Y11 Extn 1 Yrly Exam 2007

Marks

Question 1

(a) Factorise $x^2 - y^2 + x + y$ 2

(b) Solve $\dfrac{1}{x-2} > 3$ 3

(c) Find the domain and range of $y = \dfrac{1}{\sqrt{9-x^2}}$ 3

(d) Find the co-ordinates of the point which divides the interval joining (-2, 5) and (3, -4) on the number plane in the ratio 3:5 2

(e) Evaluate $\displaystyle\lim_{x \to \infty} \frac{5x^2 - 3}{2x^2 + x}$ 2

(f) In a class of 24 students, 16 study Latin, 10 study Classical Greek and 5 study neither.
What is the probability that a student chosen at random, studies both Latin and Classical Greek? 2

(g) Evaluate $\displaystyle\sum_{n=1}^{25} 5 - 2n$ 3

(h) Find the values of A and B if $A(1\text{-}2x) + B(1\text{-}3x) = 1$ for all x 2

(i) Sketch the curve showing all the information below 3

$f(-2) = f(4) = 0$

$f^{1}(-1) = f^{1}(3) = 0$

$f^{11}(-1) = f^{11}(3) = 0$

$f^{1}(x) < 0 \quad if \quad x < -1$

$f^{1}(x) > 0 \quad if \quad x > -1$

Marks

Question 2 (start a new page)

(a) Differentiate

(i) $3x^2 - \dfrac{1}{x}$ 2

(ii) $\sqrt{6x-5}$ 2

(iii) $\dfrac{3x^4 - 6}{2x+1}$ 2

(b) The third term of an arithmetic series is 10 and the seventh term is 22. Find the

(i) Sum of the first 156 terms 2

(ii) 1^{st} term greater than 1000 2

(c) If $\alpha + \beta$ are the roots of the quadratic equation $2x^2 - 6x + 5$, find

(i) $\alpha + \beta$ 1

(ii) $\alpha\,\beta$ 1

(iii) $\alpha^2 + \beta^2$ 2

(iv) $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ 2

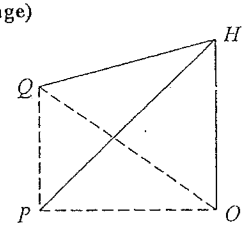
(d) Show that the line with equation $3x + 4y - 25 = 0$ is a tangent to the circle with equation $x^2 + y^2 = 25$ 2

(e) For the curve with equation $y = -x^3 + 3x^2 + 9x - 15$ find the coordinates of any

(i) Stationary points and determine their nature. 2

(ii) Inflexion points 2

Question 3 (start a new page) Marks



(a) In the diagram above, H is the top of the hill OH. O, P and Q are on ground level. The elevation from P which is due west of O to the top of the hill is 36^0. The elevation from Q which is due north of P is 27^0. The distance from P to Q is 600m. Find

(i) $\angle OPH$ and $\angle OQH$ 2

(ii) Expressions for OQ and OP in the terms of OH 3

(iii) The height of the hill correct to 2 significant figures. 3

(b) Using the formula $f^1(x) = \lim_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$

Find from first principles the derivative of $f(x) = 2x^2 - 3x$ 3

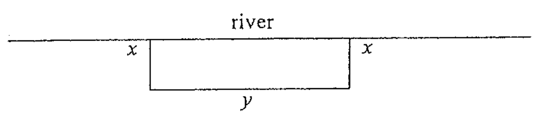
(c) For the geometric series $(1-x) + (1-x)^2 + (1-x)^3 + \ldots$

(i) Find the values of x for which the series has a limiting sum. 2

(ii) Find the value of x if the limiting sum is 0.5 2

(d) Solve $15 - x^2 - 2x > 0$ 3

(e) A rectangular field is to be formed using the river shown as one boundary.



If the length of fencing used is 100m

(i) Express y in terms of x 1

(ii) Find the maximum area that can be enclosed by the fence and the dimensions of the field. 3

Question 4 (start a new page) Marks

(a) For the parabola with equation $y = 2x^2 - 5x + 7$ find

(i) The equation of the normal at the point where $x = -3$ 3

(ii) The point of contact of the tangent which is parallel to $3x - 2y + 4 = 0$ 3

(b) A man invests \$500 at the beginning of each month in a superannuation fund. Assume that interest is paid at 12% p.a. on the investment, and payments are made for 40 years.

