SYDNEY GIRLS HIGH SCHOOL

LABOR OMNIA VINCIT

YEAR 11 MATHEMATICS

YEARLY EXAMINATION

SEPTEMBER 2003

Time allowed: 90 minutes

Topics: Chapters 1-10 (J&C)

Instructions:

- There are Four (4) questions. Questions are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work.
- Start each question on a new page. Write on one side of the paper only.

Name: ....

Ye 11 2003 Mathematics

QUESTION ONE (20 marks)

(a) Find correct to 2 decimal places

$$\frac{4.6 - 5.9}{4.6 \times 2.3}$$

(b) Evaluate 42000×9600 giving your answer in scientific notation

(c) Factorise (i) $4 - 4p^2$

(ii) $2x^2 - 7x - 15$

(d) Solve $3 - (4 - x) = 5x$

(e) Express as a fraction in simplest form $0.0\dot{3}$

(f) Expand and Simplify $(2\sqrt{3} + \sqrt{2})(3\sqrt{3} - \sqrt{2})$

(g) Given $S_n = \frac{n}{2}[2a + (n-1)d]$ find S_n when $n = 103$, $a = 5$ and $d = 1 \cdot 2$

(h) Simplify $\frac{x - 4}{x^2 - 16}$

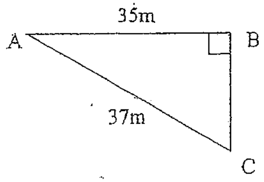
(i) Solve for x $4^x = 8$

(j) At a shoe sale, all shoes are to be sold at a discount of 15% off the marked price. What is the cost of a pair of shoes with marked price of $79.95?

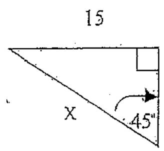
Alinta.

QUESTION TWO (20 marks)

(a) Find BC



(b) Evaluate x



(c) Express without negative indices in simplest form

$$\frac{(4x)^{-1}}{2x^3}$$

(d) Solve for x

(i) $\frac{x}{2} - \frac{1-3x}{4} = x$

(ii) $(t+4)^2 = 36$

(iii) $4(2x-7) \le 12$

(iv) $|3y+5| = 4 - y$

(v) $|7x-3| \ge 4$

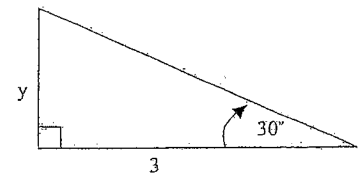
(e) Solve simultaneously

$$4y - 3x = 8$$
$$5y + 3x = 1$$

(f) Find the values of a and b if $\frac{\sqrt{2}}{3\sqrt{2}-1} = a + b\sqrt{2}$

QUESTION THREE (20 marks)

(a) Find the exact value of y



(b) Given $\tan\theta = \frac{2}{3}$ find (i) $\cos\theta$ and

(ii) $\operatorname{cosec}\theta$

(c) Simplify $\sin\theta\cot\theta$

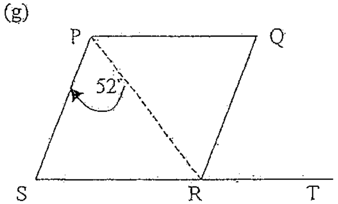
(d) If $h(t) = \begin{cases} 2 - t^3 & \text{if } t > 0 \\ t^2 + 1 & \text{if } t \le 0 \end{cases}$

find the value $h(3) + h(-4) - h(0)$

(e) State the domain and range of $y = x^2 - 12$

(f) (i) Find the sum of the interior angles of a regular hexagon.
(ii) What is the size of each exterior angle?

(g)



In the diagram PQRS is a rhombus where $\angle SPR = 52°$ and SR is produced to T.

(i) What is the value of $\angle SPQ$?
(ii) What is the value of $\angle QRT$? Give reasons.

