

SYDNEY GIRLS HIGH SCHOOL



YEAR 11

HALF-YEARLY EXAMINATIONS

MAY 2002

MATHEMATICS

Time allowed: 75 minutes

Instructions:

- There are EIGHT (8) questions.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work.
- Start each question on a new page. Write on one side of the paper only.

Name:

Question 1. 12 marks

a) Use your calculator to evaluate correct to three significant figures

i) $2500(1.015)^{24}$ ii) $\frac{(1.76)^3 - 0.65}{\sqrt{2.15}}$

b) Express as a fraction in its simplest form

i) $0.\dot{6}\dot{7}$ ii) $0.6\dot{7}$

c) Write down the answer to

i) $|4-9| - |6-9|$ ii) $32 \div 2 \div 2 \div 2$

Question 2: 12 marks

a) Simplify i) $\sqrt{50} + 2\sqrt{8}$ ii) $(2\sqrt{3})^3$

b) Express with a rational denominator i) $\frac{2}{3\sqrt{2}}$ ii) $\frac{6}{\sqrt{7}+2}$

c) If $x = \sqrt{5} + 2$, find the value of i) $x - \frac{1}{x}$ ii) $x^2 + \frac{1}{x^2}$

Question 3: 12 marks

Simplify the below expressions

i) $5 - (a - 2)$ ii) $3a(2a+1) + a(a+5)$ iii) $\frac{2a-1}{3} + \frac{a+4}{4}$

iv) $a - \frac{a+2}{2}$ v) $\frac{a-5}{5-a}$ vi) $\frac{2(3a-5)}{5} - \frac{3(a+4)}{4}$

Question 4: 12 marks

Fully factorise the following expressions

i) $x^2 - 4y^2$ ii) $x^2 - 4x - 5$ iii) $x^3 - 8y^3$

iv) $3x^2 - 5x - 2$ v) $12 - 5x - 3x^2$ vi) $x + 2y + x^2 - 4y^2$

Question 5: 12 marks

Solve the following equations to find the pronumerals

i) $x - \frac{x}{4} = 4$ ii) $3 - 2(x-1) = 7$ iii) $|2x-1| = 11$

iv) $|x+1| = 3x-1$ v) $\begin{cases} 3x - y - 5 = 0 \\ x + 3y + 25 = 0 \end{cases}$ vi) $\frac{3x+1}{2x+4} = \frac{3x+2}{2x+5}$

Question 6: 12 marks

a) Write the exact answers to the below expression

i) $\frac{4^4}{8^2}$ ii) $27^{-\frac{2}{3}}$

b) Simplify the expressions

$$\frac{25 - x^2}{x^2 + 3x - 10} \div \frac{125 - x^3}{x^2 - 4}$$

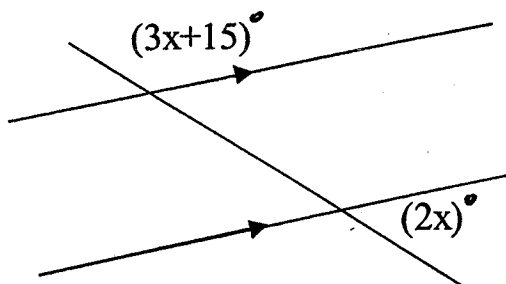
c) Solve the equation $3x^2 - 6x - 1 = 0$ and write the answer as a simplified surd

Question 7: 16 marks

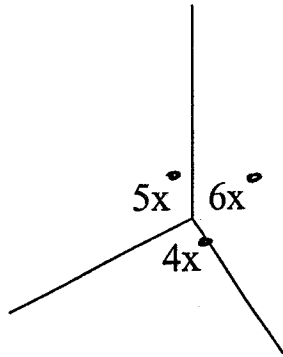
a) A regular polygon has internal angles of 165° . How many sides are there in the polygon?