(i) How much will the first \$500 invested amount to at the end of 40 years? 2

(ii) What is the total value of the investment at the end of 40 years? (No, payment is made at the end of the 40^{th} year). 3

(iii) What single amount invested for 40 years at the same interest rate Compounded monthly would result in the same total value as (ii). 2

(c) Solve $2(m+\frac{1}{m})^2 + (m+\frac{1}{m}) - 15 = 0$ 4

(d) Margaret buys two tickets in a lottery. If there are 100 tickets sold altogether, Find the probability that Margaret

(i) Wins first prize 1

(ii) Wins both prizes 2

(iii) Wins at least one prize 2

--- **End of Exam** ---

SOLUTIONS Extn 1 Year 11 Sept 2007

Question One

(a) $x^2 - y^2 + x + y$.

$(x-y)(x+y) + 1(x+y)$ 2

$= (x+y)\{x-y+1\}$

(b) $\frac{1}{x-2} > 3$

$\frac{1}{(x-2)} \times (x-2)^2 > 3(x-2)^2$

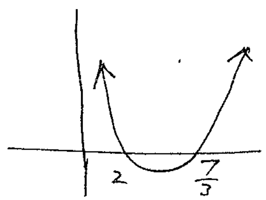
$(x-2) > 3(x-2)^2$

$0 > 3(x-2)^2 - (x-2)$ 3

$0 > (x-2)\{3(x-2)-1\}$

$0 > (x-2)(3x-7)$

$\therefore 2 < x < \frac{7}{3}$



(c) $y = \frac{1}{\sqrt{9-x^2}}$

$D_x : \{-3 < x < 3.\}$. 3

$R_y : y \geq \frac{1}{3}$

(d). $(-2, 5)$ and $(3, -4)$. $3:5$.

$x = \frac{lx_1 + kx_2}{k+l} \Rightarrow \frac{5\times -2 + 3\times 3}{3+5} = \frac{-1}{8}$ 2

$y = \frac{ly_1 + ky_2}{k+l} \Rightarrow \frac{3\times -4 + 5\times 5}{3+5} = \frac{13}{8}$.

$\left(-\frac{1}{8}, \frac{13}{8}\right)$

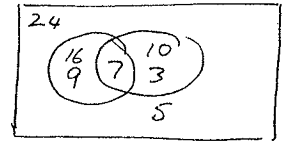
(e) $\lim_{x\to\infty} \frac{5x^2-3}{2x^2+x}$

$= \lim_{x\to\infty} \frac{\frac{5x^2}{x^2} - \frac{3}{x^2}}{\frac{2x^2}{x^2} + \frac{x}{x^2}}$ 2

$= \lim_{x\to\infty} \frac{5 - \frac{3}{x^2}}{2 + \frac{1}{x}}$

$= \frac{5}{2}$

(f)



$\therefore 16 + 10 = 19 + x$

$\therefore x = 7$ 2

P(studies both) $= \frac{7}{24}$

(g) $\sum_{n=1}^{25} 5 - 2n$ $T_1 = 5 - 2 = 3$

$T_{25} = 5 - 2\times 25 = 5 - 50 = -45$.

$\therefore S_n = \frac{n}{2}\{a + l\} = \frac{25}{2}\{3 - 45\}$. 3

$S_{25} = \frac{25}{2}\{-42\} =$

(h) $A(1-2x) + B(1-3x) = 1$.

$A - 2Ax + B - 3Bx = 1$.

$\therefore A + B = 1$ $-2A - 3B = 0$.

$2A + 3B = 0$ 2

$2A + 2B = 2$.

$B = -2$

$\therefore A = 3$

(i) + 3 marks all students

Question 2:

a i) $y = 3x^2 - \frac{1}{x}$

$= 3x^2 - x^{-1}$

$y' = 6x + x^{-2}$

$= 6x + \frac{1}{x^2}$ ②

ii) $y = \sqrt{6x-5}$

$= (6x-5)^{\frac{1}{2}}$

$y' = \frac{1}{2}(6x-5)^{-\frac{1}{2}} . 6$

$= 3(6x-5)^{-\frac{1}{2}}$

$= \frac{3}{\sqrt{6x-5}}$ ②

iii) $y = \frac{3x^4 - 6}{2x+1}$

let $u = 3x^4 - 6$ $v = 2x+1$

$u' = 12x^3$ $v' = 2$

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{12x^3(2x+1) - 2(3x^4-6)}{(2x+1)^2}$

