SYDNEY GIRLS' HIGH SCHOOL

YEAR 11 - SEPTEMBER, 1997

MATHEMATICS

3 UNIT (ADDITIONAL)

Time allowed: 75 minutes

DIRECTIONS TO CANDIDATES

- There are five (5) questions.
- Attempt ALL questions.
- All questions are of equal value.
- Start each question on a new page. Write on one side of the paper only.
- Show all necessary working. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.

i) $|2x-1|=4-3x$

ii) $-3\le 1-x<4$

iii) $\dfrac{1}{x-1}\le 2 \quad (x\ne 1)$

b) Evaluate

i) $\lim\limits_{x\to 2}\dfrac{x^2+2x-8}{x^2-4}$

ii) $\lim\limits_{x\to\infty}\dfrac{3x^2}{4x^2+2x-1}$

Question 2 **(Start a new page)**

a) Differentiate

i) $3x^5-4x+\dfrac{2}{x^2}+1$

$2x^{-2}$

ii) $(x+1)(2x-3)^4$

$4(2x-3)^3$

iii) $\dfrac{x}{1-x}$

b) Consider the function whose derivative is given by $\dfrac{dy}{dx}=x^2(2x-1)(x+1)$

Determine the nature of the stationary point at $x=0$

c) Find the equation of the line which passes through the point of intersection of $2x+3y+2=0$ and $3x+2y-2=0$ and the point (1,4).

Question 3 **(Start a new page)**

a) A geometric series has the second term 6 and the ratio of the seventh term to the sixth term is 3.

i) Find the common ratio

ii) Find the first term

iii) Calculate the sum of the first 12 terms.

b) The sum of the first 3 terms of an arithmetic series is 27, and the sum of the next three terms is 63. Find the

i) first term

ii) the common difference.

c) Find the equation of the normal to the curve $y=x^2+\dfrac{5}{x}-2$ at the point where $x=1$

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the y-axis at Q.

i) Find the coordinates of P and Q.

ii) In what ratio does L divide the interval PQ?

b) Given $y = 1 + 3x - x^3$

i) Find the stationary points and determine their nature.

ii) Find the point(s) of inflexion.

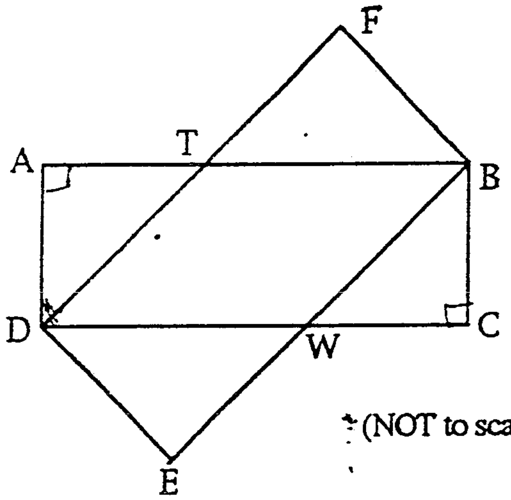
iii) Sketch the curve for $-2 \leq x \leq 3$.

iv) What is the minimum value of y for $-2 \leq x \leq 3$

Question 5 **(Start a new page)**

a) Solve $2\cos^2\theta + \sin\theta - 1 = 0$ for $0^0 \leq \theta \leq 360^0$

b)



(NOT to scale)

$y = -x^3 + 3x + 1$

$x^3 - 3x - 1 = 0$

ABCD and DEBF are two congruent rectangles with sides 3 and 7 units, as in the diagram (ie. AB=DF=7 and AD=DE=3).

i) Show that $AT = \dfrac{20}{7}$

ii) Find the area of figure DWBT.

c) Prove by Mathematical Induction that, for $n \geq 1$,

$$1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$$

END OF PAPER

YR 11 SEPTEMBER ...

Question 1.

a) i) $2x-1=4-3x$ or $2x-1=-4+3x$

$5x=5$ $\quad x=3$

$x=1$

test $x=1$

LHS $=|2-1|=1$

RHS $=4-3=1$ $\quad \therefore x=1$

test $x=3$

LHS $=|6-1|=5$

RHS $=4-9=-5$ not a soln.

