SYDNEY GIRLS HIGH SCHOOL

MATHEMATICS

Extension 2 (Circular Motion, Curve Sketching)

Year 11 November 2001

INSTRUCTIONS

- There are nine questions.
- Questions are not of equal value.
- Time Allowed: 100 minutes
- Start each question on a new page
- Write on one side of the paper only

* Use $g = 10\ ms^{-2}$

* Write non integral answers correct to one decimal place

Question 1: 8 marks

On the same set of axes, sketch the following curves for $0 \le x \le 2\pi$

i) $y = \cos 2x$

ii) $y = -2\sin x$

iii) $y = \cos 2x - 2\sin x$

Question 2: 8 marks

i) Use calculus to show $y = \dfrac{2}{x-1}$ has no stationary points and sketch the curve

ii) Determine the inverse function of $y = \dfrac{2}{x-1}$ and write it with y as the subject

iii) Sketch the inverse function on the same set of axes

iv) State the co-ordinates of any points common to both curves

Question 3: 10 marks

If $f(x) = x^2 - 2x$, sketch the following curves

i) $y = f(x)$

ii) $y = [f(x)]^2$

iii) $y = \dfrac{1}{f(x)}$

iv) $y = f|x|$

v) $y = \sqrt{f(x)}$

Question 4: 24 marks

Without the use of calculus, sketch the following curves, showing clearly any roots or asymptotes

i) $y = (x+4)^3(x+1)^2(x-2)$

ii) $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

iii) $y = 2\sin^{-1}(x-1)$

iv) $x^2 - y^2 = 9$

v) $y = 2\sin(x - \dfrac{\pi}{3})$

vi) $y = \dfrac{2x-1}{x+2}$

vii) $y = \dfrac{2x^2}{(x+1)}$

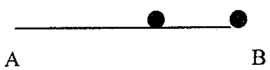
viii) $y = \dfrac{x}{(x+2)^2(x-1)}$

Question 5: 5 marks

A 3 metre string AB has a mass of 4 kg attached at point B.
If the string breaks when the speed of rotation reaches 12 rad/sec,

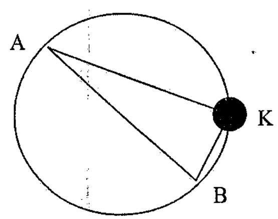
a) find the breaking strain (maximum tension) of the string

b) find the new maximum speed in rad/sec if the 4 kg mass at B is replaced by a 2kg mass and an additional 2kg mass is added to the string, 2 metres from A



Question 6: 10 marks

A 50 cm diameter disc has a mass of 2 kg at K and is attached to end points of a diameter AB by strings of length 48cm and 14 cm. If the tension in BK is 50Newtons and the mass is to remain at position K, at how many radians per second must the disc be rotating

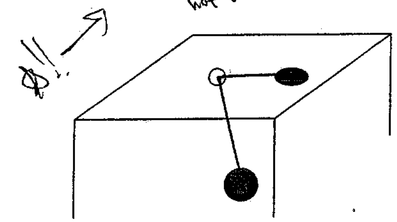


AK = 48 cm
BK = 14 cm

Question 7: 8 marks

A 1.5 metre piece of string is threaded through a hole in a smooth table And masses are placed on each end. The mass on the table is half the mass below the table. The lower mass rotates in a conical pendulum where the length of the string below the table is 1 metre and the mass moves at 2 rad/sec. The upper mass rotates in a circle about the hole in the table. Find the velocity of the upper mass in metres per second.

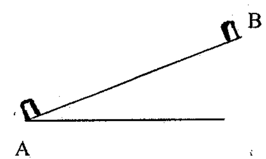
not w!



Question 8: 15 marks

A railway line at a curve is banked as shown in the below diagram.

a) Reproduce the diagram on your page showing all the forces associated with a train moving around the track; these being
   i) lateral force F along the track
   ii) reaction force N, normal to the track
   iii) the force due to circular motion
   iv) the force due to gravity
b) Resolve force F and force N into horizontal and vertical directions; Hence solving for F and N
c) If the radius of the curve is 400m and the 1.6m track is banked to Allow trains to travel at 72 km/hr; find the height B is above A
d) If a 20 tonne train travels around the corner at 108 km/hr, what is The lateral force on the rails?