$= \frac{24x^4 + 12x^3 - 6x^4 + 12}{(2x+1)^2}$

$= \frac{18x^4 + 12x^3 + 12}{(2x+1)^2}$ ②

b) $T_3 = a + 2d = 10$ ①

$T_7 = a + 6d = 22$ ②

② − ①:

$4d = 12$

$d = 3$

Sub $d=3$ into ①

$a + 2\times3 = 10$

$a = 4$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{156} = \frac{156}{2}[2\times4 + 155\times3]$

$= 78(8 + 465)$

$= 36894$ ②

ii) when $T_n > 1000$

$4 + (n-1)\times 3 > 1000$

$3n + 1 > 1000$

$3n > 999$

$n > 333$

$\therefore T_{334} > 1000$

$T_n = a + (n-1)d$

$T_{334} = 4 + 333\times3$

$= 1003$ ②

c i) $2x^2 - 6x + 5 = 0$

$\alpha + \beta = -\frac{b}{a}$

$= \frac{6}{2}$

$= 3$ ①

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{5}{2}$ ①

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 3^2 - 2\times\frac{5}{2}$

$= 9 - 5$

$= 4$ ②

iv) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$= \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{4}{(\frac{5}{2})}$

$= \frac{8}{5}$ ②

d) $x^2 + y^2 = 25$ circle centre $(0,0)$

radius $= 5$.

perp. distance from $(0,0)$ to $3x + 4y - 25 = 0$.

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \frac{|(3\times0) + (4\times0) - 25|}{\sqrt{3^2 + 4^2}}$

$= \frac{|-25|}{\sqrt{25}}$

$= \frac{25}{5}$

$= 5$ units

As perp. distance from circle centre to line = radius, line is a tangent to the circle. ②

e) i) $y = -x^3 + 3x^2 + 9x - 15$

$y' = -3x^2 + 6x + 9$

stat. pts occur when $y' = 0$

$-3x^2 + 6x + 9 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$, $x = -1$

when $x = 3$: $y = -(3)^3 + 3(3)^2 + 9(3) - 15$

$= 12$

$\therefore$ stat. pt at $(3, 12)$

when $x=-1$: $y=-(-1)^3+3(-1)^2+9(-1)-15$

$= 1+3-9-15$

$= -20$

$\therefore$ st at. pt at $(-1,-20)$

$y'' = -6x+6$

when $x=3$: $y''=-6\times3+6$

$=-12$

As $y''<0$ $\therefore$ max. at $(3,12)$

when $x=-1$: $y''=-6\times(-1)+6$

$=12$

As $y''>0$ $\therefore$ min. at $(-1,-20)$ (2)

ii) P.O.I. when $y''=0$

$-6x+6=0$

$6x=6$

$x=1$

Test $x=1$:

x	0	1	2
y''	6	0	-6

change of concavity at $x=1$.

when $x=1$: $y=-1^3+3(1)^2+9(1)-15$

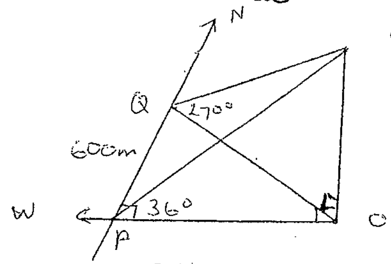
$=-1+3+9-15$

$=-4$

$\therefore$ point of inflexion at $(1,-4)$. (2)

Question Three

a)



i) $\angle OPH = 36°$ ✓

$\angle OQH = 27°$ ✓

(2)

ii) $OP = \dfrac{OH}{\tan 36°}$ or $\therefore OP = OH \tan 54°$ ✓✓

$OQ = \dfrac{OH}{\tan 27°}$ or $\therefore OQ = OH \tan 63°$ ✓ (3)

iii) $OQ^2 - OP^2 = 600^2$ ✓

$\left(\dfrac{OH}{\tan 27°}\right)^2 - \left(\dfrac{OH}{\tan 36°}\right)^2 = 360\,000$

$OH^2 = \sqrt{\dfrac{360\,000}{\dfrac{1}{\tan^2 27°} - \dfrac{1}{\tan^2 36°}}}$ ✓

(3)