QUESTION FOUR (20 marks)

(a) Show that $f(x) = x^2 - 1$ is an even function

(b) Find x if $\sin x^\circ = \cos(2x - 30)^\circ$

(c) Solve $2\sin x = -1$ for $0^\circ \leq x \leq 360^\circ$

(d) Draw neat, separate sketches of the following, showing all relevant features

(i) $x^2 + y^2 = 9$

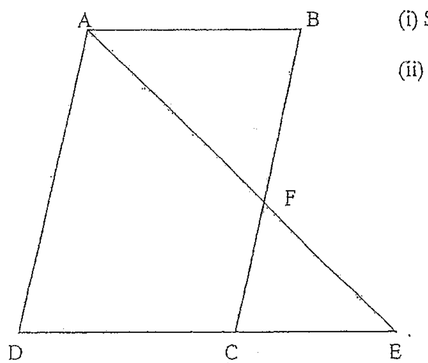
(ii) $y = -\sqrt{16 - x^2}$

(iii) $y = 2^x$

(iv) $y = |x| - 2$

(v) $y = \dfrac{1}{x - 4}$

(e) ABCD is a parallelogram, AD = 12cm, CE = 4cm, BF = 7cm

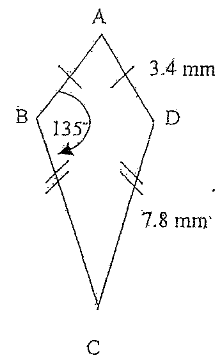


(i) Show ΔABF is similar to ΔECF

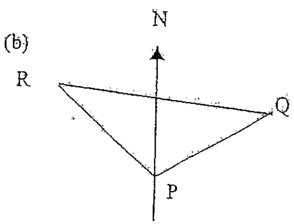
(ii) Find AB

QUESTION FIVE (20 marks)

(a) Find the area of the kite ABCD correct to 3 significant figures.



(b)



From P, a tree Q bears 057° and a tower R bears 335°

(i) Copy the diagram on your exam paper and label all information given.

(ii) Show that angle RPQ = 82°.

(iii) If the distance of the tree from P is 600m and that of the tower is 940m. Find how far the tree is from the tower.

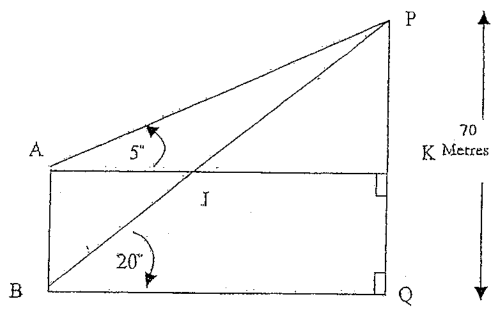
(c) Prove $\cot\theta + \tan\theta = \operatorname{cosec}\theta.\sec\theta$

(d) Graph the region

$x^2 + y^2 \leq 25$, for $x \leq 3$ and $y > -4$,

showing intersecting points.

(e) Two towers AB and PQ stand on level ground. The angles of elevation of the top of the taller tower from the top and bottom of the shorter tower are 5° and 20° respectively. The height of the taller tower is 70 metres.



(i) Explain why $A\hat{P}J = 15^\circ$

(ii) Show that $AB = \dfrac{BP\sin 15^\circ}{\sin 95^\circ}$

(iii) Show that $BP = \dfrac{70}{\sin 20^\circ}$

(iv) Hence, find the height of the shorter tower, correct to the nearest metre.