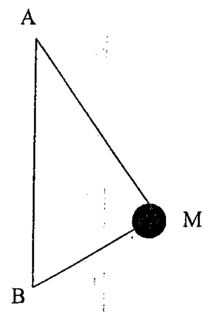


Question 9: 12 marks

Point B is 13 metres directly below point A. A mass of 5 kg is Attached to A and B by strings of 12m and 5m in length. The mass is rotated about AB

i) If the angular speed of rotation is 6 rad/sec, find the tension in both parts of the string

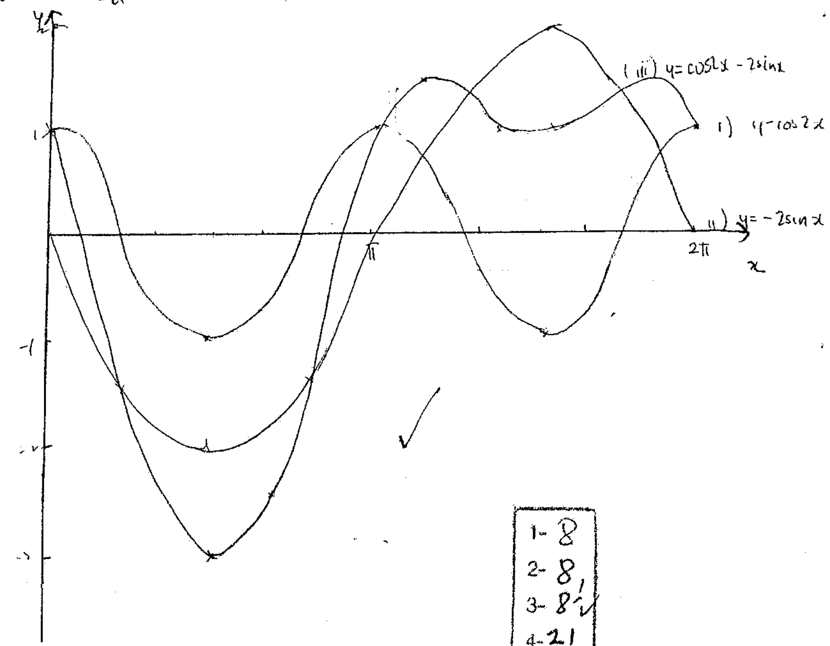
ii) What is the minimum angular speed of rotation for the lower section to maintain some tension.