$\approx 430\,m$ (2 sig figs) ✓ $\frac{}{8}$

b) $f'(x) = \lim\limits_{h\to0} \dfrac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to0} \dfrac{[2(x+h)^2-3(x+h)]-[2x^2-3x]}{h}$ ✓

$= \lim\limits_{h\to0} \dfrac{[\cancel{2x^2}+4xh+2h^2\cancel{-3x}-3h\cancel{-2x^2}\cancel{+3x}]}{h}$

$= \lim\limits_{h\to0} \dfrac{4xh+2h^2-3h}{h}$ ✓

$= \lim\limits_{h\to0} 4x+2h-3$

$= 4x-3$ (3) ✓ $\frac{}{3}$

7 a) $y = 2x^2 - 5x + 7$

i) $\frac{dy}{dx} = 4x - 5$

at $x = -3$, $\frac{dy}{dx} = -17$, $y = 40$

$\therefore N$ is $y - 40 = \frac{1}{17}(x+3)$

$17y - 680 = x + 3$

$0 = x - 17y + 683$

ii) $3x - 2y + 4 = 0$

$\frac{3}{2}x + 2 = y$, $\therefore m = 3/2$

Let $\frac{3}{2} = 4x - 5$

$13/2 = 4x$

$x = 13/8$

$y = 2\left(\frac{169}{64}\right) - 5\left(\frac{13}{8}\right) + 7$

$= \frac{169 - 260 + 224}{32}$

$= 133/32$

$\therefore$ Point is $\left(\frac{13}{8}, \frac{133}{32}\right)$

b) i) $A = P(1+r)^n$

$= 500(1.01)^{480}$

$\doteqdot \$59{,}324$

$A_{480} = 500(1.01) + \dots 500(1.01)^{480}$

$= \frac{500(1.01)\{1.01^{480} - 1\}}{.01}$

$\doteqdot \$5{,}941{,}210$

iii) $A = P(1.01)^n$

$\frac{5{,}941{,}210}{(1.01)^{480}} = P$

$\therefore P \doteqdot \$50{,}074$

c) $2\left(m + \frac{1}{m}\right)^2 + \left(m + \frac{1}{m}\right) - 15 = 0$

Let $z = m + \frac{1}{m}$

$\therefore 2z^2 + z - 15 = 0$

$\frac{(2z-5)(2z+6)}{2} = 0$

$(2z-5)(z+3) = 0$

$z = 5/2, -3$

$\therefore m + \frac{1}{m} = -3$, $m + \frac{1}{m} = 5/2$

$m^2 - 3m + 1 = 0$, $2m^2 - 5m + 2 = 0$

$m = \frac{3 \pm \sqrt{5}}{2}$, $(2m-1)(m-2) = 0$

$m = 1/2, 2$

d) i) $P(1^{st}) = 2/100 = 1/50$

ii) $P(1^{st} \, 2^{nd}) = \frac{2}{100} \cdot \frac{1}{99} = \frac{1}{4950}$

iii) $P(\text{at least } 1) = 1 - \frac{98}{100} \cdot \frac{97}{99}$

$= \frac{197}{4950}$

c) i) $-1 < \frac{(1-x)^2}{(1-x)} < 1$

$-1 < 1 - x < 1$

ie $-1 < 1 - x$, $1 - x < 1$

$x < 2$, $x > 0$

ie $0 < x < 2$

② 2

ii) $S_\infty = \frac{a}{1-r}$

$0.5 = \frac{1-x}{1-1+x}$

$0.5x = 1 - x$

$1.5x = 1$

$x = \frac{2}{3}$

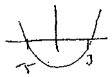
② 2

d) $15 - x^2 - 2x > 0$

$x^2 + 2x - 15 < 0$

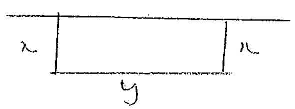
$(x+5)(x-3) < 0$

$-5 < x < 3$



③ 3

e)



i) $y = 100 - 2x$ ①

ii) $A = x(100 - 2x)$

$= 100x - 2x^2$

$\frac{dA}{dx} = 100 - 4x$ $\quad \frac{d^2A}{dx^2} = -4$ (hence a max)

for a max min $\frac{dA}{dx} = 0$

$100 - 4x = 0$

$x = 25$ ③

ie Dimensions 25m by 50m. 4

or calculate Area

Total 22