b) Find the value of the below pronumerals, giving reasons

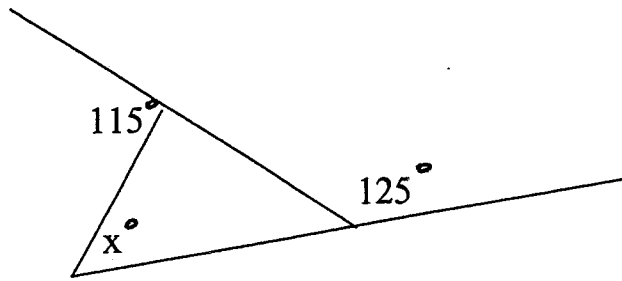
i)



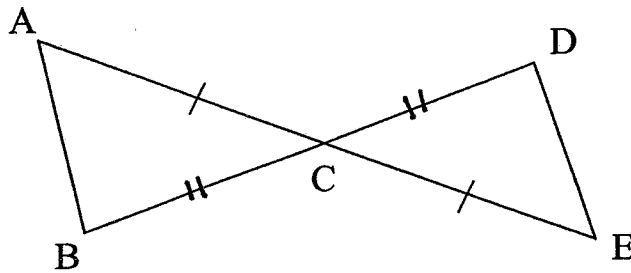
ii)



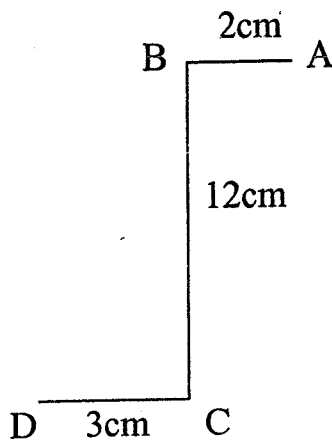
iii)



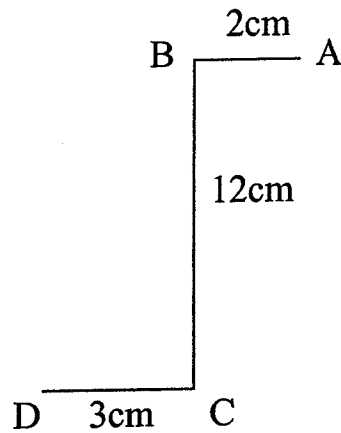
c) Prove the triangles ABC and CDE congruent and hence show that $AB \parallel DE$



d) Find the distance AD in the below diagram



Find the distance ^{of} from AD in the below diagram



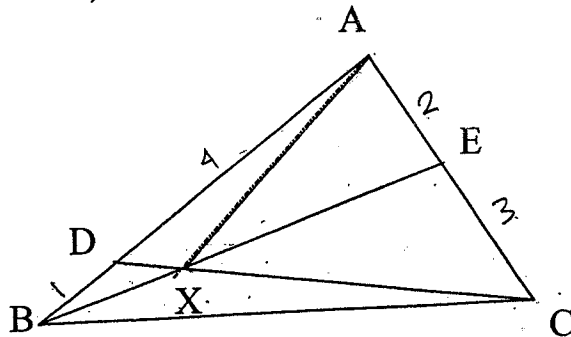
Question 8: 12 marks

a) Fully factorise the expression: $(2x+1)^2 - (x+2)^2$

b) If $\frac{6^{\frac{2}{3}} \times 10^{\frac{1}{3}}}{30} = 3^a \cdot 5^b$, find the values of a and b

c) In the below diagram, the area of triangle ABC is 85 square centimetres. If D divides AB in the ratio 4:1 and E divides AC in the ratio 2:3, find

- i) Area of triangle ADC
- ii) Area of triangle AEB
- iii) Ratio of the area of triangle ADX to triangle BDY
- iv) The area of triangle AXC



***** END OF EXAMINATION *****

$$a) i) 2500(1.015)^{24} \div 3570$$

$$ii) \frac{(1.76)^3 - 0.65}{\sqrt{2.15}} \div 3.27$$

$$b) i) \text{ Let } x = 0.676767 \dots$$

$$\therefore 100x = 67.6767 \dots$$

$$-x = -0.6767$$

$$\therefore 99x = 67 \Rightarrow x = \frac{67}{99}$$

$$ii) \text{ Let } x = 0.6777 \dots$$

$$\therefore 10x = 6.7777 \dots$$

$$-x = -0.6777$$

$$\therefore 9x = 6.1 \Rightarrow x = \frac{61}{90}$$

$$c) i) 14 - 9 - |6 - 9| = 5 - 3 = 2$$

$$ii) 32 \div 2 \div 2 \div 2 = 4$$

$$22) i) \sqrt{50} + 2\sqrt{8} = 5\sqrt{2} + 4\sqrt{2} = 9\sqrt{2}$$

$$ii) (2\sqrt{3})^3 = 8 \times 3\sqrt{3} = 24\sqrt{3}$$

$$b) i) \frac{2}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{3 \times 2} = \frac{\sqrt{2}}{3}$$