AM = 12 m
BM = 5 m

Vivian Park
Qz 127519

(1) i. $0 \le x \le 2\pi$



✓

1- 8
2- 8
3- $8\frac{1}{2}$
4- 21
5- 5
6- 9
7- 7
8- 15
9- 12
10-

$93\frac{1}{2}$

(2) i) $y = \frac{2}{x-1} = 2(x-1)^{-1}$

~~$\frac{dy}{dx} = \frac{3}{x} - 2 = 2x^{-1} - 2$~~

~~$\frac{dy}{dx} = -2$~~

$\frac{dy}{dx} = -2(x-1)^{-2}$

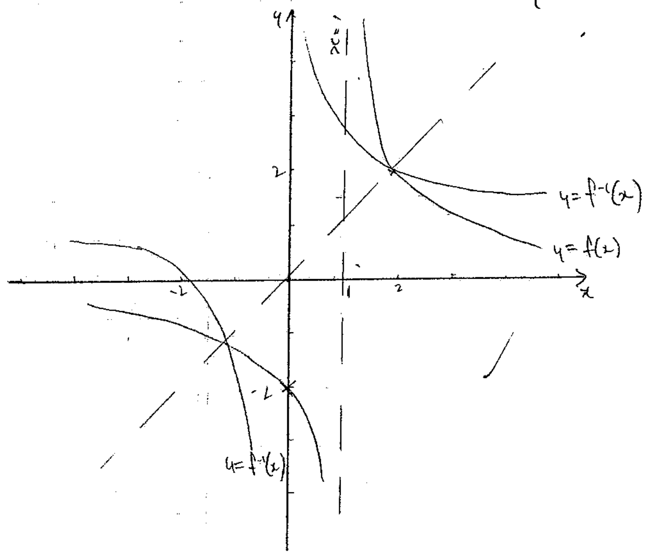
$= \frac{-2}{(x-1)^2}$

for a stny pt $\frac{dy}{dx} = 0$

~~∴ no stationary points~~

$0 = \frac{-2}{(x-1)^2}$

$0 \neq -2$ ∴ no stationary points



iii) $x = \frac{2}{y-1}$

$y - 1 = \frac{2}{x}$

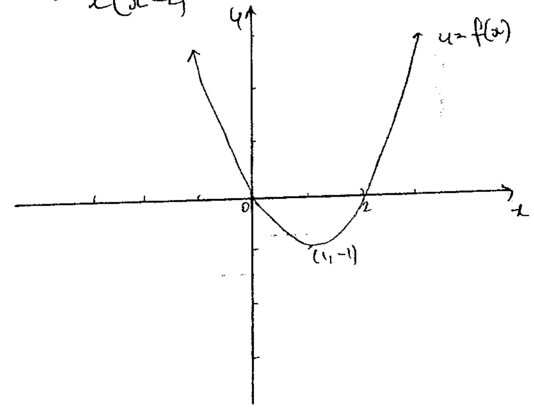
$y = \frac{2}{x} + 1$

iv) $(2,2)$ and $(-1,-1)$

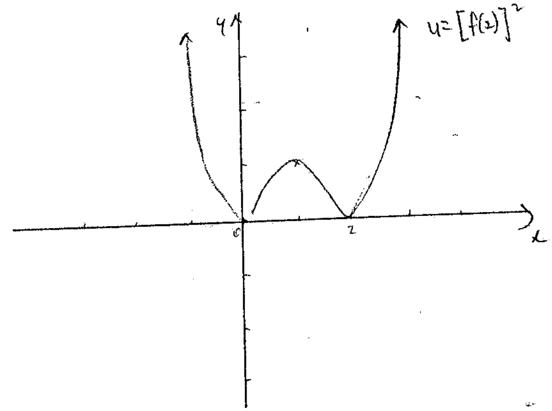
(3) $f(x) = x^2 - 2x$

$= x(x-2)$

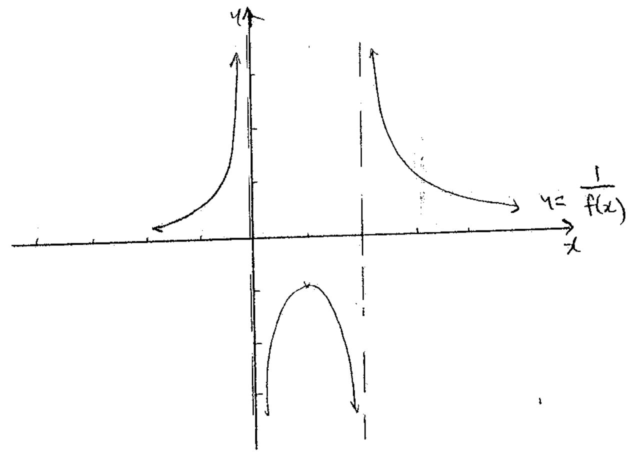
i)



ii)

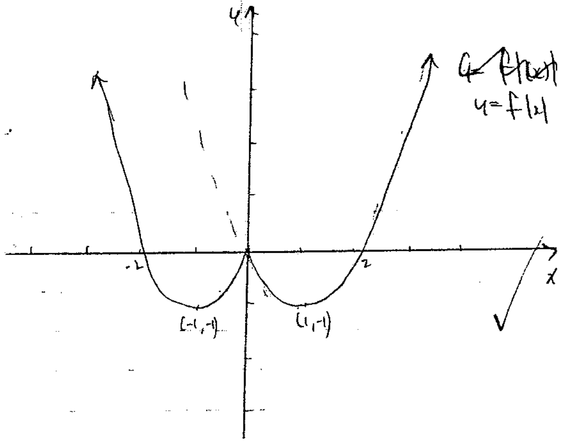


iii)

