SYDNEY GIRLS HIGH SCHOOL

YEAR 11 MATHEMATICS

HALF YEARLY

May,2004

Time allowed: 75 minutes

Topics: Chapters 1-4(J&C)

Instructions:

- There are Four (4) questions. Questions **are** of equal value.

- Attempt all questions.

- Show all necessary working. Marks may be deducted for badly arranged work.

- Start each question on a new page. Write on one side of the paper only.

Name: ……………………………………………………………..

QUESTION ONE (20 marks)

a) Find the value of $\dfrac{4.7+\sqrt{3.2}}{1+4.15^2}$, correct to 2 significant figures.

b) Simplify the following

   i) $\dfrac{\dfrac{1}{3}+\dfrac{2}{7}}{1-\dfrac{1}{3}\times\dfrac{2}{7}}$

   ii) $\dfrac{ac-ab}{b^2-c^2}$

   iii) $1-\dfrac{p-q}{p+q}$

c) Find the value of $|-8| - |-7\times2| \times |-4-2|$

d) Simplify

   i) $\dfrac{2^{-5}}{(4y)^{-1}}$

   ii) $\dfrac{8a^3b^5}{12(ab)^6}$

e) Factorise $ab-3a-7b+21$

f) Express $0.5\overset{\bullet}{4}$ as fraction in it's lowest term.

g) Simplify $\dfrac{a}{2}+\dfrac{3a}{4}-\dfrac{2a}{5}$

QUESTION TWO (20 marks)

a) Graph $-3 \leq x \leq 2$ on a number line.

b) Simplify the following (Leave the answer in surd form)

i) $\sqrt{18} \times \sqrt{6}$

ii) $\dfrac{5\sqrt{2} \times 6\sqrt{10}}{3\sqrt{5}}$

iii) $(3\sqrt{6} - 5\sqrt{2})^2$

iv) $\dfrac{3}{\sqrt{3}} + \dfrac{2}{\sqrt{2}}$

c) If $\sqrt{y} = \sqrt{63} - \sqrt{7} - \sqrt{28}$, find the value of y.

d) If $x = \sqrt{2} + 1$, find the simplest form for $x - \dfrac{1}{x}$

e) Solve $\dfrac{3x}{2} \leq x + 1$

f) Last year my rate increased by $6\dfrac{1}{4}\%$. I now pay \$1240. What did I pay before the rise?

g) Simplify $\dfrac{4}{x^2 - 7x + 10} - \dfrac{1}{x^2 - 25}$

QUESTION THREE (20 marks)

a) Solve the equation $|x - 3| = 2$

b) Factorise fully

i) $2x^2 - 128$

ii) $x^4 - 16$

c) Solve the following

i) $6x^2 + 5x - 11 = 0$

ii) $1 + \dfrac{2x + 3}{5} = x$

d) Solve $\sqrt{2 - 3x} = 5$

e) Solve simultaneously

$2x + y = 8$

$x + y = 6$

f) Factorise $x^2 - 4xy + 4y^2 - 9$

g) Solve $8^x + 8^x = 1$

QUESTION FOUR (20 marks)

a) Solve the following simultaneous equations

$$y = x^2$$

$$y = 6x - 9$$

b) Solve $|2 - 5x| < 3$

c) The sum of a number and it's reciprocal is 6. Write an equation and find the number.(Let the number be x)

d) Solve $|2x - 1| = x + 7$

e) Factorise and simplify $\dfrac{a^2 - 1}{a^3 - 1} \times \dfrac{a^2 - a}{a^2 - a - 2} \div \dfrac{a}{a^2 + a + 1}$

f) If $3\sqrt{5} + 4 + \dfrac{1}{\sqrt{5} - 2} = a + b\sqrt{5}$,find the value of a and b.

END OF PAPER

Yr 11 2U Solutions 2004 (May) Halfyearly

1) a) 0.36 (2)

b)i) $\dfrac{13}{19}$ (2)

ii) $\dfrac{-a(b-c)}{(b-c)(b+c)} = \dfrac{-a}{b+c}$ (1) ② 2

iii) $\dfrac{P+q-P+q}{P+q} = \dfrac{2q}{P+q}$ (2)

c) $8 - 14 \times 6$
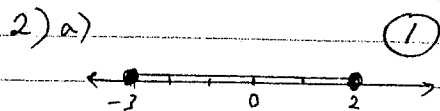$= 8 - 84$
$= -76$ (2)

d)i) $\dfrac{4y}{2^5}$
$= \dfrac{4y}{32}$
$= \dfrac{y}{8}$ (2)

ii) $\dfrac{8a^3b^5}{12a^6b^6} = \dfrac{2}{3a^3b}$ (2)

e) $a(b-3) - 7(b-3)$
$(b-3)(a-7)$ (2)

f) Let $x = 0.5444\ldots$
$10x = 5.444\ldots$
$100x = 54.4444\ldots$
$90x = 49$
$x = \dfrac{49}{90}$ (2)

g) $\dfrac{10a + 15a - 8a}{20}$
$= \dfrac{17a}{20}$ (2)