$\therefore x=1$ (4)

ii) $-3\le 1-x \cap 1-x<4$

$x\le 4 \cap x>-3$

$\therefore -3<x\le 4$ (3)

iii) $\frac{1}{x-1}(x-1)^2 \le 2(x-1)^2$

$(x-1)-2(x-1)^2\le 0$

$(x-1)[1-2(x-1)]\le 0$

$(x-1)(3-2x)\le 0$

$x<1$ or $x\ge 1\frac{1}{2}$

(4)

b) i) $\lim_{x\to 2}\frac{(x+4)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}=\frac{6}{4}$

$=\frac{3}{2}$ (2)

ii) $\lim_{x\to\infty}\frac{\frac{3x^2}{x^2}}{\frac{4x^2}{x^2}+\frac{2x}{x^2}-\frac{1}{x^2}}=\frac{3}{4}$ (2)

(since $\lim_{x\to\infty}\frac{2}{x}=0$ etc)

Question 2.

a) i) $\frac{d}{dx}=15x^4-4-4x^{-1}$

$=15x^4-4-\frac{4}{x^3}$ (2)

ii) $(x+1)4(2x-3)^3.2+(2x-3)^4.1$

$=(2x-3)^3[8(x+1)+2x-3]$

$=(2x-3)^3(10x+5)$ (3)

iii) $\frac{(1-x).1-x(-1)}{(1-x)^2}=\frac{1}{(1-x)^2}$ (3)

b) $\frac{dy}{dx}=x^2(2x-1)(x+1)$

$x=0^-$ $\quad \frac{dy}{dx}=+(-)(+)<0$

$x=0^+$ $\quad \frac{dy}{dx}=+(-)(+)<0$

$\therefore$ horizontal pt. of inflexion at $x=0$ (3)

c) $l_1+kl_2=0$

$(2x+3y+2)+k(3x+2y-2)=0$

subst $(1,4)$

$(2+12+2)+k(3+8-2)=0$

$16+9k=0$

$k=\frac{-16}{9}$

$9(2x+3y+2)-16(3x+2y-2)=0$

$18x+27y+18-48x-32y+32=0$

$30x+5y-50=0$

$6x+y-10=0$

or $2x+3y+2=0$ (1)

$3x+2y-2=0$ (2)

$\Rightarrow 4x+6y+4=0$ (1a)

$9x+6y-6=0$ (2a)

$5x-10=0$

$x=2$ $\quad \therefore y=-2$ (4)

$m=-6 \Rightarrow y-4=-6(x-1)$ $\quad (2,-2)$

$y=-6x+10$ $\quad (1,4)$

Question 3

a) $\sqrt{5}$ $ar=6$ $\quad \frac{T_7}{T_6}=3$ $\quad \therefore r=3$

i) $r=3$ (2)

ii) $a=2$ (2)

iii) $S_n=\frac{a(r^n-1)}{r-1}$

$S_{12}=\frac{2(3^{12}-1)}{2}=531440$ (2)

b) $S_3=27$

$S_6=27+63=90$

$S_n=\frac{n}{2}(2a+(n-1)d)$

$S_3=\frac{3}{2}(2a+2d)=27$ (1)

$S_6=\frac{6}{2}(2a+5d)=90$ (2)

$3(a+d)=27$ $\quad a+d=9$ (1a)

$2a+5d=30$ (2a)

$2(9-d)+5d=30$

$18-2d+5d=30$

$3d=12$

$d=4$

$a=5$ (4)

c) $y=x^2+5x^{-1}-2$

$y'=2x-5$ $\quad$ at $x=1$ $(y=4)$

$y'=-3$ $\quad \therefore m=\frac{1}{3}$ (normal)

$(1,4)$

$y-4=\frac{1}{3}(x-1)$

$3y-12=x-1$

$x-3y+11=0$ (5)