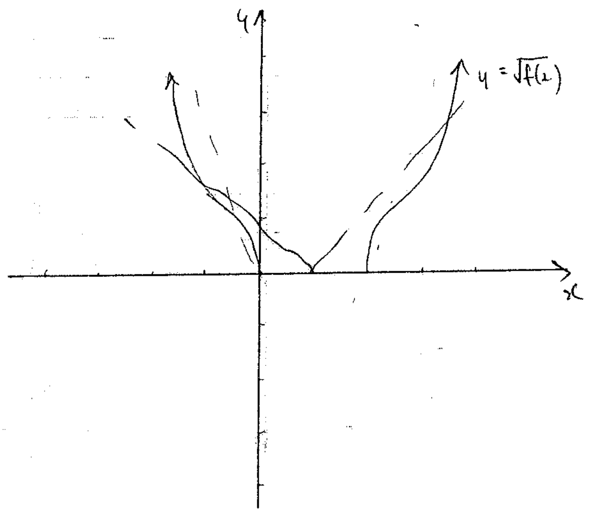


check

(iv)

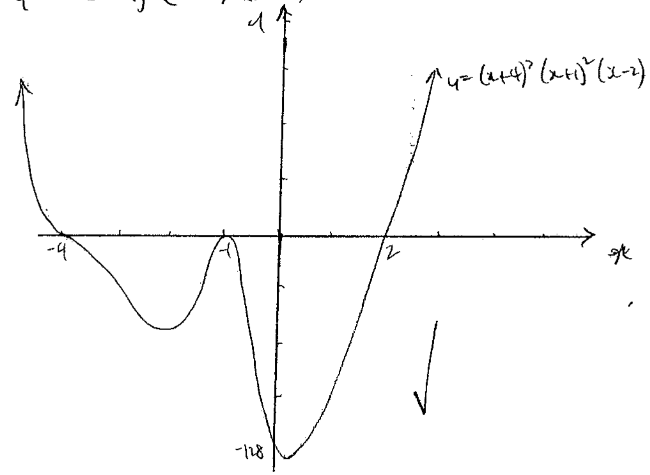


(v)