END OF PAPER

YEAR 11 - 2003

MATHEMATICS

SOLUTIONS

Question 1 (20 marks)

$\frac{4.6 - 5.9}{4.6 \times 2.3} = \frac{-1.3}{10.58}$

$= -0.12$

$42000 \times 9600 = 4.032 \times 10^8$

(i) $4 - 4p^2 = 4(1 - p^2)$

$= 4(1-p)(1+p)$

(ii) $2x^2 - 7x - 15 = (2x+3)(x-5)$

Solve

$3 - (4 - x) = 5x$

$3 - 4 + x = 5x$

$-1 = 4x$

$\therefore x = -1/4$

let $x = 0.033....$

$10x = 0.333....$

$100x = 3.333...$

$100x - 10x = 3$

$90x = 3$

$x = 3/90$

$x = 1/30$

$(2\sqrt{3} + \sqrt{2})(3\sqrt{3} - \sqrt{2})$

$18 - 2\sqrt{6} + 3\sqrt{6} - 2$

$16 + \sqrt{6}$

(g) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{103}{2}[2(5) + (103-1)1.2]$

$= 51\frac{1}{2}[10 + 122.4]$

$= 6818.6$

(h) Simplify $\frac{x-4}{x^2-16}$

$= \frac{x-4}{(x-4)(x+4)}$

$= \frac{1}{x+4}$

(i) Solve for x

$4^x = 8$

$2^{2x} = 2^3$

$2x = 3$

$x = 1\frac{1}{2}$

(j) Cost $= 0.85 \times 79.95$

$= \$67.96$

Question two. (20 marks)

(a) $BC^2 = 37^2 - 35^2$

$BC^2 = 144$

$\therefore BC = \sqrt{144}$

$= 12$ m

(b) $\sin\theta = \frac{O}{H}$

$\sin 45^\circ = \frac{15}{x}$

$x = \frac{15}{\sin 45^\circ}$

$x = 15 \div \frac{1}{\sqrt{2}}$

$x = 15 \times \sqrt{2}$

$x = 15\sqrt{2}$ OR

$x \doteq 21.2$

(c) $\frac{(4x)^{-1}}{2x^3} = \frac{1}{4x \cdot 2x^3}$

$= \frac{1}{8x^4}$

(d) Solve for x

(i) $\frac{x}{2} - \frac{1-3x}{4} = x$

$\frac{4x - 2(1-3x)}{8} = x$

$\frac{4x - 2 + 6x}{8} = x$

$\frac{10x - 2}{8} = x$

$10x - 2 = 8x$

$2x \quad 2$

$x = 1$

(ii) $(t+4)^2 = 36$

$t + 4 = 6$ or $t + 4 = -6$

$t = 2$ $\quad t = -10$

(iii) $4(2x - 7) \leq 12$

$2x - 7 \leq 3$

$2x \leq 10$

$x \leq 5$

(iv) $|3y + 5| = 4 - y$

$3y + 5 = 4 - y$ or $3y + 5 = -(4 - y)$

$4y = -1$ $\quad 3y + 5 = -4 + y$

$y = \frac{-1}{4}$ $\quad 2y = -9$

$y = -4\frac{1}{2}$

(v) $|7x - 3| \geq 4$

$7x - 3 \geq 4$ or $7x - 3 \leq -4$

$7x \geq 7$ $\quad 7x \leq -1$

$x \geq 1$ $\quad x \leq -\frac{1}{7}$

(e) $4y - 3x = 8$ ... ①

$5y + 3x = 1$ ... ②

Add ① + ②

$9y = 9$ $\quad 4 - 3x = 8$

$y = 1$ $\quad -3x = 4$

$x = -\frac{4}{3}$

(f) $\frac{\sqrt{2}}{3\sqrt{2} - 1} \times \frac{3\sqrt{2} + 1}{3\sqrt{2} + 1} = \frac{6 + \sqrt{2}}{18 - 1}$

$= \frac{6 + \sqrt{2}}{17}$

$\therefore a = 6/17$ and $b = 1/17$

Question three (20 marks)

$\tan 30^\circ = \frac{y}{3}$

$3 \times \tan 30^\circ = y$

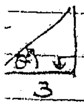
$\therefore y = 3 \times \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}}$