$$ii) \frac{6(\sqrt{7}-2)}{(\sqrt{7}+2)(\sqrt{7}-2)} = \frac{6(\sqrt{7}-2)}{7-4} = \frac{6(\sqrt{7}-2)}{3} = 2(\sqrt{7}-2)$$

$$c) x = \sqrt{5} + 2$$

$$x - \frac{1}{x} = \sqrt{5} + 2 - \left(\frac{1}{\sqrt{5}+2}\right) \left(\frac{\sqrt{5}-2}{\sqrt{5}-2}\right)$$

$$= \sqrt{5} + 2 - (\sqrt{5}-2)$$

$$= \sqrt{5} + 2 - \sqrt{5} + 2$$

$$= 4$$

$$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$$

$$= 16 + 2$$

$$= 18$$

$$Q3 i) 5 - (a-2) = 5 - a + 2 = 7 - a$$

$$ii) 3a(2a+1) + a(a+5)$$

$$= 6a^2 + 3a + a^2 + 5a$$

$$= 7a^2 + 8a$$

$$iii) \frac{2a-1}{3} + \frac{a+4}{4}$$

$$= \frac{4(2a-1) + 3(a+4)}{12}$$

$$= \frac{8a - 4 + 3a + 12}{12}$$

$$= \frac{11a + 8}{12}$$

$$iv) a - \frac{a+2}{2}$$

$$= \frac{2a - (a+2)}{2}$$

$$= \frac{2a - a - 2}{2}$$

$$= \frac{a-2}{2}$$

$$v) a - 5 = -1$$

$$vi) \frac{2(3a-5)}{5} - \frac{3(a+4)}{4}$$

$$= \frac{8(3a-5) - 15(a+4)}{20}$$

$$= \frac{24a - 40 - 15a - 60}{20}$$

$$= \frac{9a - 100}{20}$$

$$Q4 i) x^2 - 4y^2 = (x-2y)(x+2y)$$

$$ii) x^2 - 4x - 5 = (x-5)(x+1)$$

$$iii) x^3 - 8y^3 = (x-2y)(x^2 + 2xy + 4y^2)$$

$$iv) 3x^2 - 5x - 2 = \frac{-6}{3} \frac{(3x-6)(3x+1)}{3} = (x-2)(3x+1)$$

$$v) 12 - 5x - 3x^2 = \frac{-36}{12} \frac{(12-9x)(12+4x)}{12} = (4-3x)(3+x)$$

$$vi) x+2y + x^2 - 4y^2$$

$$= 1(x+2y) + (x+2y)(x-2y)$$

$$= (x+2y)(1+x-2y)$$

5i) $x - \frac{3x}{4} = 4$

$\therefore 4x - x = 16$
 $3x = 16$ $x = 5\frac{1}{3}$

ii) $3 - 2(x-1) = 7$

$3 - 2x + 2 = 7$
 $-2x + 5 = 7$
 $-2x = 2$ $\therefore x = -1$

iii) $|2x-1| = 11$

$+(2x-1) = 11$ $-(2x-1) = 11$
 $2x = 12$ $-2x + 1 = 11$
 $x = 6$ $-2x = 10$
 $x = -5$

$\therefore x = -5, 6$

iv) $|x+1| = 3x-1$

$\therefore +(x+1) = 3x-1$ $-(x+1) = 3x-1$
 $x+1 = 3x-1$ $-x-1 = 3x-1$
 $2 = 2x$ $0 = 4x$
 $1 = x$ $x = 0$