1/2 / 2

(4) i) $y = (x+4)^3(x+1)^2(x-2)$

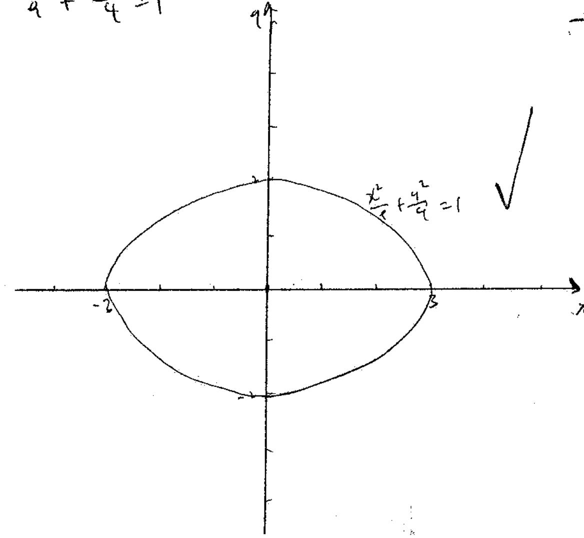


check →

(ii) $\frac{x^2}{4} + \frac{y^2}{4} = 1$

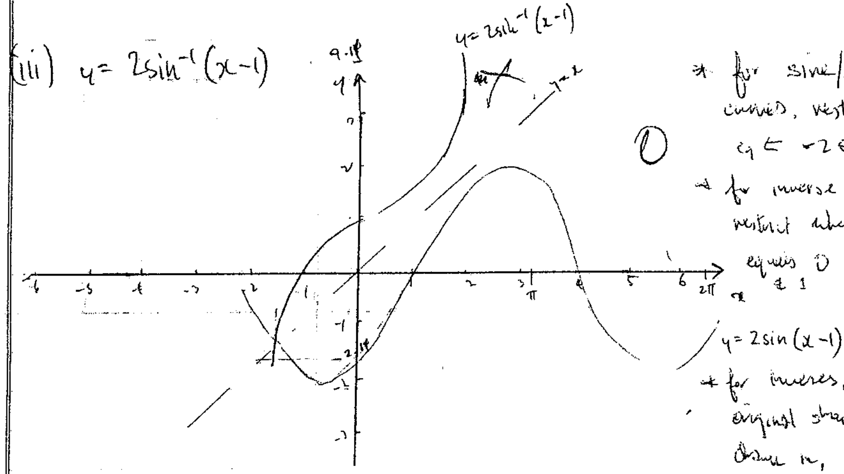
check

3



3

(iii) $y = 2\sin^{-1}(x-1)$

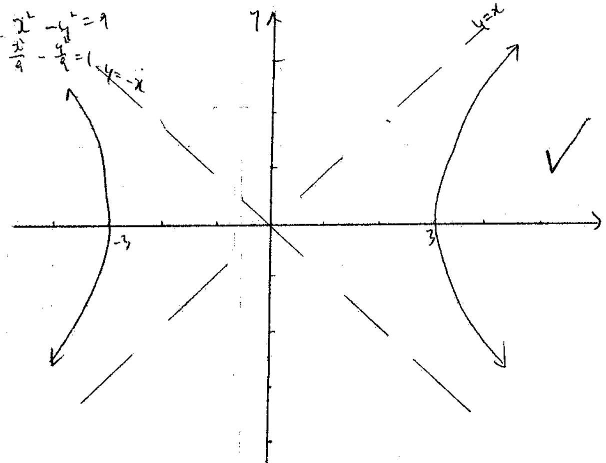


0

* for sine/cosine/tan curves, restrict the eq to $-2 \leq y \leq 2$
* for inverse curves restrict where base equals 0 eq ≤ 1
* for inverses, the original straight is drawn in, it doesn't help anyway.