$y = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3}$

$= \sqrt{3}$

$\therefore y = \frac{3}{\sqrt{3}}$ or $y = \sqrt{3}$

$\tan\theta = \frac{2}{3}$



$x^2 = 3^2 + 2^2$

$x^2 = 9 + 4$

$x = \sqrt{13}$

i) $\cos\theta = \frac{3}{\sqrt{13}}$

) $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

$= \frac{\sqrt{13}}{2}$

Simplify

$\sin\theta \cdot \cot\theta = \sin\theta \cdot \frac{\cos\theta}{\sin\theta}$

$= \cos\theta$

$h(3) = 2 - t^3 = 2 - (3)^3$

$= -25$

$h(-4) = t^2 + 1 = (-4)^2 + 1$

$= 17$

$h(0) = t^2 + 1 = 0 + 1$

$= 1$

$\therefore h(3) + h(-4) - h(0) =$

$-25 + 17 - 1 = -9$

(e) $y = x^2 - 12$

Domain: all real x

Range: $y \geq -12$

(f) (i) Angle sum $= (n-2)\,180^\circ$

hexagon $= (6-2)\,180^\circ$

$= 720^\circ$

(ii) Exterior angle $= 180 - \frac{720}{6}$

$= 180^\circ - 120^\circ$

$= 60^\circ$

OR

Exterior angle $= \frac{360}{6}$

$= 60^\circ$

(g) (i) $\angle SPQ = 2 \times 52^\circ$

$= 104^\circ$

(ii) $\angle SRQ = 104^\circ$

opposite angle of rhombus

$\angle QRT = 180 - 104$

$= 76^\circ$

(straight angle).

Question Four (20 marks)

(a) $f(x) = f(-x)$

$f(-x) = x^2 - 1$

$\therefore f(x) = f(-x) = x^2 - 1$

is an even function

(b) $x + 2x - 30 = 90$

$3x - 30 = 90$

$3x = 60$

$x = 20$

(c) $2\sin x = -1$ S | A, T | C

$\sin x = -\frac{1}{2}$

$x = 30^\circ$

$\therefore x_1 = 180 + 30$

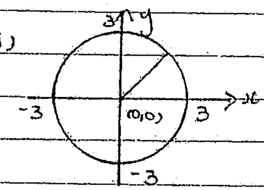
$= 210^\circ$

$x_2 = 360 - 30$

$= 330^\circ$

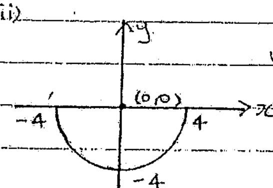
(d)

(i)



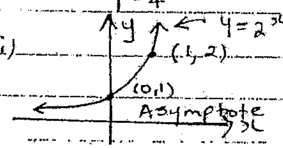
$x^2 + y^2 = 9$

(ii)

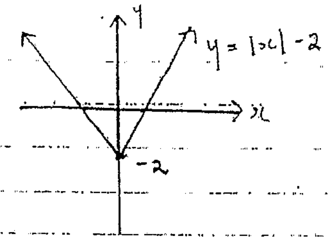


$y = -\sqrt{16 - x^2}$

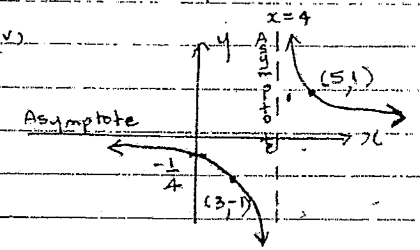
(iii)



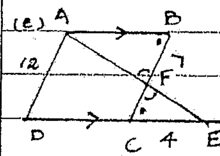
(iv)



(v)



(e)



(i) DATA: ABCD is a parallelogram

AIM: Prove $\Delta ABF ||| \Delta ECF$

Proof: In ΔABF and ΔECF

$A\hat{B}C = B\hat{C}E$ (alternate angles parallel lines - opposite sides of parallelogram)