Check: $\checkmark x=1$, $RHS = 3-1 = 2 > 0$
 $\therefore x=1$ is valid
 $\checkmark x=0$, $RHS = 0-1 = -1 < 0$
 $\therefore x \neq 0$
 $\therefore x=1$ only

v) $3x - y + 5 = 0$
 $x + 3y + 25 = 0$

x(1) by 3: $9x - 3y - 15 = 0$
 Add: $10x + 10 = 0$
 $x = -1$

Sub in 2: $-1 + 3y + 25 = 0$ $\therefore y = -8$

$2x+4 = 2x+5$

$(3x+1)(2x+5) = (3x+2)(2x+4)$
 $6x^2 + 17x + 5 = 6x^2 + 16x + 8$
 $x = 3$

6a) i) $\frac{4^{-4}}{8^{-2}} = \frac{8^2}{4^4} = \frac{64}{256} = \frac{1}{4}$

ii) $27^{-2/3} = \left(\frac{1}{\sqrt[3]{27}}\right)^2 = \frac{1}{9}$

b) $\frac{25-x^2}{x^2+3x-10} \div \frac{125-x^3}{x^2-4}$
 $= \frac{(5+x)(5-x)}{(x+5)(x-2)} \times \frac{(x+2)(x-2)}{(5-x)(25+5x+x^2)}$
 $= \frac{x+2}{x^2+5x+25}$

c) $3x^2 - 6x - 1 = 0$
 $\therefore x = \frac{6 \pm \sqrt{36+12}}{6}$
 $= \frac{6 \pm \sqrt{48}}{6}$
 $= \frac{6 \pm 4\sqrt{3}}{6} = \frac{3 \pm 2\sqrt{3}}{3}$

7. a) Ext angle = 150° (adv. line)
 No. of sides = $\frac{360}{15}$ (sum of ext \angle 's)
 $= 20$

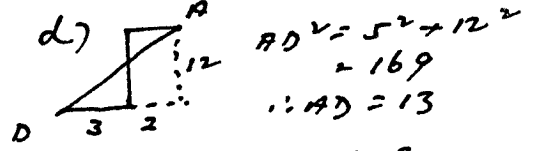
b) $3x + 15 + 2x = 180$ (vert opp \angle 's become co-interior with the parallel)

$15x = 360$

$x = 20$

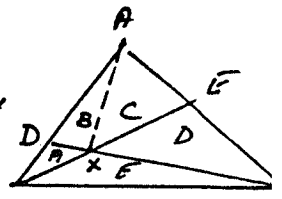
iii) $115 + y = 180$ (adv. ang. l)
 $\therefore y = 65$
 $65 + x = 125$ (ext \angle of Δ)
 $\therefore x = 60$

c) In Δ 's ABC, CDE
 $AC = CE$ (data)
 $BC = DC$ (data)
 $\angle ACB = \angle DCE$ (vert opp)
 $\therefore \Delta ACB \cong \Delta ECD$ (SAS)
 $\therefore \angle BAC = \angle DEC$ (corr. \angle 's of cong Δ)
 $\therefore AB \parallel DE$ (alt \angle 's equal)



8a) $(2x+1)^2 - (x+2)^2 = \{2x+1+x+2\} \{2x+1-x-2\}$
 $= 3(x+1)(x-1)$
 b) $\frac{6^{\sqrt{3}} \cdot 10^{\sqrt{3}}}{30} = \frac{3^{\sqrt{3}} \cdot 2^{\sqrt{3}} \cdot 5^{\sqrt{3}} \cdot 2^{\sqrt{3}}}{2 \times 3 \times 5}$
 $= 3^{-1/3} \cdot 5^{-2/3} \therefore a = -1/3, b = -2/3$

c) $\angle ADC = \frac{4}{5} \times 85 = 68$
 $\angle AEB = \frac{2}{5} \times 85 = 34$
 $\angle ADX : \angle BDX = 4 : 1$



iv) $A+B+C+D+E = 85$ B
 $A+B+C = 34$ $\therefore D+E = 51$
 $B+C+D = 68$ $\therefore A+E = 17$
 $B = 4A, D = 3C/2$
 $\therefore 5A + C = 34$ $\rightarrow C = 34 - 5A$
 $\therefore 8A + 5(34 - 5A) = 136$