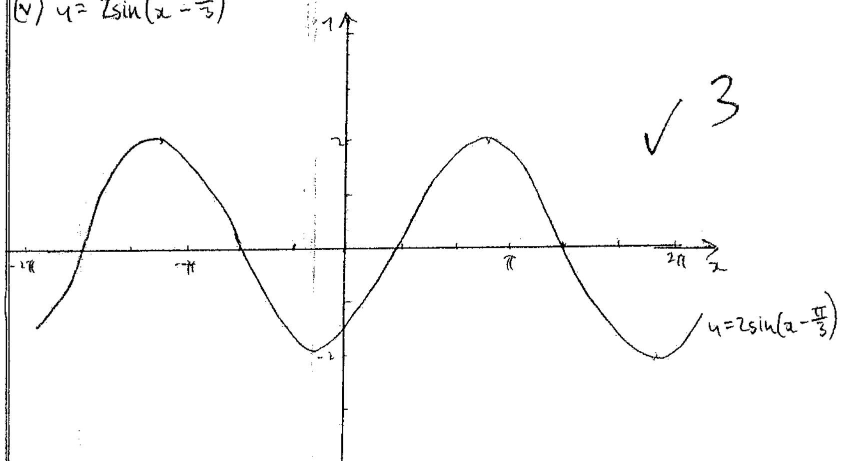
(iv) $x^2 - y^2 = 9$

$\frac{x^2}{9} - \frac{y^2}{9} = 1$



✓ 3

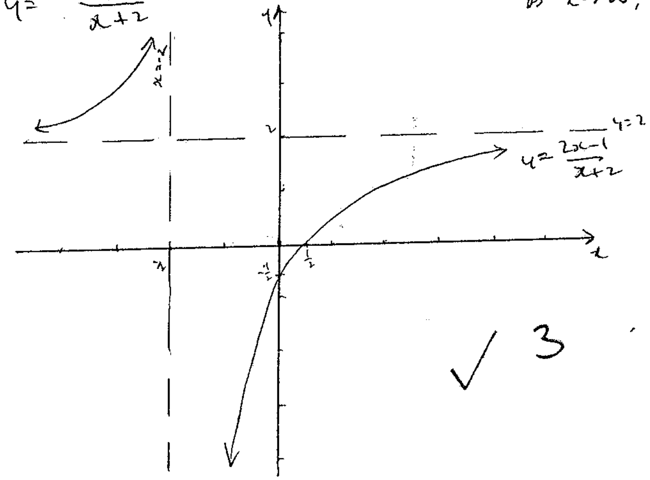
(v) $y = 2\sin(x - \frac{\pi}{3})$



✓ 3

(vi) $y = \frac{2x-1}{x+2}$

as $x \to \infty$, $y \to 2$

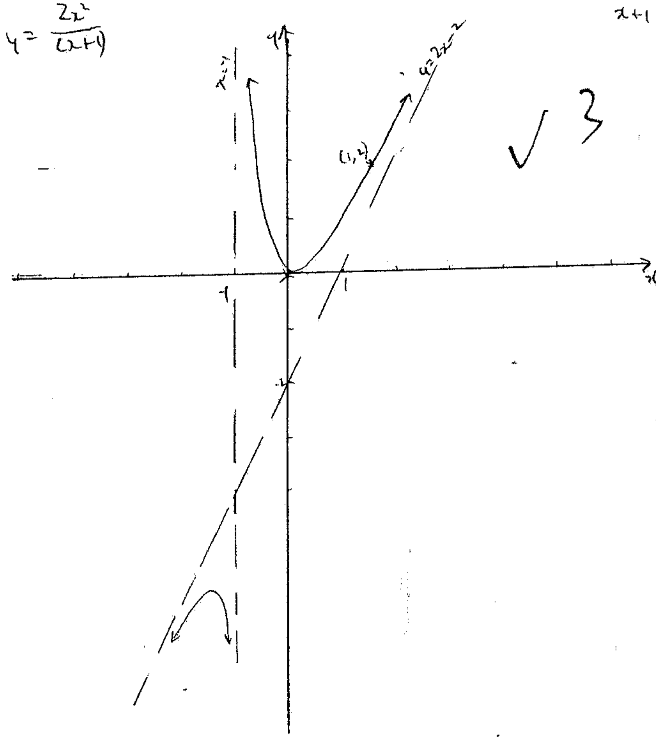


✓ 3

check

(vii) $y = \frac{2x^2}{(x+1)}$

$x+1 \overline{)\,2x^2 + 0 + 0}$ quotient $2x - 2 + \frac{2}{x+1}$
$2x^2 + 2x$
$-2x + 0$
$-2x - 2$
2



✓ 3

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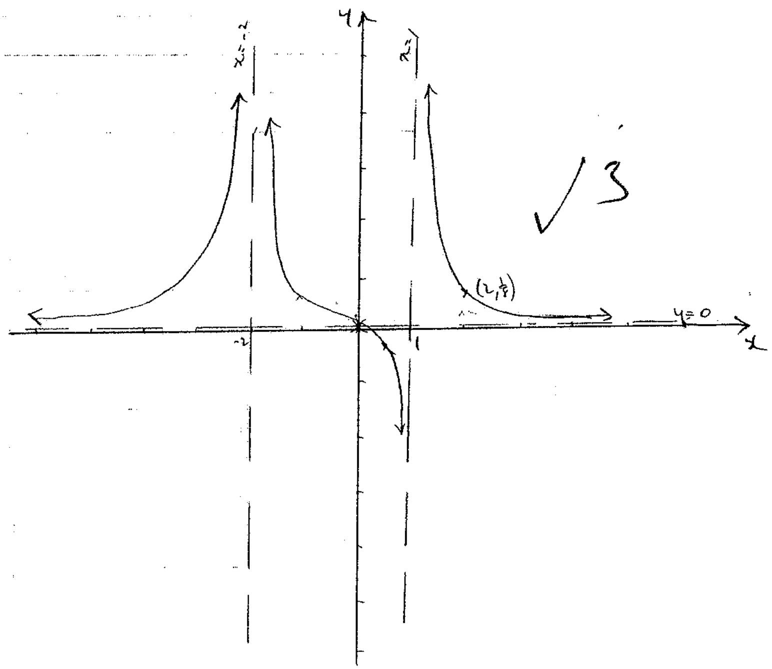
(v iii) $y = \dfrac{x}{(x+2)^2(x-1)}$