2) a) (1)

b)i) $3\sqrt{2} \times \sqrt{6}$
$= 3\sqrt{12}$
$= 3 \times 2\sqrt{3}$
$= 6\sqrt{3}$ (2)

ii) $\dfrac{30\sqrt{20}}{3\sqrt{5}}$
$= \dfrac{10\sqrt{4}}{20}$ (2)

iii) $54 - 30\sqrt{12} + 50$
$= 104 - 30 \times 2\sqrt{3}$
$= 104 - 60\sqrt{3}$ (2)

iv) $\dfrac{3\sqrt{2} + 2\sqrt{3}}{\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}}$
$= \dfrac{3\sqrt{12} + 2\sqrt{18}}{6}$
$= \dfrac{6\sqrt{3} + 6\sqrt{2}}{6}$
$= \sqrt{3} + \sqrt{2}$ (2)

c) $3\sqrt{7} - \sqrt{7} - 2\sqrt{7} \leq \sqrt{y}$
$\sqrt{0} \leq \sqrt{y}$
$y = 0$ (2)

d) $x - \dfrac{1}{x}$

$\dfrac{1}{x} = \dfrac{1}{\sqrt{2}+1}$
$= \dfrac{1}{\sqrt{2}+1} \times \dfrac{\sqrt{2}-1}{\sqrt{2}-1}$
$= \dfrac{\sqrt{2}-1}{2-1}$
$= \sqrt{2}-1$

$x - \dfrac{1}{x} = \sqrt{2}+1 - (\sqrt{2}-1)$
$= \sqrt{2}+1 - \sqrt{2}+1$
$= 2$ (2)

e) $3x \leq 2x + 2$
$x \leq 2$ (2)

f) $\dfrac{106.25x}{100} = 1240$
$x = \$1167.06$ (2)

g)
$\dfrac{}{(x-2)(x-5)} - \dfrac{}{(x-5)(x+5)}$
$= \dfrac{4(x+5) - 1(x-2)}{(x-2)(x-5)(x+5)}$
$= \dfrac{4x+20 - x + 2}{(x-2)(x-5)(x+5)}$
$= \dfrac{3x+22}{(x-2)(x-5)(x+5)}$ (3)

Q3.

a) $x-3=2$ or $x-3=-2$
$x=5$ or $x=1$ (2)

b)i) $2(x^2 - 64)$
$= 2(x-8)(x+8)$ (2)

ii) $(x^2-4)(x^2+4)$
$(x-2)(x+2)(x^2+4)$ (2)

c)i) $6x^2 + 5x - 11 = 0$
$6x^2 + 11x - 6x - 11 = 0$
$x(6x+11) - (6x+11) = 0$
$(x-1)(6x+11) \leq 0$
$x = 1$ or $-\dfrac{11}{6}$ (3)

ii) $5 + 2x + 3 = 5x$
$8 = 3x$
$x = 2\tfrac{2}{3}$ (2)

d) $2 - 3x = 25$
$-23 = 3x$
$x = -\dfrac{23}{3}$
$x = -7\tfrac{2}{3}$ (2)

e) $2x + y = 8$ ——— ①
$x + y = 6$ ——— ②
② - ①
$-x = -2$
$\boxed{x = 2}$ sub into ②
$2 + y = 6$
$\boxed{y = 4}$ (3)

f) $(x - 2y)^2 - 9$

$(x - 2y - 3)(x - 2y + 3)$ ②

g) $2(8^x) = 1$

$8^x = \frac{1}{2}$

$2^{3x} = 2^{-1}$

$3x = -1$

$x = -\frac{1}{3}$ ②

4) a)

$y = x^2$ ——— ①

$y = 6x - 9$ ——— ②

sub ① into ②

$x^2 = 6x - 9$

$x^2 - 6x + 9 = 0$

$(x - 3)^2 = 0$

$\boxed{x = 3}$ sub into ②

$y = 18 - 9$

$\boxed{y = 9}$ ③

b)

$2 - 5x < 3$ or

$2 - 5x \geq 3$

$-5x < 1$

$x > -\frac{1}{5}$ ③

or

$-5x \geq -5$

$x < 1$ $-\frac{1}{5} < x < 1$

c) $x + \frac{1}{x} = 6$

$x^2 + 1 = 6x$

$x^2 - 6x + 1 = 0$

$x^2 - 6x + 9 = -1 + 9$

$(x - 3)^2 = 8$

$x - 3 = \pm 2\sqrt{2}$

$x = \pm 2\sqrt{2} + 3$

$x = 2\sqrt{2} + 3$ or $-2\sqrt{2} + 3$ ③

d) $2x - 1 = x + 7$

or $2x - 1 = -x - 7$

$x = 8$ or $3x = -6$

$x = -2$

Test $x = 8$

$LHS = |2 \times 8 - 1|$

$= 15$

$RHS = 8 + 7$

$= 15$

$LHS = RHS$ ∴ $x = 8$

is a solution

Test $x = -2$

$LHS = |2 \times -2 - 1|$ ④

$= 5$

$RHS = -2 + 7$

$= 5$

$LHS = RHS$ ∴ $x = -2$

is a solution

∴ solution $x = 8$ or -2

f)

$\dfrac{(a-1)(a+1)}{(a-1)(a^2+a+1)} \times \dfrac{a(a-1)}{(a-2)(a+1)} \times \dfrac{a^2+a+1}{a}$

$= \dfrac{a-1}{a-2}$ ④

g) $3\sqrt{5} + 4 + \dfrac{1}{\sqrt{5} - 2} \times \dfrac{\sqrt{5} + 2}{\sqrt{5} + 2}$

$3\sqrt{5} + 4 + \dfrac{\sqrt{5} + 2}{5 - 4}$

$= 3\sqrt{5} + 4 + \sqrt{5} + 2$

$= 4\sqrt{5} + 6$

∴ $a = 6$ ③

$b = 4$