$A\hat{F}B = E\hat{F}C$ (vertically opposite)

$\therefore B\hat{A}E = C\hat{E}F$ (by subtraction and angle sum of Δ)

$\therefore \Delta ABF ||| \Delta ECF$ (equiangular)

(ii) Since Δ's are similar sides are proportional hence

$\frac{CF}{BF} = \frac{CE}{BA} = \frac{EF}{AF}$

$CF = 12 - 7$ $BF = 7\text{cm}$ $CE = 4$

$= 5\text{cm}$ $AB = ?$

$\frac{5}{7} = \frac{4}{AB}$

$AB = 4 \times 7 - 5\frac{3}{5}$ or 5.6cm

Question 5 (20 marks)

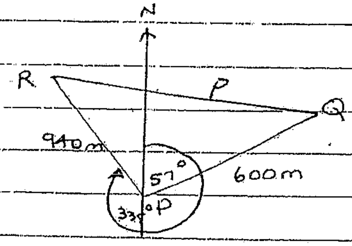
(a) Area $= 2 \times \frac{1}{2} ac \sin D$

$= 2 \times \frac{1}{2} \times 7.8 \times 3.4 \times \sin 135^\circ$

$= 18.7525$

$= 18.8 \text{ mm}^2$ (3 sig. fig.)

(b)(i)



(i) $R\hat{P}Q = 57^\circ + (360^\circ - 335^\circ)$

$= 57^\circ + 25^\circ$

$= 82^\circ$

(ii) $p^2 = q^2 + r^2 - 2qr\cos P$

$= (940)^2 + (600)^2 - 2(940)(600)\cos 82^\circ$

$p^2 = 1086612.74$

$p = \sqrt{1086612.74}$

$p \doteqdot 1042.4 \text{ m}$

(c) Prove

$\cot\theta + \tan\theta = \text{cosec}\,\theta.\sec\theta$

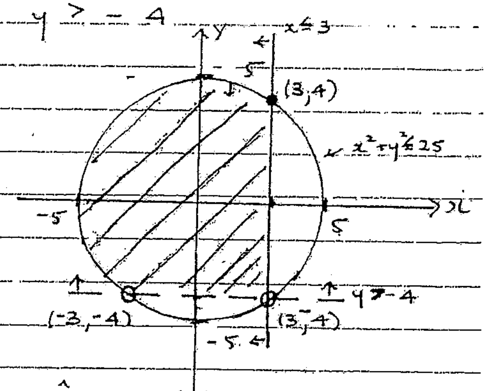
L.H.S $\cot\theta + \tan\theta$

$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$

$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta.\cos\theta}$

$= \frac{1}{\sin\theta.\cos\theta} = \text{cosec}\,\theta.\sec\theta$

(d) $x^2 + y^2 \leq 25$, $x \leq 3$ and $y \geq -4$



(e) (i) $A\hat{B}Q = 90^\circ$ (angle with ground)

$A\hat{B}P = 90^\circ - 20^\circ$

$= 70^\circ$

$B\hat{A}P = 90^\circ + 5^\circ$

$= 95^\circ$

$\therefore A\hat{P}J = 180^\circ - (95 + 70^\circ)$

$= 180^\circ - 165^\circ$

$= 15^\circ$ (angle sum of ΔAPB)

(ii) $\frac{p}{\sin P} = \frac{a}{\sin A}$

$\frac{AB}{\sin 15^\circ} = \frac{BP}{\sin 95^\circ}$

$AB = \frac{BP \times \sin 15^\circ}{\sin 95^\circ}$

(iii) In ΔBPQ

$\sin\theta = \frac{O}{H}$

$\sin 20^\circ = \frac{PQ}{BP}$

$BP = \frac{70}{\sin 20^\circ}$

(iv) $AB = \frac{BP \times \sin 15^\circ}{\sin 95^\circ}$