✓3

$x=2$, $y = \dfrac{2}{(4)^2(1)} = \dfrac{2}{16} = \dfrac{1}{8}$

$x=10$, $y = \dfrac{10}{(12^2)(9)}$ = small

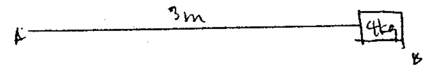
$x=1.5$, $y=$

$x=\frac{1}{2}$, $y = \dfrac{0.5}{(2.5)^2(0.5)}$

$x=-1$, $y = \dfrac{-1}{(1)(-2)} = \dfrac{1}{2}$

$x=-10$, $y = \dfrac{-10}{(-8)^2(-11)} =$

(5)

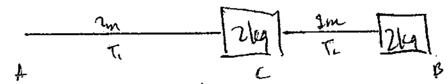


12 rad/sec

(a) $T = mr\omega^2$
$= 4 \times 3 \times 12^2$
$= ~~432~~ \; 1728N$

✓ 2

(b)



$\omega = ?$

$T_1 = 2 \times 2 \times \omega^2$
$= 4\omega^2$

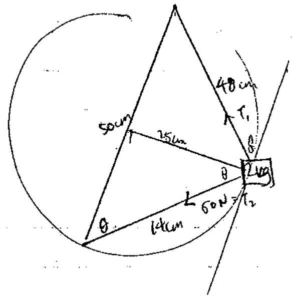
$T_2 = 2 \times 3 \times \omega^2$
$= 6\omega^2$

$T_{total} = 10\omega^2 = 1728N$
$\omega^2 = 172.8$
$\omega \doteqdot 13.15$ rad/sec

✓3

$\frac{5}{5}$

(6)



$\omega = ?$

resolving tangentially $T_1\cos\theta = T_2\sin\theta$ (1)

resolving radially $T_1\sin\theta + T_2\cos\theta = mr\omega^2$ (2)

now $\sin\theta = \frac{48}{50}$

and $\cos\theta = \frac{14}{50}$

and $T_2 = 50N$

$\therefore$ at fem (1) $\frac{14}{50}T_1 = (50)\left(\frac{48}{50}\right)$

$T_1 = (50)\left(\frac{48}{50}\right)\left(\frac{50}{14}\right)$

$= 171\frac{3}{7} = \frac{1200}{7}$ ✓ (3)

(3) into (2) $\left(\frac{1200}{7}\right)\left(\frac{48}{50}\right) + (50)\left(\frac{14}{50}\right) = 2 \times 0.25 \times \omega^2$

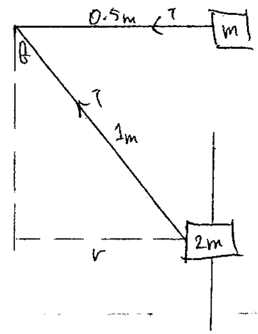
$\frac{164\frac{4}{7} + 35}{0.5} = \omega^2$

$\omega^2 = 399\frac{1}{7}$

$\omega \doteq 19.98$ rad/sec

9/10

(7)



$\omega = ?$

2 rad/sec

at m $T = mR\omega^2 = 0.5m\omega^2$ (1)

at 2m resolving vertically $T\cos\theta = 2mg$ (2)

horizontally $T\sin\theta = 2mr\omega^2$

$T\sin\theta = 8mr$ (3)

~~(1) into (2) $0.5m\omega^2\cos\theta = 2mg$, $\omega^2\cos\theta = 4g$~~

(1) into (3) $0.5m\omega^2\sin\theta = 8mr$

now $\sin\theta = \frac{r}{1}$

$= r$

$\therefore 0.5\cancel{m}\omega^2\cancel{r} = 8\cancel{m}\cancel{r}$

$0.5\omega^2 = 8$

$\omega^2 = 16$

$\omega = 4$ rad/sec ✓

require ✓

$\frac{T\sin\theta}{T\cos\theta} = \frac{8mr}{4mg} = \frac{2r}{g}$

7/8

(3) ÷ (2) $\tan\theta = \frac{4}{9} = \frac{r}{h}$

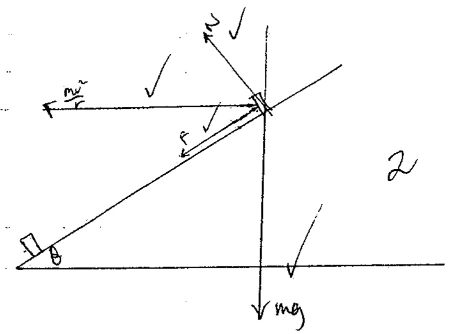
$\therefore h = \frac{9}{4}$

$\doteq 2.45$m

$\therefore \cos\theta \doteq \frac{2.45}{1}$

not req

(8) (a)



