^2$
$= 2 - 4t^2$

e)



$\cos\alpha = \frac{\sqrt{5}}{3}$

f) $(2x+5) + (3x+10) = 90$
$5x + 15 = 90$
$5x = 75$
$x = 15$

Question 2:

a) i. D: all real x
R: all real y

ii. D: all real x
R: $y \geq 4$

iii. D: $-2 \leq x \leq 2$
R: $0 \leq y \leq 2$

b) $f(x) = 3x^2 - 4x$
$f(-x) = 3(-x)^2 - 4(-x)$
$= 3x^2 + 4x$
$-f(x) = -3x^2 + 4x$

As $f(x) \neq f(-x)$ and $f(-x) \neq f(-x)$, $f(x)$ is neither even nor odd.

c) i. In ΔACE and ΔPCE:
$\angle CEA = \angle CEP$ $(AE \perp CT$; given$)$
$\angle ACE = \angle PCE$ $(CT$ bisects $\angle ACB$; given$)$
CE is common
$\therefore \Delta ACE \equiv \Delta PCE\,(AAS)$

ii. Corresponding sides in congruent triangles

iii. $AM = BM$ (M is midpt of AB; given)
$AE = PE$ (proven)
$\therefore EM \,||\, PB$ (equal intercepts)
or use "a line joining the midpoints of two sides of a triangle is parallel to the third side and half its length".

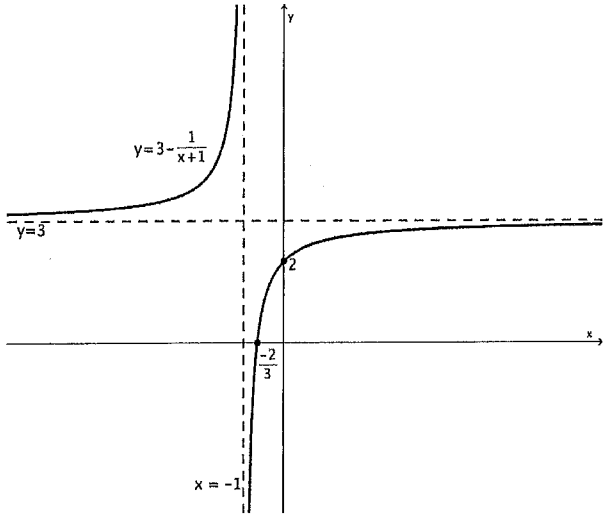
Question 3:

a) $y = 3 - \dfrac{1}{x+1}$

Vertical asymptote: $x = -1$

Horizontal asymptote: $y = 3$

x intercept $(y=0)$: $x = -\dfrac{2}{3}$ $\quad$ y intercept $(x=0)$: $y = 2$



b) i) $y = x^2 ---(1)$ and $y = x + 2 ---(2)$

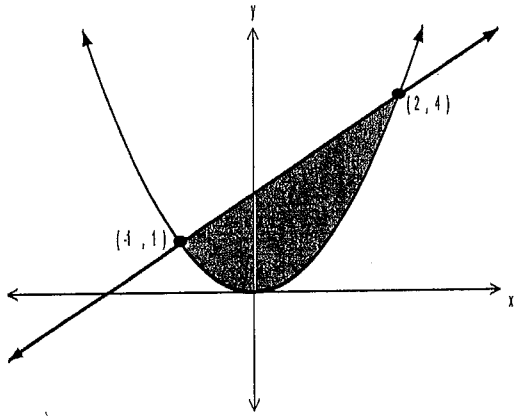
$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2, x = -1$$

When $x = 2: y = 4$

When $x = -1: y = 1$

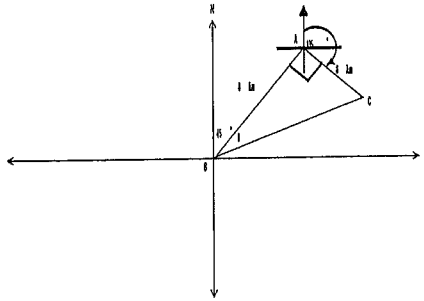
$\therefore$ Points of intersection are $(2,4)$ and $(-1,1)$

ii)



iii) See ii)

c) i)



ii) $\tan\theta = \dfrac{6}{8}$

$\theta = 36.8699...^0$

$\cong 36^0 52'$

Bearing of C from B: $45^0 + 36^0 52'$

$\cong 082^0 T$ (nearest degree)

Question 4:

a) $x^2-4x+y^2+4y+4=0$

$(x-2)^2+(y+2)^2=4$

Centre: $(2,-2)$

Radius: 2 units

b) i)

$f(-2)=(-2)^2$

$=4$

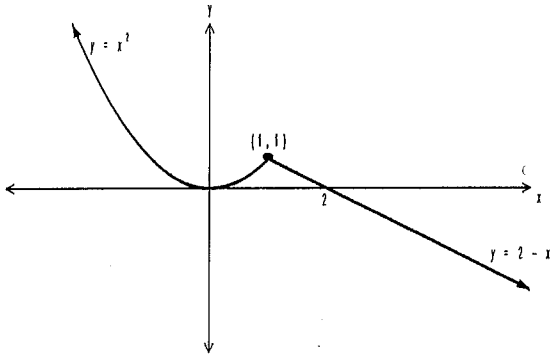
$f(3)=2-3$

$=-1$

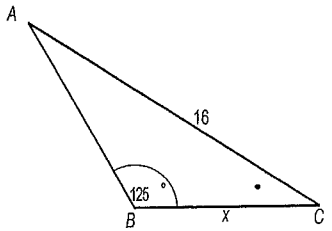
$f(-2)+f(3)=4-1$

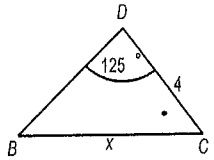
$=3$

ii)



c) i)





$\angle BDC=125^0$ (adj. supp. $\angle$s)

$\therefore \angle ABC=\angle BDC$

In ΔABC and ΔBDC:

$\angle ABC=\angle BDC$ (proven)

$\angle ACB=\angle BCD$ (common)

$\therefore \Delta ABC \,|||\, \Delta BDC$ (equiangular)

ii)

$\frac{x}{4}=\frac{16}{x}$ (corres. sides similar Δs in same ratio)

$x^2=64$

$x=8$ $(\text{as } x>0)$

d)

$\frac{4}{x+1}<3$

$4(x+1)<3(x+1)^2$

$3(x+1)^2-4(x+1)>0$

$(x+1)\left[3(x+1)-4\right]>0$

$(x+1)(3x-1)>0$

$x<-1 \text{ or } x>\frac{1}{3}$