Question 4

a) $y = 12x^{-1}$

$\frac{dy}{dx} = -12x^{-2}$

$= \frac{-12}{x^2}$

at $x = 3$ $\quad m = \frac{-12}{9} = \frac{-4}{3}$

$y - 4 = \frac{-4}{3}(x - 3)$

$y = -\frac{4x}{3} + 8$

$x = 0, \quad y = 8 \quad Q(0, 8)$

$y = 0, \quad x = 6 \quad P(6, 0)$

$l : m \quad PQ \quad \overset{x_1\ y_1}{(6, 0)} \quad \overset{x_2\ y_2}{(0, 8)}$ (4)

$\frac{lx_2 + mx_1}{l + m} = 3$

$\frac{0 + 6m}{l + m} = 3 \Rightarrow 6m = 3l + 3m$

$3m = 3l$

$\therefore \frac{m}{l} = 1$

L divides PQ in ratio 1:1

(2)

b) $y = 1 + 3x - x^3$

$\frac{dy}{dx} = 3 - 3x^2$

$\frac{d^2y}{dx^2} = -6x$ (4)

Stat. Pts $\frac{dy}{dx} = 0 \quad 3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$x = -1, \ \frac{d^2y}{dx^2} > 0 \ \therefore$ min st. pt $(-1, -3)$

$x = 1 \ \frac{d^2y}{dx^2} < 0 \ \therefore$ max st. pt $(1, 3)$.

Pts of inflexion:

$\frac{d^2y}{dx^2} = 0$

$x = 0$

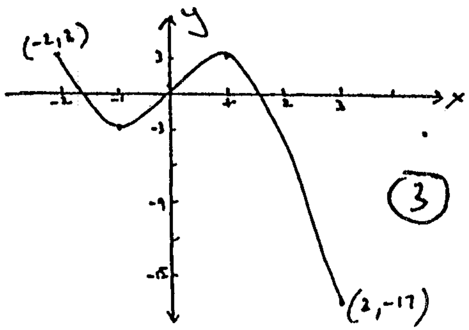
x	0^-	0	0^+
$\frac{d^2y}{dx^2}$	+	0	−

change in concavity

$\therefore$ pt. of inflexion at $(0, 1)$ (1)

$x = -2: \ y = 1 - 6 + 8$
$= 3$

$x = 3 \quad y = 1 + 9 - 27$
$= -17$ (1)



(3)

$\therefore$ minimum value is -1

Question 5.

a) $2(1 - \sin^2\theta) + \sin\theta - 1 = 0$

$2\sin^2\theta - \sin\theta - 1 = 0$

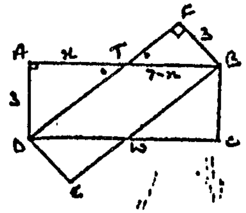
$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$2\sin\theta = -1 \qquad \sin\theta = 1$

$\sin\theta = -\frac{1}{2} \qquad \theta = 90$

$\theta = 210°, 330°, 90°$ (5)

b)



Let $AT = x \quad \therefore TB = 7 - x$

$\Delta ADT \equiv \Delta FBT$ (AAS)

$\therefore TB = DT = 7 - x$

By Pyth. Thm in ΔADT

$(7 - x)^2 = 3^2 + x^2$

$49 - 14x + x^2 = 9 + x^2$

$\therefore 14x = 40$

$x = \frac{40}{14} = \frac{20}{7}$

Area of parallelogram (5)

$= DW \cdot AD$

$= (7 - \frac{20}{7}) \cdot 3 = \frac{29}{7} \cdot 3 = 12\frac{3}{7} \ u^2$

c) step 1: verify for n=1

$1^2 = \frac{1}{3} \cdot 1(1)(3)$

step 2: assume true for n=k

$1 + 3^2 + \dots + (2k-1)^2 = \frac{1}{3}k(2k-1)(2k+1)$

Prove true for n=k+1

$1 + 3^2 + \dots + (2k-1)^2 + (2k+1)^2 = \frac{1}{3}(k+1)(2k+1)(2k+3)$

$LHS = \frac{1}{3}k(2k-1)(2k+1) + (2k+1)^2$

$= \frac{1}{3}(2k+1)[k(2k-1) + 3(2k+1)]$

$= \frac{1}{3}(2k+1)(2k^2 - k + 6k + 3)$

$= \frac{1}{3}(2k+1)(2k^2 + 5k + 3)$ ...... (5)