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(b) resolving vertically $N\cos\theta = F\sin\theta + mg$ ①

horizontally $N\sin\theta + F\cos\theta = mr\omega^2$ ②

①× cos θ $N\cos^2\theta = F\sin\theta\cos\theta + mg\cos\theta$ ③

②× sin θ $N\sin^2\theta + F\cos\theta\sin\theta = mr\omega^2\sin\theta$ ④

④+③ $N = mr\omega^2\sin\theta + mg\cos\theta$

$= m\left(\frac{v^2}{r}\sin\theta + g\cos\theta\right)$

①× sin θ $N\sin\theta\cos\theta = F\sin^2\theta + mg\sin\theta$ ⑤

②× cos θ $N\sin\theta\cos\theta + F\cos^2\theta = mr\omega^2\cos\theta$ ⑥

⑥−⑤ $F = mr\omega^2\cos\theta - mg\sin\theta$

$= m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)$

2

(c) $r = 400m$, $1.6m$, $v = 72 km/hr = 20 m/s$, $h = ?$

∴ $F = 0$

∴ $0 = m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)$

$\frac{\cancel{m}v^2}{r}\cos\theta = \cancel{m}g\sin\theta$

$\frac{v^2}{r}\cos\theta = g\sin\theta$

$\frac{v^2}{rg} = \tan\theta$

$\tan\theta = \frac{20^2}{400 \times 9.8}$

$\theta \doteq 5.83°$

now $\sin\theta = \frac{h}{hyp}$

$\sin 5.83° = \frac{h}{1.6}$

$h = 0.16m = 16cm$

(d) $m = 20\ 000kg$, $v = 108 km/h = 30 m/s$, $F = ?$

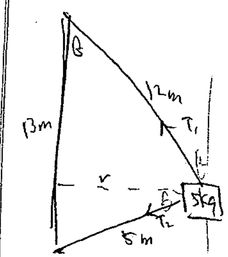
$F = m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)$

$= 20\ 000\left(\frac{30^2}{400}\cos 5.83° - (9.8)(\sin 5.83°)\right)$

$= 24,858 N$

15/15

(9)



i) $\omega = 6 rad/sec$, $T = ?$

now $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\theta = 22°37'$

resolving vertically $T_1\cos\theta = T_2\sin\theta + mg$ ①

horizontally $T_1\sin\theta + T_2\cos\theta = mr\omega^2$ ②

from ① $T_1\frac{12}{13} = T_2\frac{5}{13} + 50$

$T_1 = T_2\frac{5}{12} + 54\frac{1}{6}$ ③

③ into ② $\left(T_2\frac{5}{12} + 54\frac{1}{6}\right)\left(\frac{5}{13}\right) + T_2\frac{12}{13} = mr\omega^2$

$\frac{25}{156}T_2 + 20\frac{5}{6} + T_2\frac{12}{13} = 5 \times 4.6 \times 6^2$

$\left[\sin 22°37' = \frac{r}{12},\ r = 4.6\right]$

$\frac{13}{12}T_2 = 828 - 20\frac{5}{6}$

$T_2 = 756\frac{1}{13} N$

∴ $T_1 = T_2\frac{5}{12} + 54\frac{1}{6}$

$= 364\frac{8}{13} N$

8

ii) if $T_2 = 0$ then

$T_1\cos\theta = mg$ ①

$T_1\sin\theta = mr\omega^2$ ②

②÷① $\tan\theta = \frac{r\omega^2}{g}$

$\tan 22°37' = \frac{4.6 \times \omega^2}{9.8}$

$\omega = 0.942 rad/sec$

∴ $\omega > 0.94 rad/sec$ for T_2 exist.

4

12/12